Tactical Ambulance Location and Relocation

Brigitte Werners
Dirk Degel, Lara Wiesche, Sebastian Rachuba

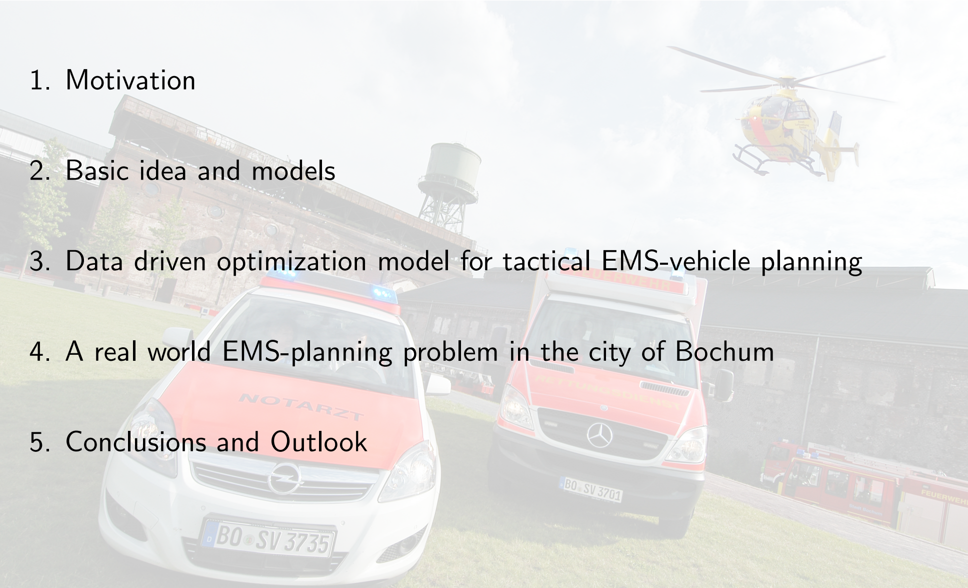
Management, Operations Research and Accounting – Ruhr-University Bochum, Germany

CHOIR in Practice, University of Twente, the Netherlands

November 22, 2013

OPERATIONS RESEARCH PROF. DR. BRIGITTE WERNERS | Stadt Bochum Feuerwehr und Rettungsdienst | ifu Institut für Unternehmensführung | Stiftung Zukunft NRW | RUHR UNIVERSITÄT BOCHUM RUB

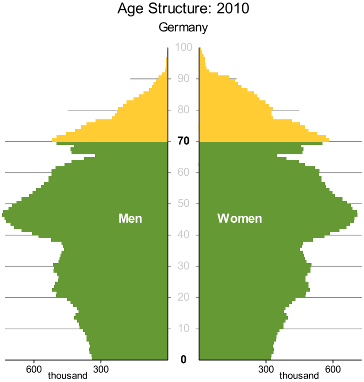
Outline

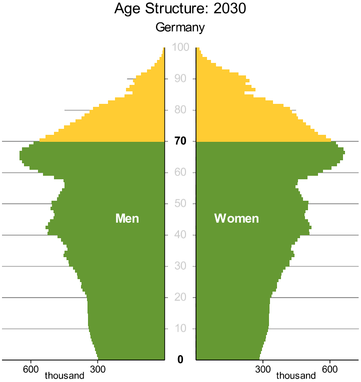


1. Motivation
2. Basic idea and models
3. Data driven optimization model for tactical EMS-vehicle planning
4. A real world EMS-planning problem in the city of Bochum
5. Conclusions and Outlook

Motivation

- Aging population



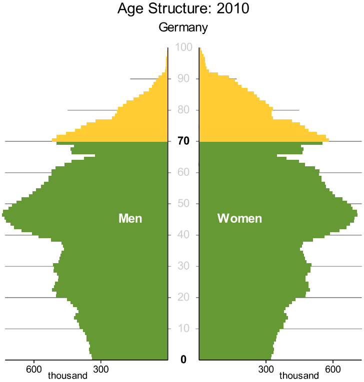


- Age class of $[70, \cdot\,]$ causes 50 % of EMS operations
- EMS demand increases

source: Statistisches Bundesamt 2013

Motivation

- Aging population



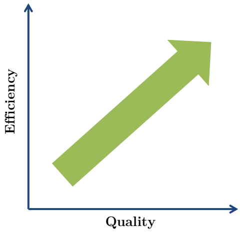
- Age class of $[70, \cdot\,]$ causes 50 % of EMS operations
- EMS demand increases

source: Statistisches Bundesamt 2013

Motivation

1. Access to emergency medical services (EMS) is crucial
2. Ensuring optimal supply quality
   - Short arrival time
   - Coverage of entire demand area
   - High degree of achievement

   $\Longrightarrow$ Dependent on location and number of ambulances
3. Efficiency
   - Reduction of fixed costs (avoid overcapacity)
   - Utilization of ambulances



Quality criteria and objectives

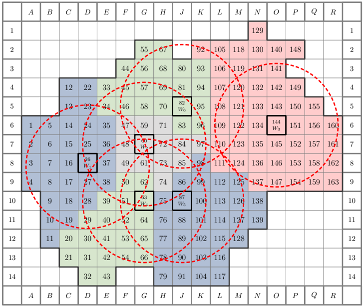
Evaluation of emergency medical services (objective of EMS provider):

- Degree of achievement (ex post):

$$\frac{\#\text{ operations within a time standard } T}{\#\text{ total number of operations}}$$

Objectives in literature (ex ante):

- Coverage: Single coverage, double coverage, busy fraction models, queuing models, hypercube models



Optimal quality in emergency medical services

Research project: 2 years (work in progress)

Stiftung Zukunft NRW

Stadt Bochum
Feuerwehr und Rettungsdienst

Focus:

- Analysis of required resources for EMS
- Analysis of time-dependent demand and speed fluctuations for EMS
- Tactical and strategic planning horizon
- Stochastic influences, uncertain parameters (demand, speed)

Goal:

- Dynamic (and robust) optimization model

$\Longrightarrow$ IT-based decisions support tool for local EMS providers

Decision support tool

Decision support

Optimization	Simulation	Forecast
location, allocation and resource planning	degree of achievement, resources, utilization	trends, development
empirical demand, required/adequate coverage	evaluations	population, experts

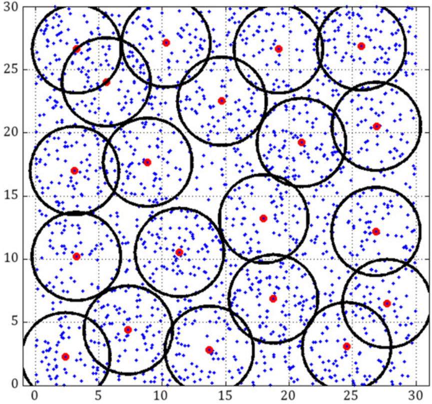
Decision support tool

Decision support

Optimization	Simulation	Forecast
location, allocation and resource planning	degree of achievement, resources, utilization	trends, development
empirical demand, required/adequate coverage	evaluations	population, experts

Basic covering location models

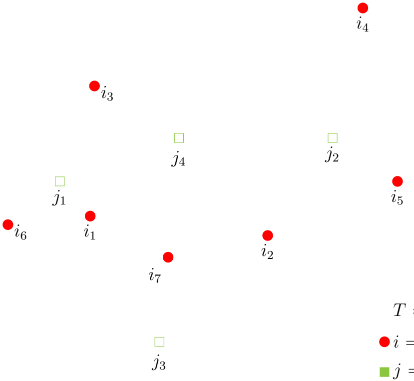
Idea: Demand nodes i have to be covered within a time standard T



source: Zarandi et al. 2011–The large scale maximal covering location problem; page 1565

Set Covering Problem (SCP)

Toregas et al. 1971



T = response time

i = demand node $i \in I$

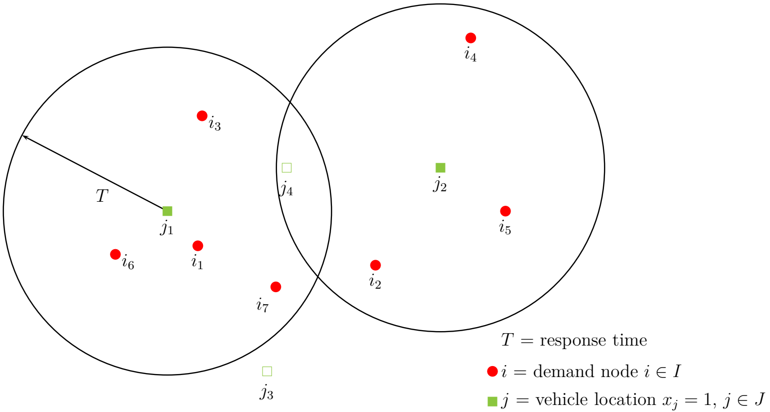
j = vehicle location $x_j = 1,\ j \in J$

j = vehicle location $x_j = 0,\ j \in J$

$$\mathcal{N}_i := \{j \in J \mid t_{ij} \leq T\}$$

Set Covering Problem (SCP)

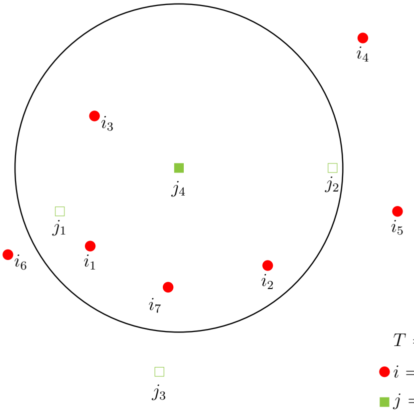
Toregas et al. 1971



$$\mathcal{N}_i := \{j \in J \mid t_{ij} \leq T\}$$

Maximal Covering Location Problem (MCLP)

Church, ReVelle 1974



T = response time

i = demand node $i \in I$

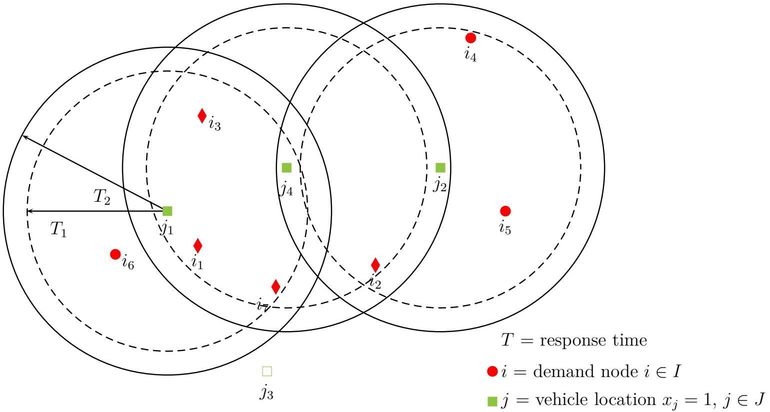
j = vehicle location $x_j = 1$, $j \in J$

j = vehicle location $x_j = 0$, $j \in J$

$$\mathcal{N}_i := \{j \in J \mid t_{ij} \leq T\}$$

Double Standard Model (DSM)

Gendreau et al. 1997



$$\mathcal{N}_i^{T_\ell} := \{j \in J \mid t_{ij} \leq T_\ell,\, \ell \in \{1, 2\}\} \qquad p = 3$$

Double Standard Model (DSM)

Gendreau et al. 1997

$$
\begin{aligned}
\max \quad & \sum_{i \in I} d_i x_i^2 \\
\text{s. t.} \quad & \sum_{j \in \mathcal{N}_i^{T_2}} y_j \geq 1 && \forall i \in I \\
& \sum_{i \in I} d_i x_i^1 \geq \alpha \sum_{i \in I} d_i \\
& x_i^1 \geq x_i^2 && \forall i \in I \\
& \sum_{j \in \mathcal{N}_i^{T_1}} y_j \geq x_i^1 + x_i^2 && \forall i \in I \\
& \sum_{j \in J} y_j = p \\
& x_i^1, x_i^2 \in \{0,1\} && \forall i \in I \\
& y_j \in \mathbb{N}_0 && \forall j \in J
\end{aligned}
$$

- d_i: demand at node i
- $\mathcal{N}_i^{T_\ell} := \{j \in J \mid t_{ij} \leq T_\ell\}$

  $T_1 < T_2$
- p: number of ambulances

$$
x_i^k = \begin{cases} 1, & \text{if demand node } i \text{ is covered } k \in \{1,2\} \text{ times} \\ 0, & \text{else.} \end{cases}
$$

y_j: number of ambulances at node j

Double Standard Model (DSM)

Gendreau et al. 1997 — Limitations: (1) static consideration (2) fixed double coverage

$$\max \sum_{i \in I} \boldsymbol{d}_i \boldsymbol{x}_i^2$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{N}_i^{T_2}} y_j \geq 1 \qquad \forall i \in I$$

$$\sum_{i \in I} \boldsymbol{d}_i x_i^1 \geq \alpha \sum_{i \in I} \boldsymbol{d}_i$$

$$x_i^1 \geq \boldsymbol{x}_i^2 \qquad \forall i \in I$$

$$\sum_{j \in \mathcal{N}_i^{T_1}} y_j \geq x_i^1 + \boldsymbol{x}_i^2 \qquad \forall i \in I$$

$$\sum_{j \in J} y_j = \boldsymbol{p}$$

$$x_i^1, \boldsymbol{x}_i^2 \in \{0,1\} \qquad \forall i \in I$$

$$y_j \in \mathbb{N}_0 \qquad \forall j \in J$$

- d_i: demand at node i
- $\mathcal{N}_i^{T_\ell} := \{j \in J \mid \boldsymbol{t}_{ij} \leq T_\ell\}$

  $T_1 < T_2$
- p: number of ambulances

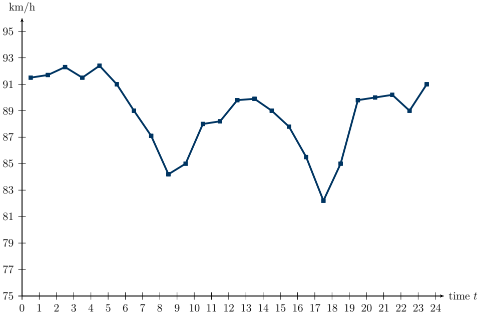
  (fleet size)

$$x_i^k = \begin{cases} 1, & \text{if demand node } i \text{ is covered } k \in \{1, \mathbf{2}\} \text{ times} \\ 0, & \text{else.} \end{cases}$$

y_j: number of ambulances at node j

Extensions of the Double Standard Model

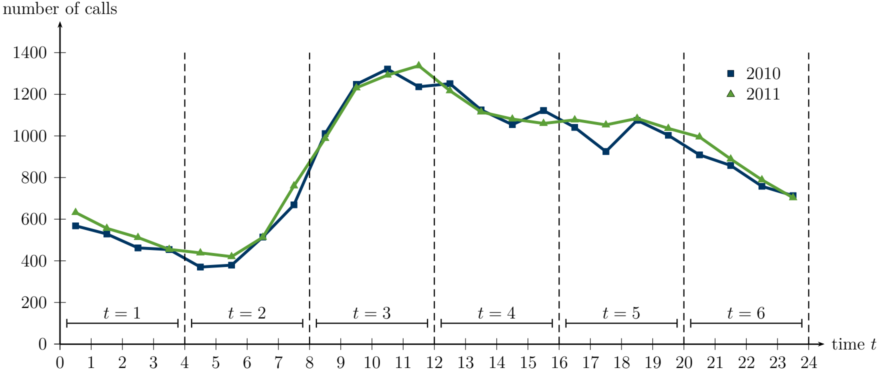
Time-dependent parameter: speed



- Time-dependent speed on (city-)motorways (example of the city of Vienna)

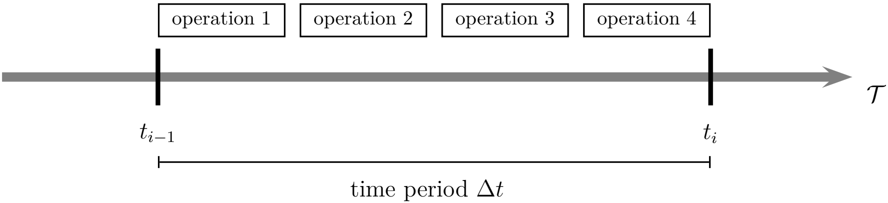
  Source: Kritzinger et al. (2011), S. 71

Time-dependent parameter: demand



$\Longrightarrow$ Dynamic considerations required

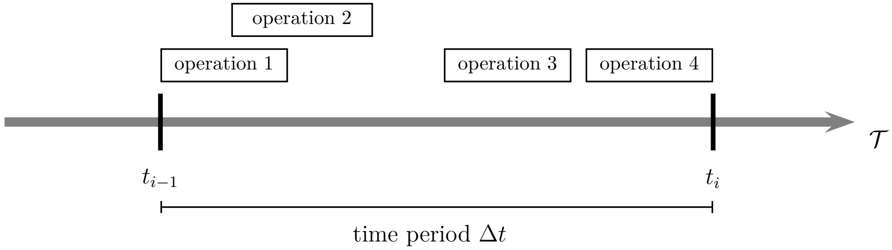
Time-dependent parameter: degree of required coverage



ξ : number of ambulances, X : number of parallel operations

$$P(\{X \leq \xi\}) \geq \beta \quad (= 0.95)$$

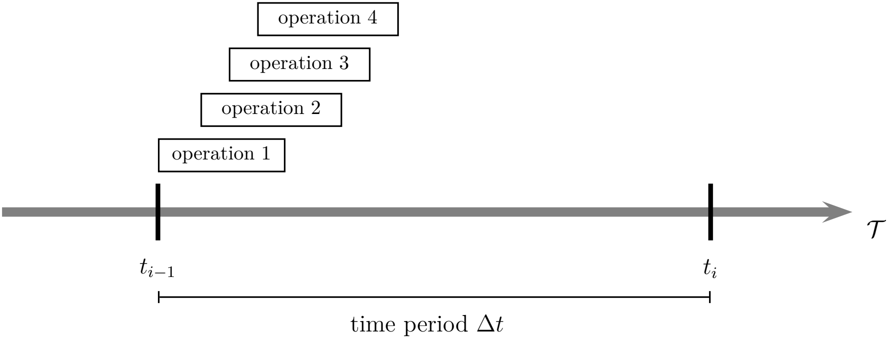
Time-dependent parameter: degree of required coverage



ξ : number of ambulances, X : number of parallel operations

$$P(\{X \leq \xi\}) \geq \beta \quad (= 0.95)$$

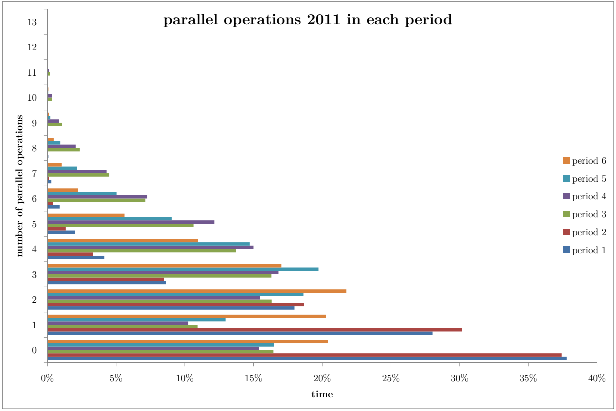
Time-dependent parameter: degree of required coverage



ξ : number of ambulances, X : number of parallel operations

$$P(\{X \leq \xi\}) \geq \beta \quad (= 0.95)$$

Time-dependent parameter: degree of required coverage



ξ : number of ambulances, X : number of parallel operations

$$P(\{X \leq \xi\}) \geq \beta \quad (= 0.95)$$

Extensions of the Double Standard Model

Existing:

- **Speed** is time-dependent and location-dependent
- Empirical investigation (e. g. Schmid/Doerner (2010); Wiesche 2012)

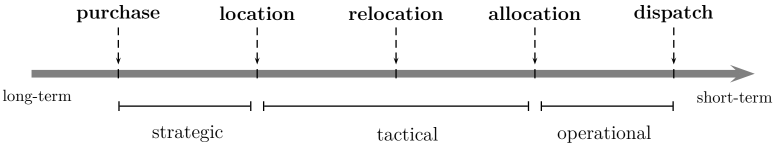
Extensions of the Double Standard Model

Existing:

- **Speed** is time-dependent and location-dependent
- Empirical investigation (e. g. Schmid/Doerner (2010); Wiesche 2012)

Additional:

- Time- and location-dependent **demand**
- **Empirically required coverage** for each demand node/period
- **Additional and flexible** ambulance stations
- **Dynamic** and **flexible allocation** of ambulances to stations



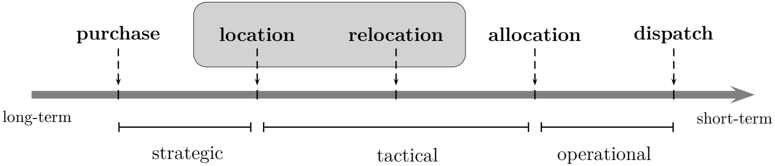
Extensions of the Double Standard Model

Existing:

- **Speed** is time-dependent and location-dependent
- Empirical investigation (e. g. Schmid/Doerner (2010); Wiesche 2012)

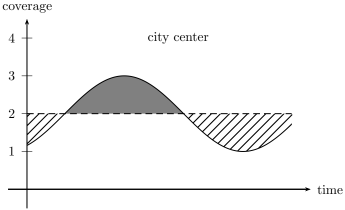
Additional:

- Time- and location-dependent **demand**
- **Empirically required coverage** for each demand node/period
- **Additional and flexible** ambulance stations
- **Dynamic** and **flexible allocation** of ambulances to stations



New optimization approach

Objectives (1)



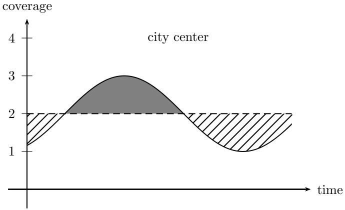
- Observed: differences between required and actual coverage
- Maximize the empirically required coverage:

$$x_{it}^{k} := \begin{cases} 1, & \text{if demand node } i \text{ is covered } k\text{-times in period } t, \\ 0, & \text{otherwise.} \end{cases}$$

- $k = \mathbf{e}(it)$ is determined empirically ($\leftarrow$ parallel operations)

New optimization approach

Objectives (1)



- Observed: differences between required and actual coverage
- Maximize the empirically required coverage: $x_{it}^{\mathrm{e}(it)} \in \{0,1\}$

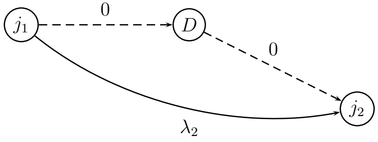
$$\max \quad \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} d_{it} x_{it}^{\mathrm{e}(it)}$$

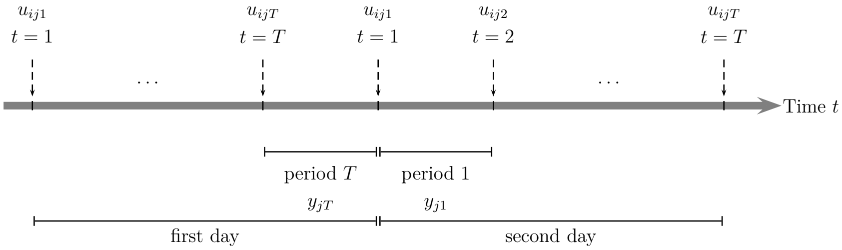
- $k = \mathrm{e}(it)$ is determined empirically ($\leftarrow$ parallel operations)

New optimization approach

Objectives (2)

- Re-locations due to variations of demand/speed (penalty costs)
- Utilization of flexible ambulance stations (penalty costs)





New optimization approach

Objectives (2)

- Re-locations due to variations of demand/speed (penalty costs)
- Utilization of flexible ambulance stations (penalty costs)

$$\min \quad \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} u_{ijt}$$

$$\min \quad \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{F}} y_{jt}$$

$$y_{Dt} = p - p_t \qquad \forall t \in \mathcal{T}$$

$$y_{jt} + \sum_{i \in \mathcal{J} \cup \{D\}} u_{ij(t+1)} - \sum_{i \in \mathcal{J} \cup \{D\}} u_{ji(t+1)} = y_{j(t+1)} \qquad \forall j \in \mathcal{J} \cup \{D\}, \forall t \in \mathcal{T} \setminus \{T\}$$

$$y_{jT} + \sum_{i \in \mathcal{J} \cup \{D\}} u_{ij1} - \sum_{i \in \mathcal{J} \cup \{D\}} u_{ji1} = y_{j1} \qquad \forall j \in \mathcal{J} \cup \{D\}$$

$$u_{ijt} \in \{0, 1\} \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \cup \{D\}, \forall t \in \mathcal{T}$$

$$y_{jt} \in \mathbb{N}_0 \qquad \forall j \in \mathcal{J}, \forall t \in \mathcal{T}$$

$$\vdots$$

New optimization approach

Constraints

- covering constraints
- location constraints
- relocation constraints
- allocation constraints
- demand constraints

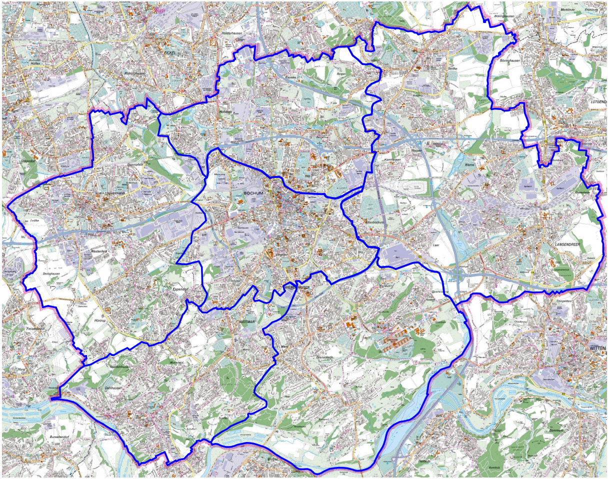
Bochum

Coordinates

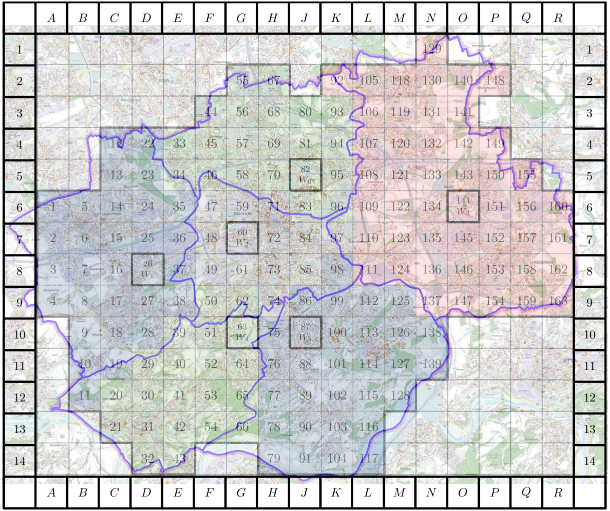
$51^\circ\,28'\,55''N \quad 7^\circ\,12'\,57''E$

- 16th biggest city in Germany
- Area: 145.4 km^2 (56.1 sq mi)
- Population: ca. 375,000
- Population density: 2,577$/km^2$
- Services: about 21,000 operations per year
- Services per 1,000 inhabitants per year: about 56 operations
- **14** ambulances

The city area of Bochum



The city area of Bochum



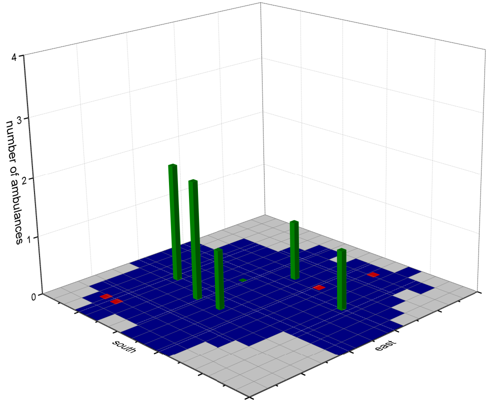
The city area of Bochum

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													129					1
2							55	67		92	105	118	130	140	148			2
3						44	56	68	80	93	106	119	131	141				3
4			12	22	33	45	57	69	81	94	107	120	132	142	149			4
5			13	23	34	46	58	70	82 W_6	95	108	121	133	143	150	155		5
6	1	5	14	24	35	47	59	71	83	96	109	122	134	144 W_3	151	156	160	6
7	2	6	15	25	36	48	60 W_2	72	84	97	110	123	135	145	152	157	161	7
8	3	7	16	24 W_1	37	49	61	73	85	98	111	124	136	146	153	158	162	8
9	4	8	17	27	38	50	62	74	86	99	112	125	137	147	154	159	163	9
10		9	18	28	39	51	63 W_4	75	87 W_5	100	113	126	138					10
11		10	19	29	40	52	64	76	88	101	114	127	139					11
12		11	20	30	41	53	65	77	89	102	115	128						12
13			21	31	42	54	66	78	90	103	116							13
14				32	43			79	91	104	117							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

Model results

Usage of flexible ambulance stations and relocations

Period $t = 1$, number of ambulances 7

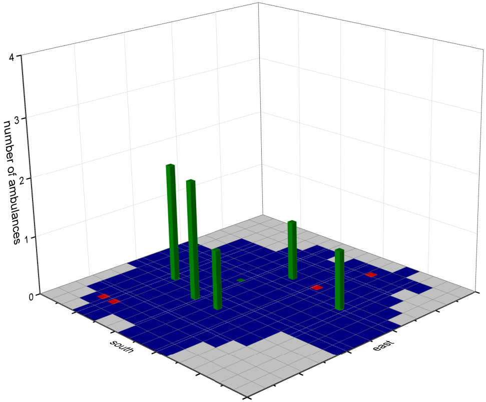


main stations, flexible stations

Model results

Usage of flexible ambulance stations and relocations

Period $t = 2$, number of ambulances 7

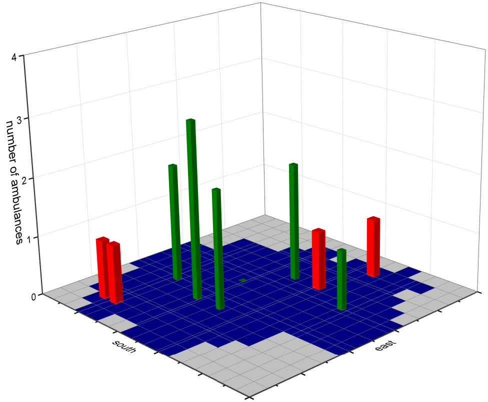


main stations, flexible stations

Model results

Usage of flexible ambulance stations and relocations

Period $t = 3$, number of ambulances 14

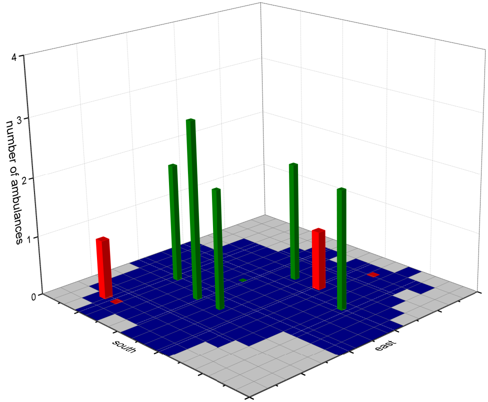


main stations, flexible stations

Model results

Usage of flexible ambulance stations and relocations

Period $t = 4$, number of ambulances 13

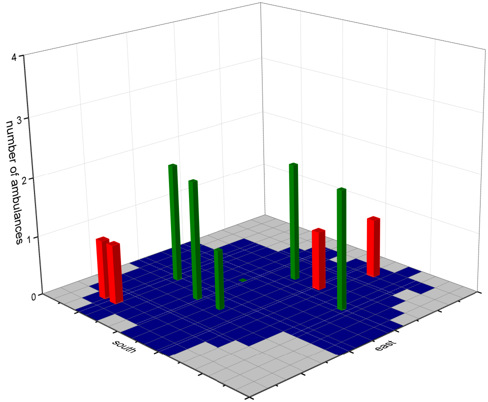


■ main stations, ■ flexible stations

Model results

Usage of flexible ambulance stations and relocations

Period $t = 5$, number of ambulances 13

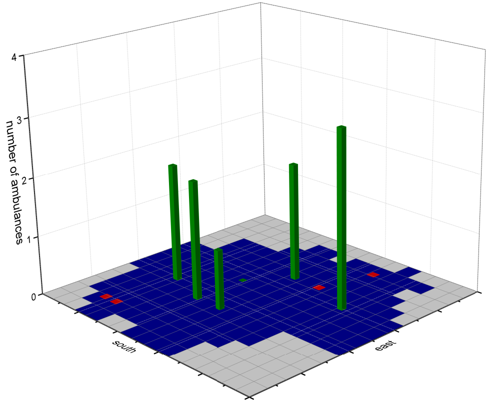


main stations, flexible stations

Model results

Usage of flexible ambulance stations and relocations

Period $t = 6$, number of ambulances 10



main stations, flexible stations

Model results

Difference between resulting and required degree of coverage (period 8-12 a.m.)

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													−2					1
2							−2	−1		−1	−2	−2	−2	−2	−2			2
3						−2	−1	−1	−1	−1	−1	−2	2	2				3
4			−2	−2	−2	3	3	3	−1	−1	−1	3	2	2	2			4
5			1	1	5	3	3	3	3	−1	3	3	2	2	2	2		5
6	−2	1	1	5	5	5	2	2	3	3	3	3	2	2	2	2	2	6
7	1	1	1	5	5	5	6	4	3	3	3	2	2	2	2	2	2	7
8	1	1	1	5	6	5	6	4	4	4	−1	2	2	2	2	2	−2	8
9	1	1	1	2	6	7	7	3	4	0	−1	−1	2	2	2	−2	−2	9
10		1	1	2	2	7	4	4	0	0	−1	−1	−2					10
11		−2	1	2	2	0	0	0	0	0	−1	−1	−2					11
12		−2	−2	−2	−1	−1	0	0	0	−1	−1	−2						12
13			−2	−2	−2	−1	−1	0	−1	−1	−2							13
14				−2	−2			−2	−2	−2	−2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

status quo

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													−1					1
2							−2	0		2	0	0	−1	−1	−1			2
3						−2	0	0	2	2	2	0	2	1				3
4			−2	−2	−2	0	0	1	1	2	2	4	2	2	0			4
5			0	0	0	0	0	1	1	1	4	4	2	1	0	0		5
6	−2	0	0	0	0	1	−1	0	1	1	3	3	1	1	0	0	0	6
7	0	0	0	0	0	1	3	3	2	2	3	1	1	0	0	0	0	7
8	0	0	0	0	2	1	3	3	3	3	1	1	0	0	0	0	−2	8
9	0	0	0	2	2	4	4	1	2	2	0	0	0	0	0	−2	−2	9
10		1	2	4	3	4	2	2	2	2	0	0	−2					10
11		0	2	4	4	3	2	2	2	2	0	0	−2					11
12		0	0	0	2	2	3	2	2	0	0	−2						12
13			0	0	0	2	1	2	0	0	−2							13
14				0	0			−2	−2	−2	−2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

max double coverage

Model results

Difference between resulting and required degree of coverage (period 8-12 a.m.)

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													-2					1
2							-2	-1		-1	-2	-2	-2	-2	-2			2
3						-2	-1	-1	-1	-1	-1	-2	2	2				3
4			-2	-2	-2	3	3	3	-1	-1	-1	3	2	2	2			4
5			1	1	5	3	3	3	3	-1	3	3	2	2	2	2		5
6	-2	1	1	5	5	5	2	2	3	3	3	3	2	2	2	2	2	6
7	1	1	1	5	5	5	6	4	3	3	3	2	2	2	2	2	2	7
8	1	1	1	5	6	5	6	4	4	4	-1	2	2	2	2	2	-2	8
9	1	1	1	2	6	7	7	3	4	0	-1	-1	2	2	2	-2	-2	9
10		1	1	2	2	7	4	4	0	0	-1	-1	-2					10
11		-2	1	2	2	0	0	0	0	0	-1	-1	-2					11
12		-2	-2	-2	-1	-1	0	0	0	-1	-1	-2						12
13			-2	-2	-2	-1	-1	0	-1	-1	-2							13
14				-2	-2			-2	-2	-2	-2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

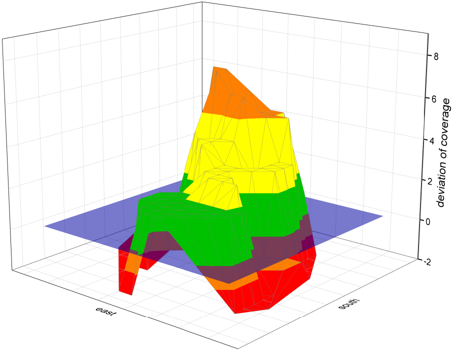
status quo

	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	
1													-1					1
2							-2	1		3	0	0	-1	-1	-1			2
3						-2	1	1	3	3	3	0	2	1				3
4			-2	-2	-2	1	1	2	2	3	3	5	2	2	0			4
5			0	0	0	1	1	2	2	2	5	5	2	1	0	0		5
6	-2	0	0	0	0	2	0	1	2	2	4	4	1	1	0	0	0	6
7	0	0	0	0	0	0	3	3	3	3	4	1	1	0	0	0	0	7
8	0	0	0	0	1	0	2	3	3	4	1	1	0	0	0	0	-2	8
9	0	0	0	1	1	3	3	0	1	1	0	0	0	0	0	-2	-2	9
10		1	2	3	2	3	1	1	1	1	0	0	-2					10
11		0	2	3	3	2	1	1	1	1	0	0	-2					11
12		0	0	0	1	1	2	1	1	0	0	-2						12
13			0	0	0	1	0	1	0	0	-2							13
14				0	0			-2	-2	-2	-2							14
	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	

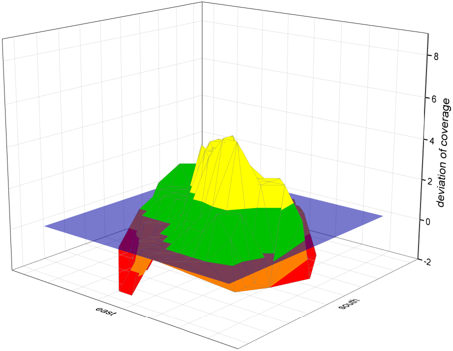
max empirically required coverage

Coverage level

Difference to empirically required coverage (period 8-12 a.m.)

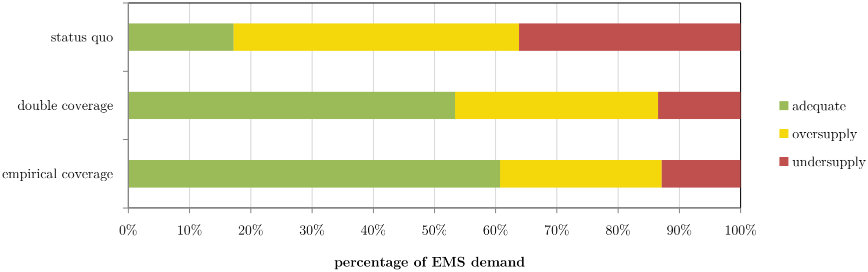


current allocation



optimal allocation

Difference between the empirically required coverage in percentage EMS demand



Conclusions and outlook

Improvements & further research

- Quality improvements:
  - Consideration of flexible ambulance stations is recommended
  - More suitable coverage according to empirical demands
- Efficiency of resource utilization:
  - Reduced number of ambulances
  - Constant quality, with cost reduction
  - Higher quality, with equal costs
- Integration of uncertainties (demand, driving speed $\rightarrow$ robust models)
  - Robust Uncertain Set Covering Problem (considers uncertainty in driving speed)
- Heuristic solution approach for large instances

Contact

Prof. Dr. Brigitte Werners
Management, Operations Research and Accounting

Faculty of Management and Economics
Ruhr-University Bochum
Universitaetsstrasse 150
44780 Bochum
E-mail: or@rub.de

OPERATIONS RESEARCH PROF. DR. BRIGITTE WERNERS
Stadt Bochum Feuerwehr und Rettungsdienst
ifu Institut für Unternehmensführung
Stiftung Zukunft NRW
RUHR UNIVERSITÄT BOCHUM RUB

References

Dirk Degel, Lara Wiesche, Sebastian Rachuba, and Brigitte Werners.
Dynamic ambulance location providing suitable coverage for time-dependent demand.
Cayirli T.; Gunal, M. M.; Gunes, E. D.; Ormeci, E. L. (eds.): Operational Research Applied to Health Services (ORAHS) 2013 Conference Proceedings, 2013.

Dirk Degel, Lara Wiesche, Sebastian Rachuba, and Brigitte Werners.
Time-dependent ambulance allocation considering data-driven empirical necessary coverage.
Technical report, Ruhr-University Bochum, Bochum, 2013/01 2013.

Simon Heussen, Brigitte Werners, Dirk Degel, Sebastian Rachuba, Lara Wiesche, and Pascal Lutter.
Projekt: optimale Versorgungsqualität im Rettungswesen—Optimierung von Rettungswachenstandorten.
Brandschutz: Deutsche Feuerwehr-Zeitung, 67. Jahrgang(September 2013):701–703, 2013.

Brigitte Werners, Dirk Degel, Sebastian Rachuba, Lara Wiesche, and Simon Heußen.
Bedarfsplanung für kommunale Rettungsdienste: Wo sollte jedes Fahrzeug stehen?
Rettungsdienst (www.skverlag.de), 4:38–41, 2013.