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# Emergency OR, or NOT?

Robust optimization of the OR schedule to deal with emergency surgery

(offline operational level)

# Research motivation

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The arrival of **emergency surgeries** is the most important source of **disturbances** in the OR

➔ leads to: overtime, surgery cancellations, waiting time, reduced OR utilization

Options to deal with emergency surgery:

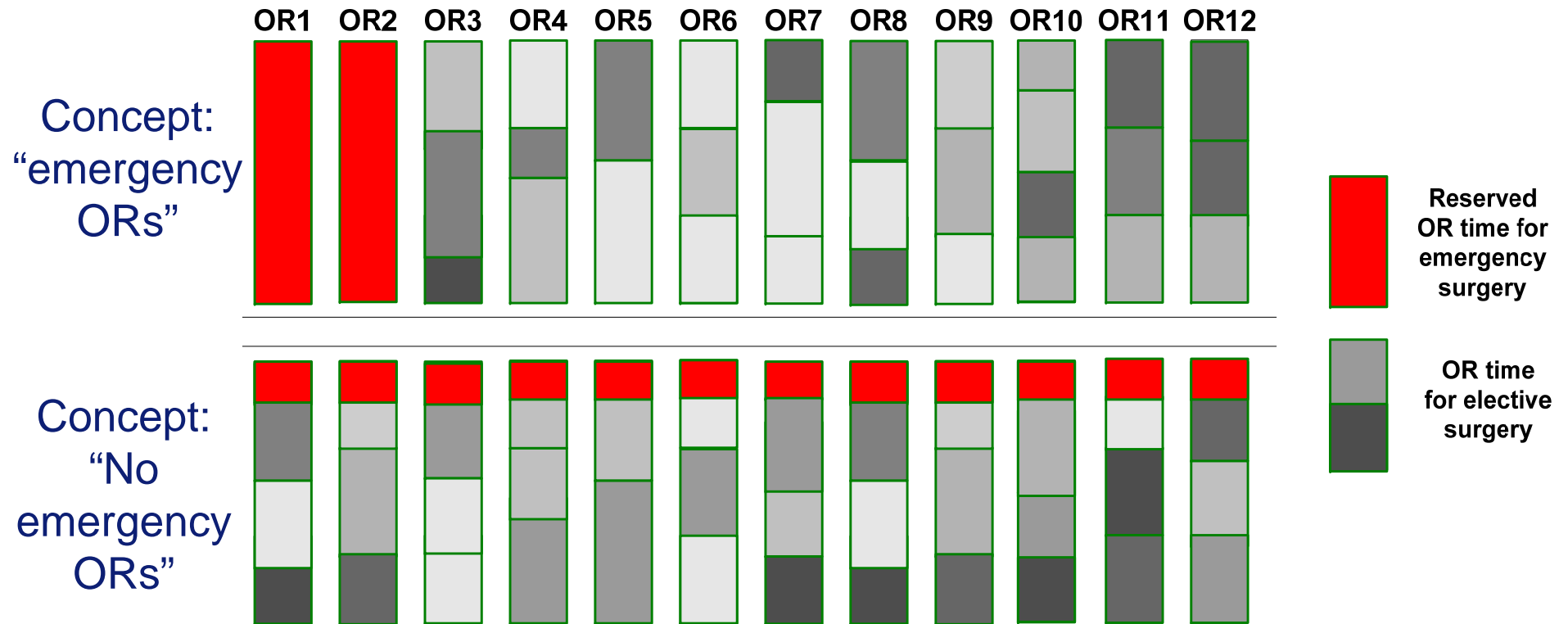
**Dedicated emergency ORs**

vs.

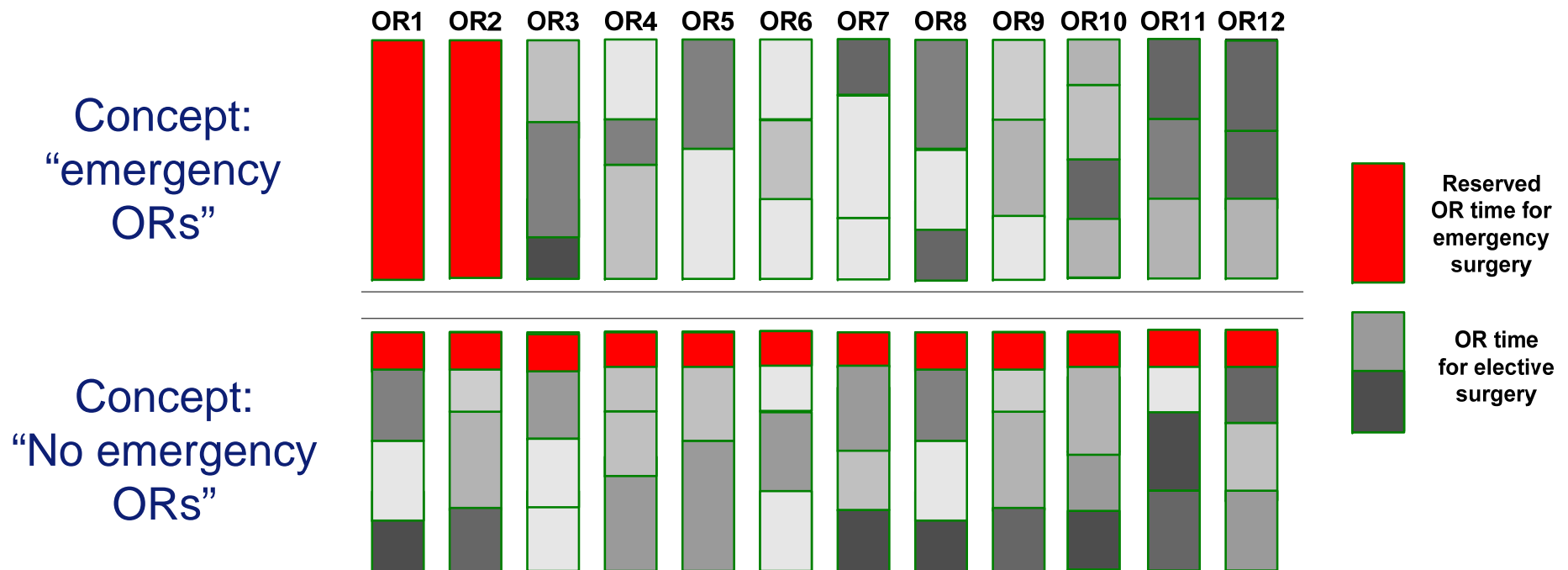
**Schedule emergency surgery in elective ORs**

# Emergency OR, or not?

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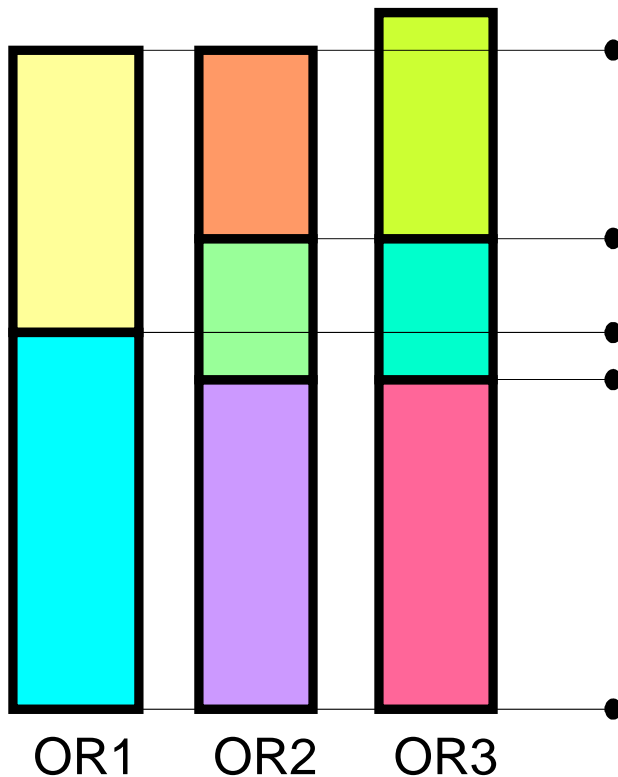
# Emergency OR, or not?



*Result of simulation:* emergency OR has **worse** performance w.r.t.: emergency surgery waiting time, overtime, OR utilization

# Problem description

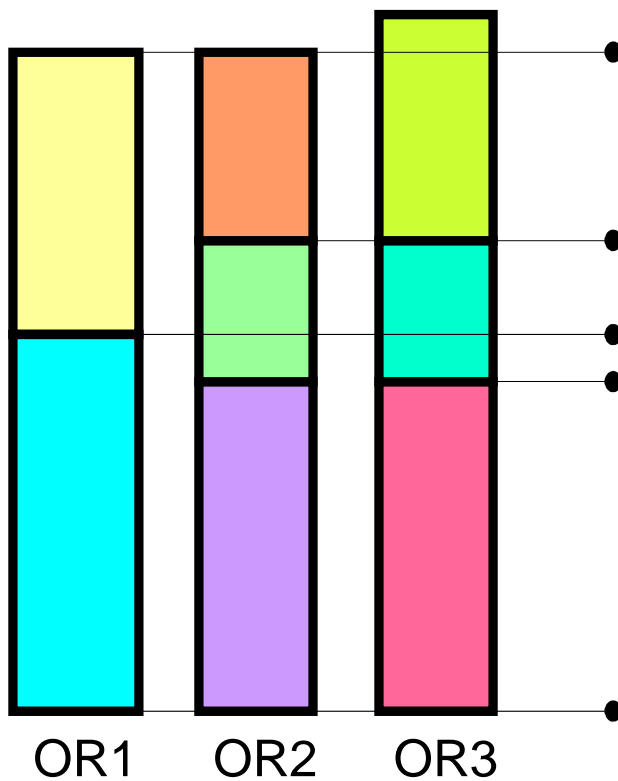
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- Usually: longest surgeries are planned first
- As a result: first break-in-moment (BIM) is far away

# Problem description

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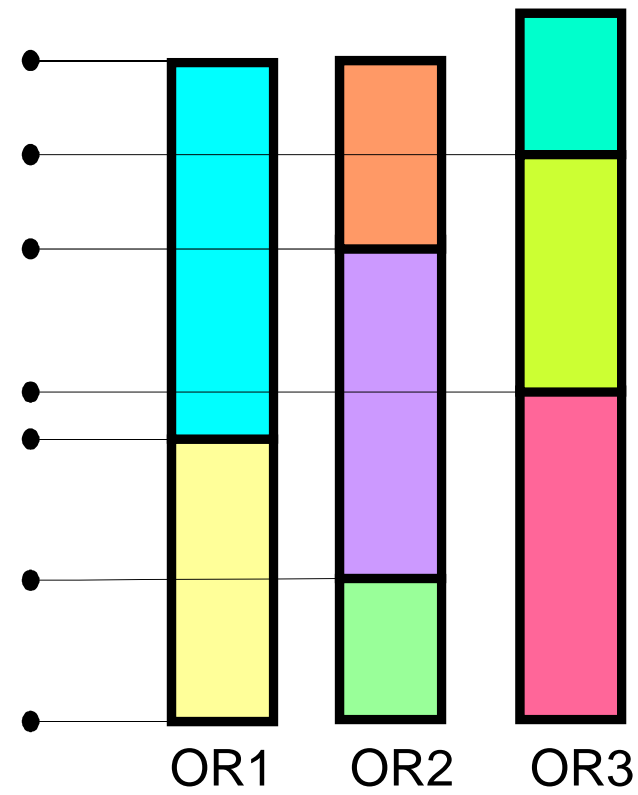


OR1

OR2

OR3

*Before*



OR1

OR2

OR3

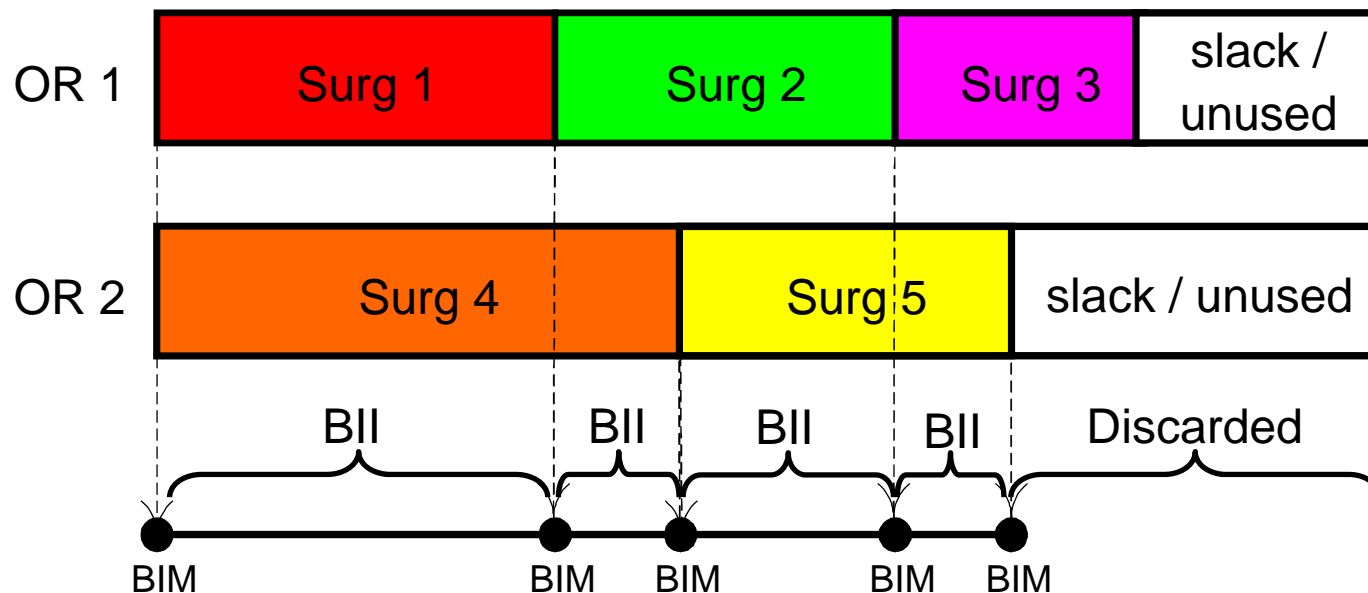
*After*

# Solution approach

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- Goal: spread “Break-In-Moments” between elective surgeries as evenly as possible
- Problem is NP-hard in the strong sense
  - Proof by reduction from 3-partition
- Input: an elective surgery schedule for a given week
- Optimization: constructive + local search heuristics

# Constructive heuristic



BIM = Break-in-Moment

BII = Break-in-Interval

The problem is to find “min max BII”



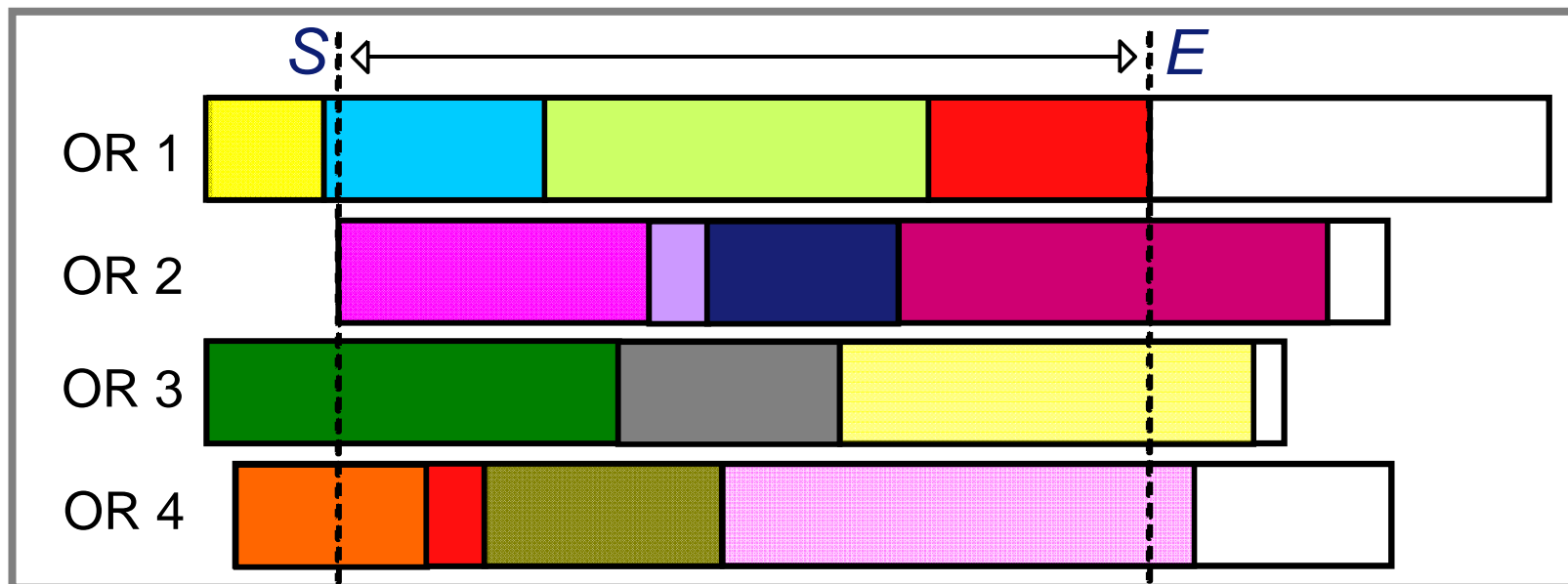
# Lower bound to “min max BI”

$$\lambda = \frac{E - S}{1 + \underbrace{\sum_{j \in J} (M_j - 1)}_{\substack{\text{\#BIMs} \\ \text{\#BIs}}}}$$

*E*: earliest OR end time

*S*: latest OR start time

*M<sub>j</sub>*: number of surgeries in OR *j*

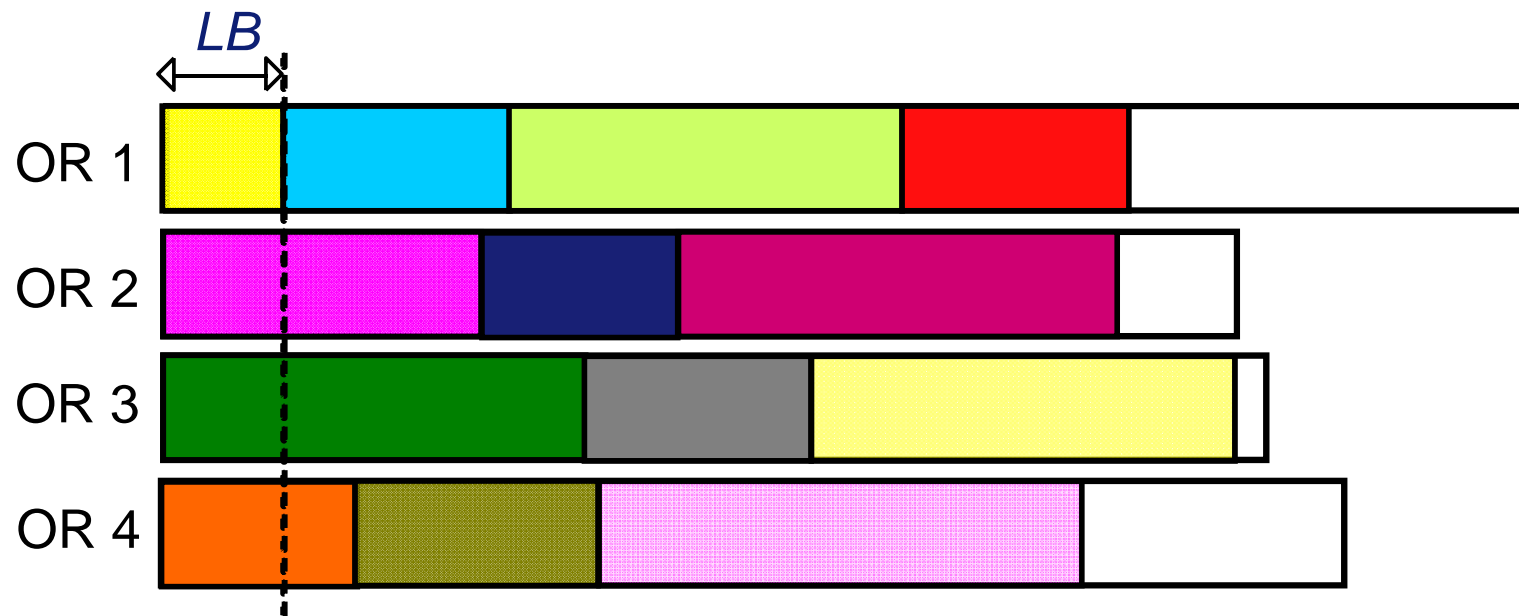


# Lower bound to “min max BII”

*Observation:*

The surgery with the shortest expected duration also forms a lower bound to “min max BII”

→ Lexicographic optimization



# Constructive heuristic

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First calculate  $\lambda$ : a lower bound to “min max BII”

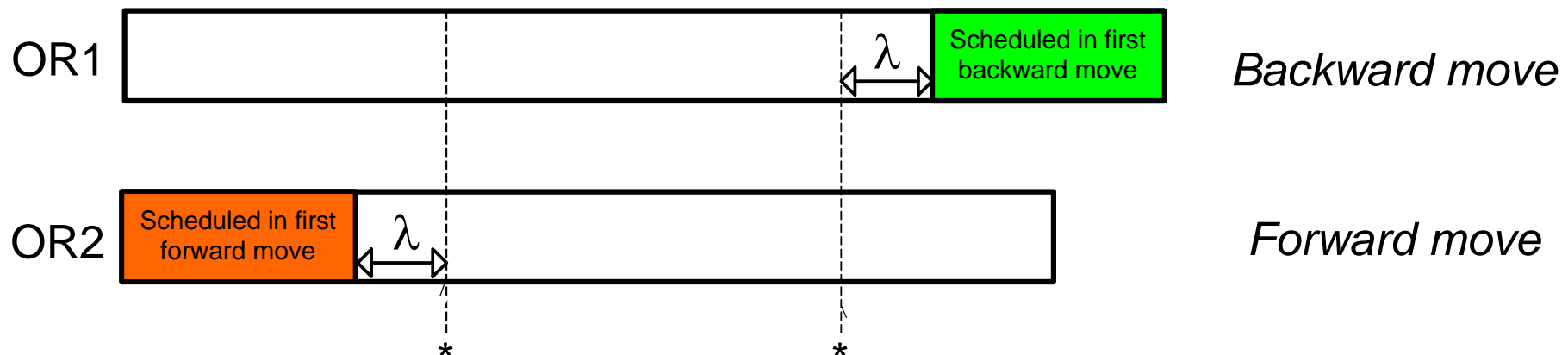
$$\lambda = \frac{E - S}{1 + \sum_{j \in J} (M_j - 1)}$$

$E$ : earliest OR end time

$S$ : latest OR start time

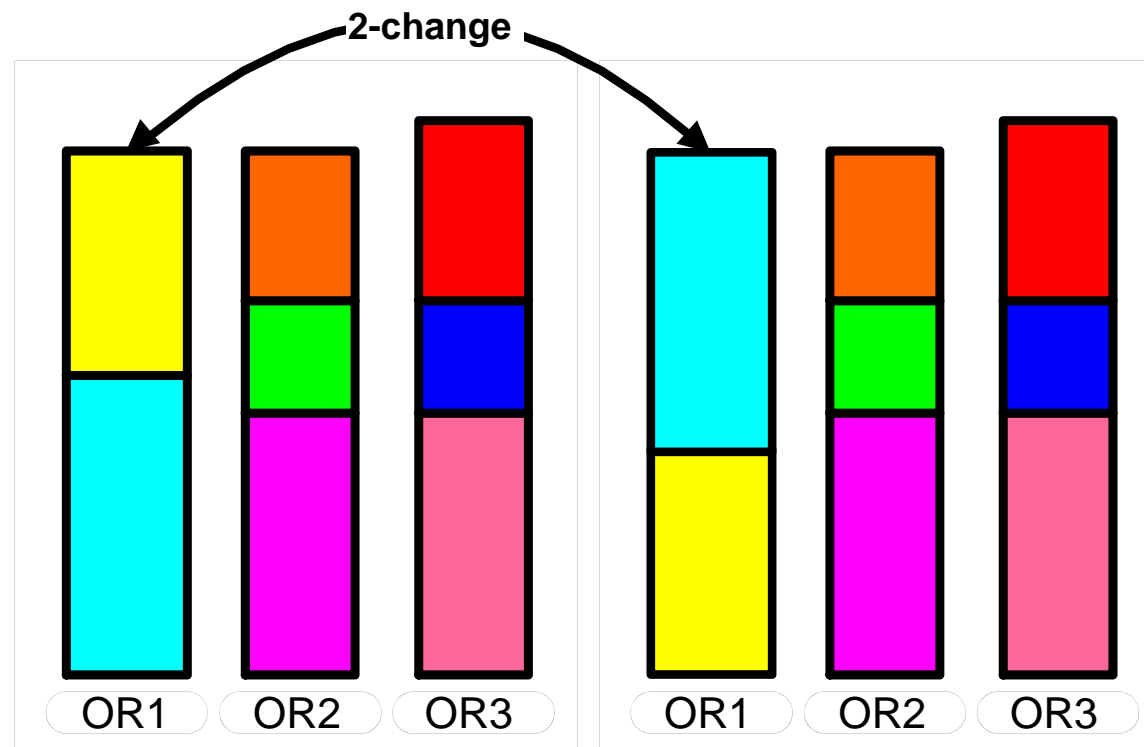
$M_j$ : number of surgeries in OR  $j$

Iteratively schedule a surgery forward or backward closest to \*



# Local search method: swapping surgeries

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# Local search method: “Shifting bottleneck” approach (SB)

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- Sort the ORs on non-increasing number of surgeries
- In iteration  $i$ , add OR  $i$  in an optimal way (enumerate all sequences)

Optional local search extension (**SB+**)

**REPEAT**

**FOR** “every OR  $i$ ” **DO**

Extract OR  $i$ , and insert optimally

**UNTIL** “no improvement was found”

# Simulation results operational problem

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Waiting time less than:	First emergency procedure		Second emergency procedure		Third emergency procedure	
	No BII opt.	BII opt.	No BII opt.	BII opt.	No BII opt.	BII opt.
<b>10 minutes</b>	28.8%	48.6%	34.9%	44.9%	40.4%	46.2%
<b>20 minutes</b>	53.0%	75.8%	56.9%	73.6%	63.0%	69.8%
<b>30 minutes</b>	70.5%	90.9%	71.8%	87.2%	76.3%	86.7%

*Case mix Academic Hospital*

# Results after simulation

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“Emergency surgery in elective program” instead of “emergency ORs” yields:

- Improved OR utilization (3.1%)
- Less overtime (21%)

Break-in-moment optimization yields:

- Reduced waiting time for emergency surgery, especially for the first arrival  
(patients helped within 10 minutes: from 28.8% → 48.6%)

# Assignment

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## **BIM (break-in-moment)-optimization problem:**

### **Given:**

elective surgery schedule for a certain number of ORs

### **Objective:**

sequence the surgeries within each operating room in such a way, that the maximum break-in interval ('max-BII') is minimized

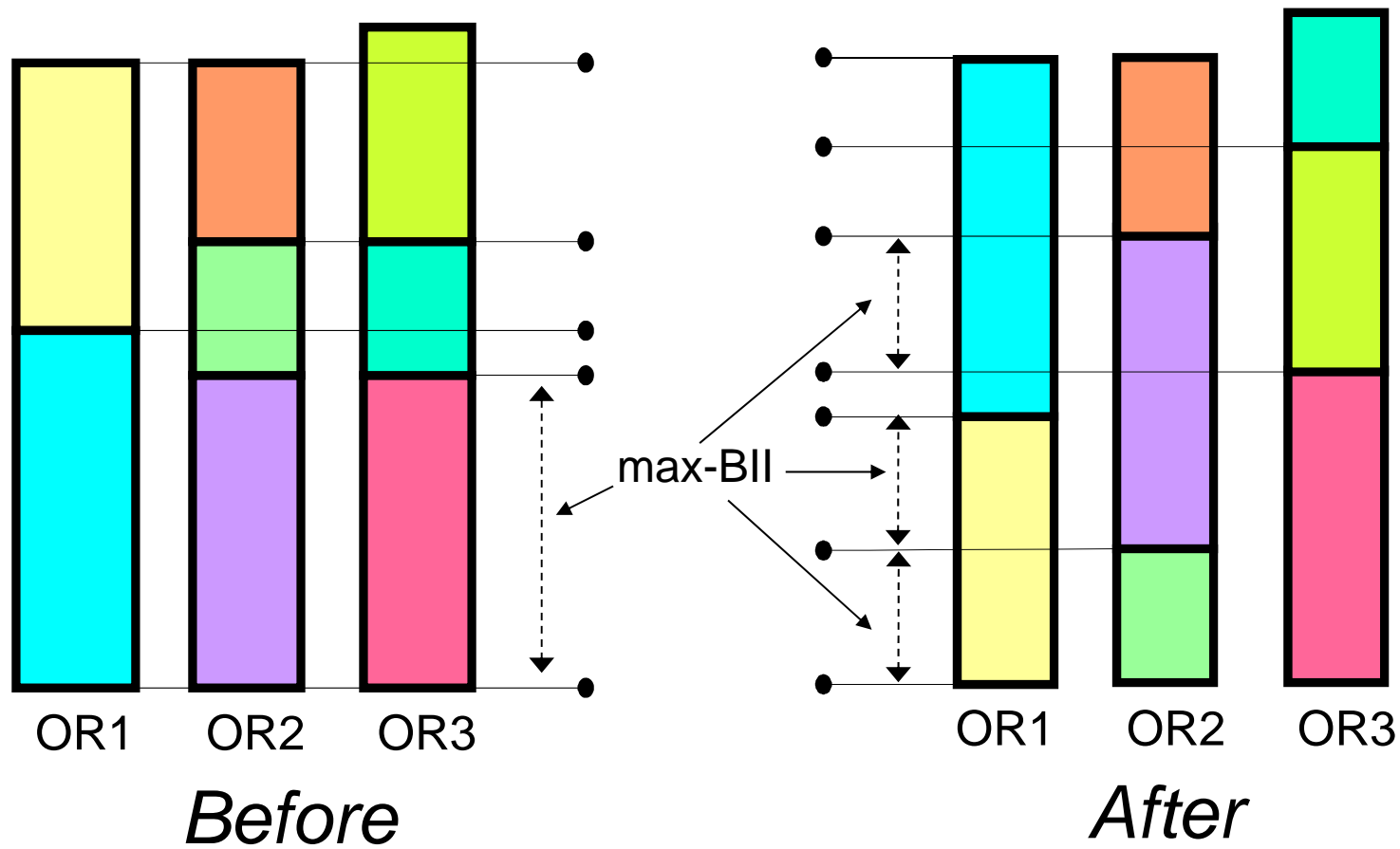
### **Assignment:**

Try to formulate a (mixed) integer linear program (ILP) of the BIM-optimization problem



# Assignment, illustration

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# Instrument tray optimization

# Instrument trays for surgery

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- Each surgery requires dozens of instruments, most of which are re-used after sterilization
- Stochastic requirements per surgery type
- Instruments are expensive
- Diversity of instruments is enormous
- Sterilization is expensive ( $\pm$  €1 per instrument)



# Instrument trays for surgery



- Most hospitals use “instrument trays”
- There are:
  - “surgery type-specific trays”
  - “base trays”
  - “add-on trays”

- Instruments remain in their tray (are sterilized together)
- Rarely used instruments are kept in inventory



# Problems with instrument trays

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- Instrument trays “evolve”
- Many instruments are outdated
- Many instruments are not used during surgery
- Missing instruments must be collected from a storage space (takes time → another tray is opened)
- The more types of trays → the more inventory (€ € €)
- Preparing trays “to order” is very hard

# Instrument trays: potential savings

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- **Potential savings:**
  - Unnecessary sterilizations, repairs, replacements
  - Unnecessary inventory
  - Location of inventory
  - Required instruments not in tray(s)
  - Time required for gathering instruments
  - Time required for counting instruments
- **Elske Florijn (MSc student from UT):**
  - In AMC, 21% of the instruments are obsolete
    - €2.3 million sterilization costs per year
    - Repair costs
    - Handling costs
  - €150.000 / year sterilization cost savings when 12 out of the 550 trays types contents are optimized
- **Problem: data collection is very hard**



# Appointment Scheduling

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- Planned arrivals
  - Appointments (elective)

- Unplanned arrivals
  - Emergencies
  - Semi-urgent
  - Self-referrals
  - One-stop shops
  - Calls for appointments

## *Examples:*

- Outpatient department
- Pre-operative screening
- Emergency department
- Radiology department
- Casting department

- Mathematical process: inhomogeneous Poisson process

# Elective appointment scheduling

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- Performance evaluation of given schedules
- Optimization of appointment schedules: appointment scheduling
- Current practice: equally spaced appointments
  - Late start (waiting of doctor) due to late arrival patient
  - Tardiness
- (To what extent) is walk-in possible?
  - Performance?
  - Conditions?



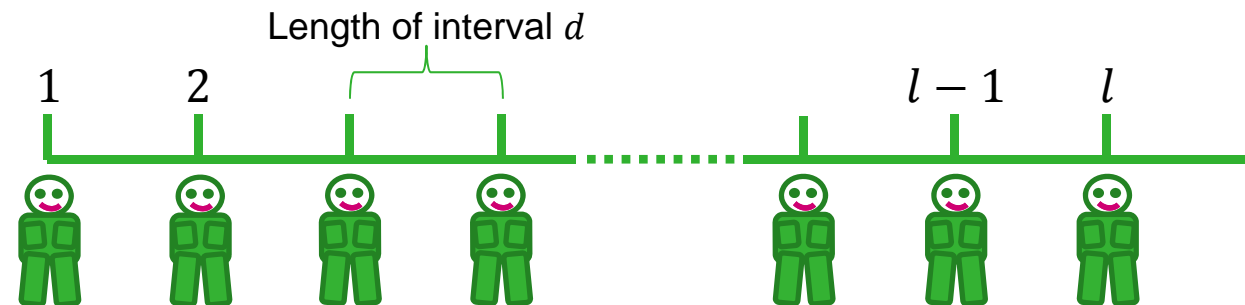
# Appointment scheduling model

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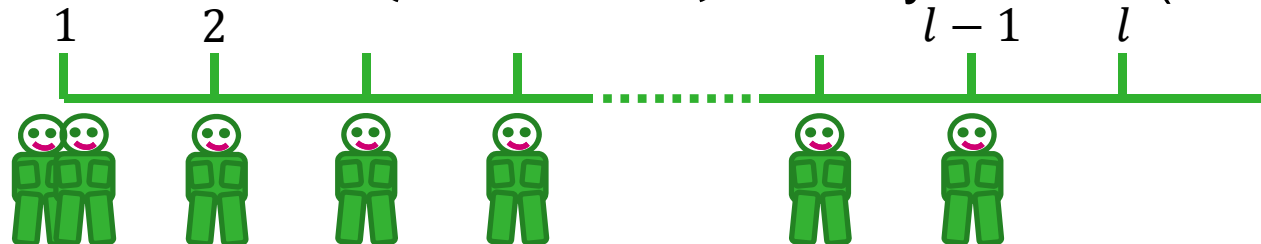
- **Parameters:**
  - $l$ : number of intervals
  - $d$ : length of interval
  - $n$ : total number of patients
  - $\beta$ : average service time
- **Parameters (not considered)**
  - $\alpha$ : no-show percentage (solution: consider overbooking)
  - $\lambda$ : arrival rate of emergencies (non-preemptive priority)
  - Punctuality data
- **Decision variables:**
  - $x_t$ : number of patients scheduled at time  $t$

# Examples

- Number of intervals  
Number of patients  
Patients scheduled at time  $t$
- $l = n, x = (1, 1, 1, \dots, 1, 1)$



- $l = n, x = (2, 1, 1, \dots, 1, 0)$ : Bailey-Welch (1952)



# Appointment scheduling objective

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- Waiting time of patients
- Idle time doctor
  - (alternatively: resource utilization)
- Makespan of the schedule
- Tardiness

# Solution methods

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- Performance evaluation:
  - Simulation (discrete event, Monte Carlo)
    - Allows full generality
  - Markov chain approach
    - No early/late arrivals
- Optimization:
  - Local search using Monte Carlo simulation to evaluate solutions

# Markov chain approach

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- 2 versions:
  - Exponential service durations; number of service completions during interval  $\sim \text{Poisson}(d/\beta)$
  - General (integer) service durations
- $\pi_i^-$  = distribution of # patients just before  $i$
- $\pi_i^+$  = distribution of # patients just after  $i$
- Recursion for  $\pi_i^+$  and  $\pi_i^-$  (depending on decision vector  $x$ )
- Performance metrics can be derived from  $\pi_i^+$ ,  $\pi_i^-$ , and  $x$

# Optimization method

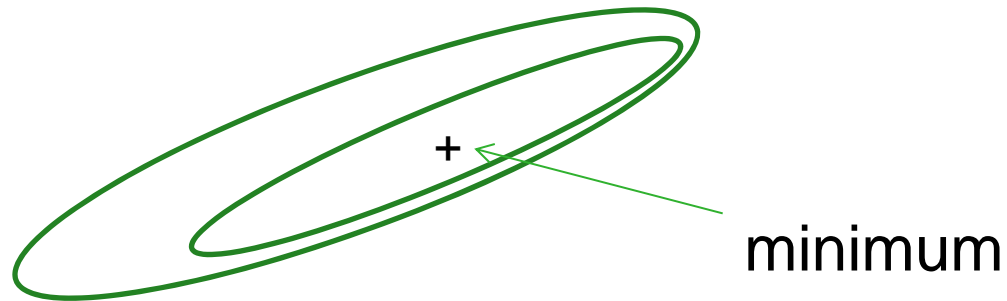
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- Problem: find global optimum on a  $(l - 1)$ -dimensional grid (the # of appointments at time  $l$  can be derived from this);  $n$  fixed
- Local search: shift on or more patients
- Leads to local optimum
- Can we do more?
  - Functional properties of objective function
  - Choice of local search neighborhood

# About convexity

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- Consider a 2-d convex function. What is the minimum?

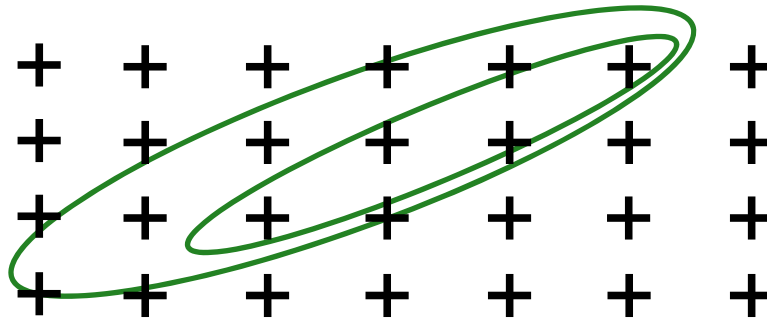


- Example method: *steepest descent*
  - Guarantees global minimum

# About convexity

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- What is the minimum **grid point**?



- Rounding continuous solution does not work
- Straightforward local search does not work
- No simple search on the grid
  - UNLESS: *multimodularity* property



# Multimodularity

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- $f$  multimodular if:

$$f(x + v) + f(x + w) \geq f(x) + f(x + v + w)$$

- with  $v, w$  different vectors of the form:

$$v_0 = (-1, 0, \dots, 0)$$

$$v_1 = (1, -1, 0, \dots, 0)$$

$$v_2 = (0, -1, 1, 0, \dots, 0)$$

...

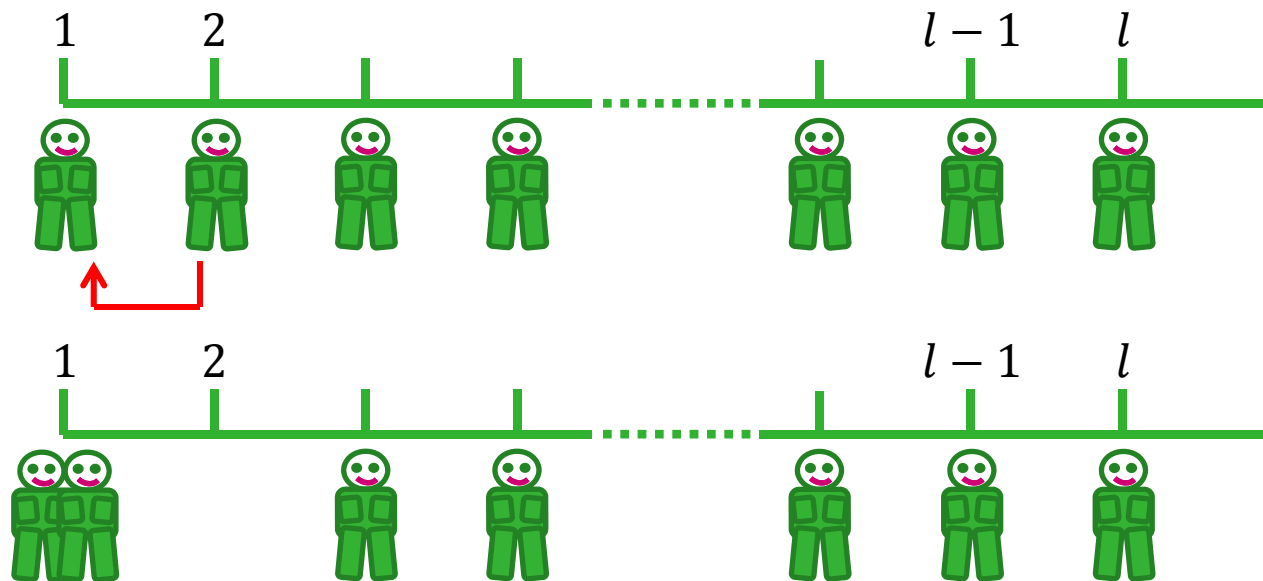
$$v_{m-1} = (0, \dots, 0, 1, -1)$$

$$v_m = (0, \dots, 0, 1)$$

# Interpretation of vectors $v_0, \dots, v_m$

$x + v$  is a shift of patterns

Example:  $x = (1, \dots, 1)$   
 $v_1 = (1, -1, 0, \dots, 0)$   
 $x + v_1 = (2, 0, 1, \dots, 1)$



# Multimodularity interpretation

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- $f$  multimodular if:

$$f(x + v) + f(x + w) \geq f(x) + f(x + v + w)$$

- Interpretation:

Better two changes ( $f(x + v + w)$ ) or none ( $f(x)$ )  
than one by one ( $f(x + v)$  and  $f(x + w)$ )

# Multimodularity

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- THEOREM: objective is multimodular
- PROOF: coupling of service durations
  - Details: G.C. Kaandorp, G.M. Koole, Optimal outpatient appointment scheduling, in: Health Care Management Science 10, 2007

➔ Local search converges to global optimum when neighborhood equals all combinations of vectors of the form  $v_0, \dots, v_m$

- High complexity; works well with fewer vectors

# Literature

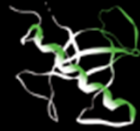
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- G.C. Kaandorp, G.M. Koole, Optimal outpatient appointment scheduling, in: Health Care Management Science 10, 2007
- Anke Hutzschenreuter, BMI paper, 2004 (Google for “Anke Hutzschenreuter BMI”)
- Welch & Bailey, Appointment systems in hospital outpatient departments, The Lancet, 1952

UNIVERSITY OF TWENTE.

# An exact approach for relating recovering surgical patient workload to the master surgical schedule

Peter Vanberkel



Centre for Healthcare Operations, Infrastructure and Research  
The Netherlands Cancer Institute - Antoni van Leeuwenhoek Hospital

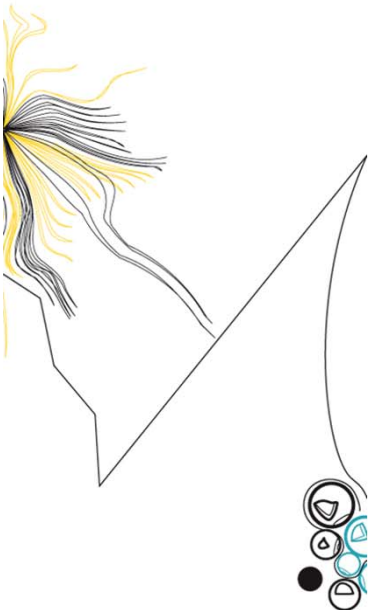





# Outline

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- Motivation / Background
- The Master Surgical Schedule (MSS)
- Model: Ward workload as a function of the MSS
- Application

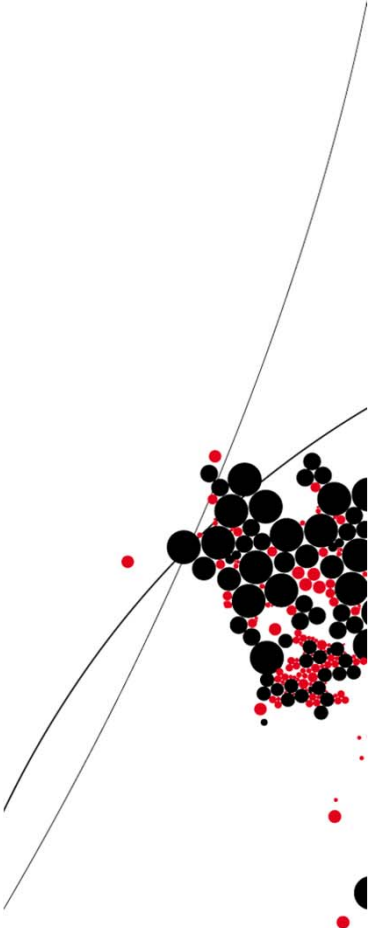




# Motivation / Background > Problem Description

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## **Netherlands Cancer Institute - Antoni van Leeuwenhoek Hospital**

- 
- 550 scientists and scientific support personnel
  - 53 medical specialists,
  - 180 beds,
  - Out-patient clinics receive 24,000 new patients each year,
  - 5 operating rooms
  - 9 irradiation units.
  - OR 6 to open.



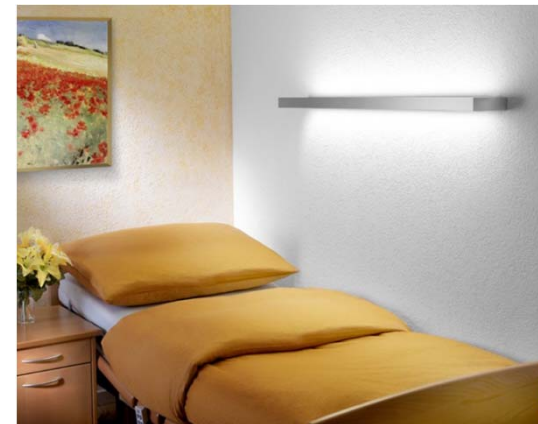
# Motivation / Background

## > The OR-Ward Relationship

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### OR 6 opened in 2009

- How will this impact the rest of the hospital, particularly the Wards?
  - Occupancy Rate
  - Admission rates / Discharge rates
  - Frequency of treatments





# Motivation / Background

## > The OR-Ward Relationship

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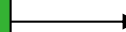
Waiting  
Patients



OR



Wards



Exit

- **Upstream of the OR:** Sufficient patient buffer to prevent 'starving'
- **In the OR:** Physician schedules, equipment...
- **Downstream of the OR:** Our Focus



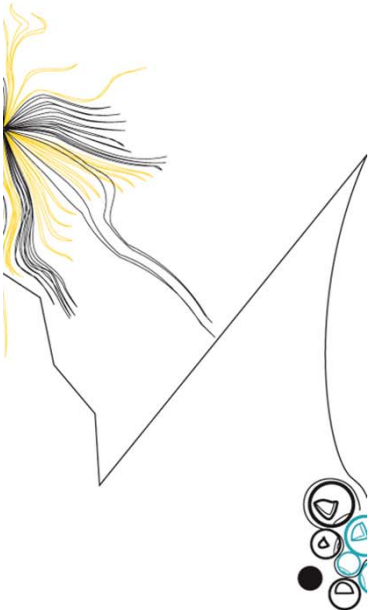
# Motivation / Background

## > The OR-Ward Relationship

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### Patient Flow (day of surgery)



- **Morning of Surgery:** Patient is admitted to the ward
- **Time of Surgery:** Patient has anesthesia, surgery, PACU
- **After Surgery:** Patient admitted to Ward and recover for LOS
- **After Recovery:** Patient is discharged home





# The Master Surgical Schedule

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- Surgical department activity is dictated by the MSS.
    - What specialties get what OR blocks? (Not patient specific)
    - Typically cyclical
    - Organizes the OR: Accounts for potential resource conflicts within the OR, e.g. physician schedules, equipment, etc.
  - Dictates the arrival pattern of recovering Surgical Patients to the wards
- 
- 



# The Master Surgical Schedule

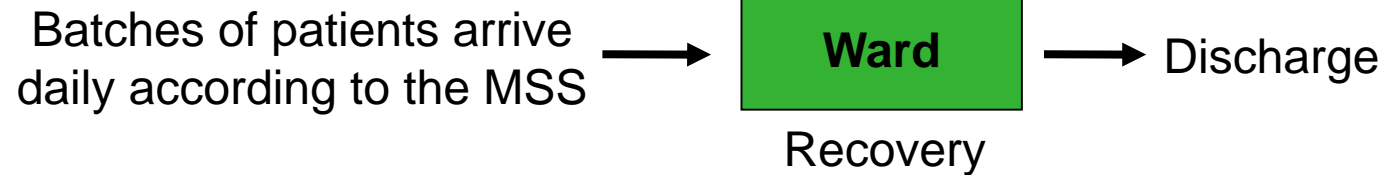
	Mon	Tue	Wed	Thu	Fri
OR1	Chi (KLM)	CHI (VWL)	CHI (vwl/rur) HIPEC	Chi (nie)	Chi (VRP)
OR2	KNO	CHI (RUT)	Urologie (hbs)	RT	Urologie (MND)
OR3	KNO	Plas Chi	KNO	KNO	Plas Chi
OR4	CHI (COR)	Gyne	Chi Mamma	Plas Chi	Gyne
OR5	RT	CHI (SND/WOS)	RT (vwl/rur)	Urologie (pel/bex)	Urologie (P&B)
OR6	Urologie (P&B)	CHI (VWL)	Gyne	Chi (ODB)	Chi (Cor/rur)

**Goal:** Directly derive ward workload metrics from the MSS



# Model: Ward workload as a function of the MSS

## Conceptual Model Scheme



## Assumptions

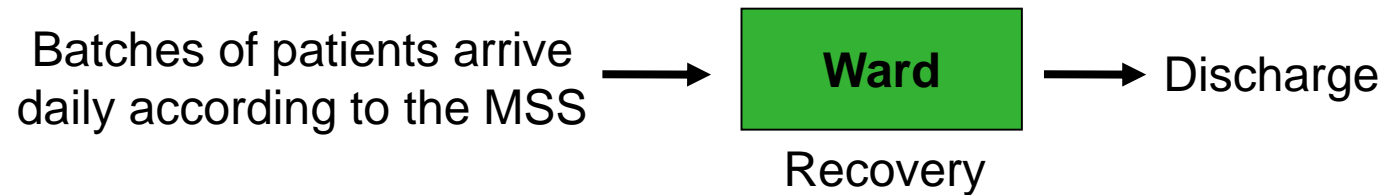
- No cancellations due to lack of ward space (extra nurses will be called in)
  - Acceptable Risk of “calling in a nurse” is  $\approx 10\%$
- Time scales is days.
- Count patients on the day of admission, not on the day of discharge



# Model: Ward workload as a function of the MSS

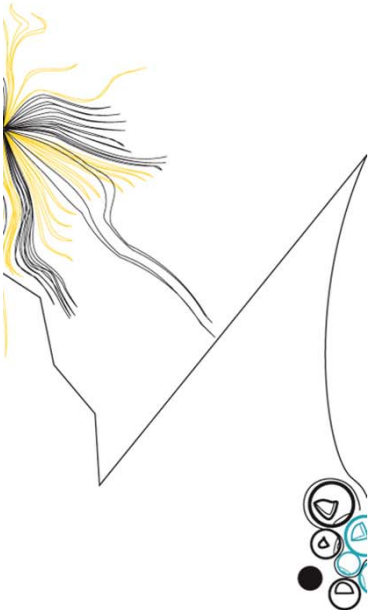
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## Conceptual Model Scheme



## Metrics

- 1) Recovering Patients in the Hospital
- 2) Ward occupancy
- 3) Rates of admissions and discharges
- 4) Patients in recovery day  $n$



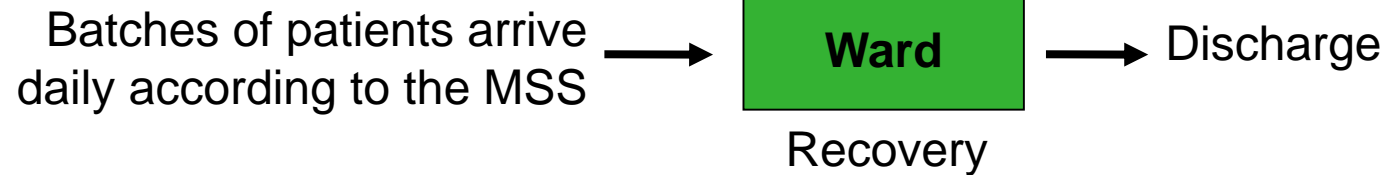




# Model: Ward workload as a function of the MSS

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## Conceptual Model Scheme



## Data

- For each surgical specialty
  - Empirical Distributions of Cases/Block (batch size)
  - Empirical Distribution of Length of Stay (LOS)





## Model: Ward workload as a function of the MSS

### > Metric 1: Recovering Patients in the Hospital

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#### Recovering patients in the hospital on the day of surgery ( $t = 0$ )

- Consider the influence of a single specialty in isolation
  - Let  $c(x)$  be a random variable for the number of completed surgeries
  - $c(x)$  also describes the batch size of admissions to the ward
- Finally,  $c(x)$  represents the number of recovering patients in the hospital on the day of surgery ( $t = 0$ )

## Model: Ward workload as a function of the MSS

### > Metric 1: Recovering Patients in the Hospital

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#### Recovering patients in the hospital on the days after surgery ( $t > 0$ )

- Let  $d(t)$  be the probability a patient who is in the hospital on day  $t$ , is discharged on day  $t$
- Let  $h_t(x)$  be the probability of  $x$  recovering patients on day  $t$ , then:

$$h_t(x) = \begin{cases} c(x) & \text{when } t = 0 \\ \sum_{k=x}^c \binom{k}{x} (d(t))^{k-x} (1 - d(t))^x h_{t-1}(k) & \text{otherwise} \end{cases}$$

$k - x$  are discharged   ←    $x$  are not discharged   ←    $k$  recovering patients on previous day

# Model: Ward workload as a function of the MSS

## > Metric 1: Recovering Patients in the Hospital

---

### Recovering patients in the hospital (all specialties)

- Consider a given MSS in isolation

	Mon	Tue	Wed	Thu	Fri
<b>OR1</b>	Chi (KLM)	CHI (VWL)	CHI (vwl/rur) HIPEC	Chi (nie)	Chi (VRP)
<b>OR2</b>	KNO	CHI (RUT)	Urologie (hbs)	RT	Urologie (MND)
<b>OR3</b>	KNO	Plas Chi	KNO	KNO	Plas Chi
<b>OR4</b>	CHI (COR)	Gyne	Chi Mamma	Plas Chi	Gyne
<b>OR5</b>	RT	CHI (SND/WOS)	RT (vwl/rur)	Urologie (pel/bex)	Urologie (P&B)
<b>OR6</b>	Urologie (P&B)	CHI (VWL)	Gyne	Chi (ODB)	Chi (Cor/rur)

- Each block generates patients for the ward. The number of patients is distributed according to  $h_t(x)$
- Since patients do not interfere with each other during recovery, the aggregate number of patients can be computed with discrete convolutions

$$C(x) = \sum_{k=0}^{\tau} A(k)B(x - k)$$

## Model: Ward workload as a function of the MSS

### > Metric 1: Recovering Patients in the Hospital

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#### Recovering patients in the hospital (all specialties)

- Let  $H_{t'}(x)$  be the probability of  $x$  patients on day  $t'$  for all specialties
  - $t' = 1$  is the first day of the MSS cycle

$$H_{t'}(x) = h_n^{block1} \star h_n^{block2} \star \dots$$

- Where:
  - $\star$  is a discrete convolution  $\rightarrow P(C = x) = \sum_{k=0}^x P(A = k)P(B = x - k)$

$n$  is a function of  $t'$  and the weekday the block falls on (this ensures the arrival of patients are offset to reflect the day of surgery)

# Model: Ward workload as a function of the MSS

## > Metric 1: Recovering Patients in the Hospital

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### Recovering patients in the hospital (all specialties, recurring MSS)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri
OR1	Chi (KLM)	CHI (VWL)	CHI (vwl/rur) HIPEC	Chi (nie)	Chi (VRP)			Chi (KLM)	CHI (VWL)	CHI (vwl/rur) HIPEC	Chi (nie)	Chi (VRP)			Chi (KLM)	CHI (VWL)	CHI (vwl/rur) HIPEC	Chi (nie)	Chi (VR
OR2	KNO	CHI (RUT)	Urologie (hbs)	RT	Urologie (MND)			KNO	CHI (RUT)	Urologie (hbs)	RT	Urologie (MND)			KNO	CHI (RUT)	Urologie (hbs)	RT	Urologi (MND)
OR3	KNO	Plas Chi	KNO	KNO	Plas Chi			KNO	Plas Chi	KNO	KNO	Plas Chi			KNO	Plas Chi	KNO	KNO	Plas Ci
OR4	CHI (COR)	Gyne	Chi Mamma	Plas Chi	Gyne			CHI (COR)	Gyne	Chi Mamma	Plas Chi	Gyne			CHI (COR)	Gyne	Chi Mamma	Plas Chi	Gyne
OR5	RT	CHI (SND/WOS)	RT (vwl/rur)	Urologie (pel/bex)	Urologie (P&B)			RT	CHI (SND/WOS)	RT (vwl/rur)	Urologie (pel/bex)	Urologie (P&B)			RT	CHI (SND/WOS)	RT (vwl/rur)	Urologie (pel/bex)	Urologi (P&B)
OR6	Urologie (P&B)	CHI (VWL)	Gyne	Chi (ODB)	Chi (Cor/rur)			Urologie (P&B)	CHI (VWL)	Gyne	Chi (ODB)	Chi (Cor/rur)			Urologie (P&B)	CHI (VWL)	Gyne	Chi (ODB)	Chi (Cor/

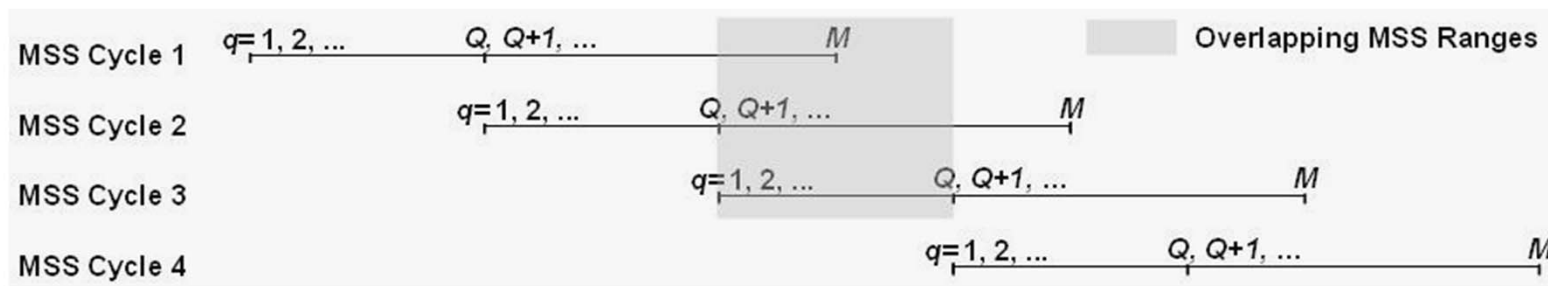
- With recurring MSS, patients from different MSS cycles will overlap
- MSS is cyclic, i.e. the MSS does not change from week to week



## Model: Ward workload as a function of the MSS

### > Metric 1: Recovering Patients in the Hospital

Recovering patients in the hospital (all specialties, recurring MSS)



- Let  $H_q(x)$  be the 'steady state' distribution for the number of patients recovering in the hospital on any day  $q$  of the MSS

$$H_q(x) = H_q \star H_{q+Q} \star H_{q+2Q} \star \dots \star H_{q+2[M/Q]Q}$$




## Model: Ward workload as a function of the MSS

### > Metric 2: Ward occupancy

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	Mon	Tue	Wed	Thu	Fri
OR1	Chi (KLM)	CHI (VWL)	CHI (vwf/rur) HIPEC	Chi (nie)	Chi (VRP)
OR2	KNO	CHI (RUT)	Urologie (hbs)	RT	Urologie (MND)
OR3	KNO	Plas Chi	KNO	KNO	Plas Chi
OR4	CHI (COR)	Gyne	Chi Mamma	Plas Chi	Gyne
OR5	RT	CHI (SND/WOS)	RT (vwf/rur)	Urologie (pel/bex)	Urologie (P&B)
OR6	Urologie (P&B)	CHI (VWL)	Gyne	Chi (ODB)	Chi (Cor/rur)

 Ward A  
 Ward B

For ward specific results, when computing  $H_t(x)$  only consider OR blocks for the ward of interest.



## Model: Ward workload as a function of the MSS

### > Metric 3: Rates of admissions and discharges

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- Admission Rate:
  - Modify  $h_t(x)$  as follows, and then continue with the convolutions:

$$h_t(x) = \begin{cases} c(x) & \text{when } t = 0 \\ 0 & \text{otherwise} \end{cases}$$

- Discharge Rate:
  - Compute  $h'_t(x)$  as follows:

$$h'_t(x) = \sum_{k=x}^c \binom{k}{x} (d(t))^x (1 - d(t))^{k-x} h_t(k)$$

and then set  $h_t(x) = h'_t(x)$  and continue with the convolutions








## Model: Ward workload as a function of the MSS

### > Metric 4: Patients in Recovery day $n$

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- Keep “day of surgery” index throughout computations
    - index by  $t$  and  $t'$
  - Meaningfulness of Metric
    - For some well defined patient groups the recovery “activities” are precisely defined for each recovery day
    - For example: The majority of patients who receive lung cancer surgery are discharged on day 8. On each day the activities of care are stated.

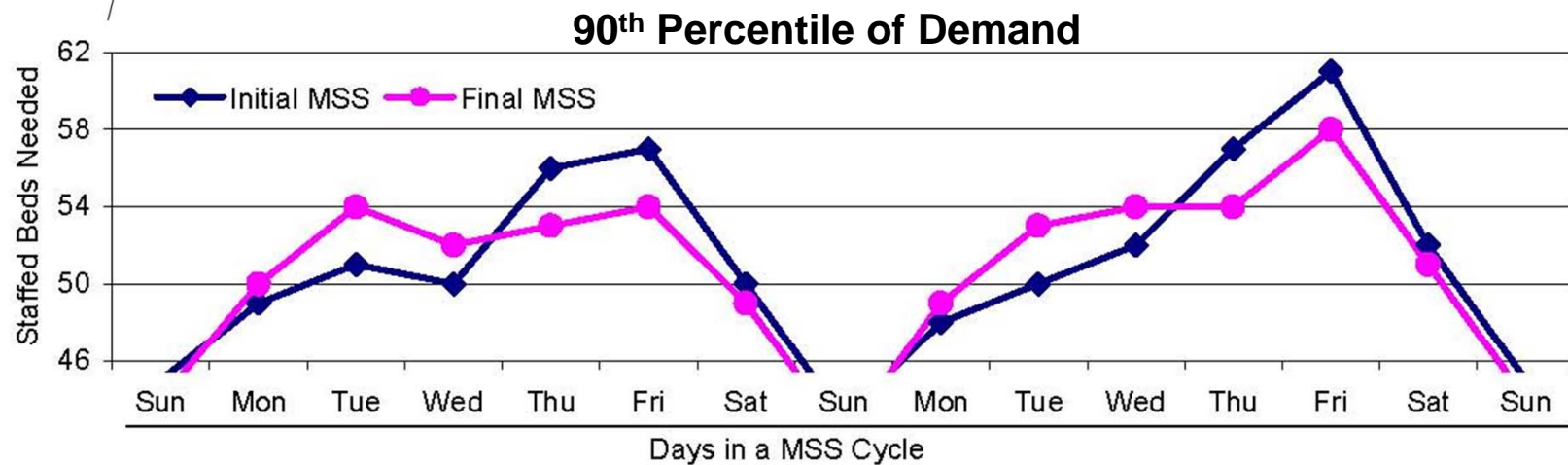


# Application: NKI/AVL Amsterdam

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- Evaluation model, not an optimization model
- Manual process
  - Staff from the OR proposed the MSS
  - The model was used to evaluate the proposal
  - Staff from the OR and Wards debated the proposal and made suggestions for modifications
    - This continued until all parties agreed to the MSS
- Advantages of Manual Process
  - Enhanced user “buy-in”
  - Staff from both groups developed intuition for how changing the assignment of Specialties to OR block impacted the wards
  - Began to understand the impact of the OR constraints

# Example Result



## Initial MSS

- 1/10 days required 61 staffed beds
- 4/10 days required  $> 54$  staffed beds
- 2/10 days required  $< 50$  staffed beds
- Other days required b/w 50 & 54

## Final MSS

- 1/10 days required 58 staffed beds
- 9/10 days required b/w 50 & 54
- Further discussion is ongoing to change physician schedules to eliminate peak in week 2