
Volume 37
Issue 3
June 2009
ISSN 0305-0483
ELSEVIER
omega
The International Journal of Management Science
www.elsevier.com/locate/omega

Available online at www.sciencedirect.com
ScienceDirect

Omega 37 (2009) 510–521

omega
The International Journal of Management Science

www.elsevier.com/locate/omega

Unraveling quantity discounts☆

Fredo Schotanus[a,*], Jan Telgen[a], Luitzen de Boer[b]

[a]*Department of Operational Methods for Production and Logistics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands*
[b]*Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Alfred Getz vei 1, Sentralbygg II, N 7491 Trondheim, Norway*

Received 22 September 2006; accepted 7 September 2007
Available online 18 September 2007

Abstract

We consider the situation in which a buying organization deals with a discrete quantity discount schedule offered by a selling organization. Furthermore, the buying organization can negotiate with the selling organization about the lot size and purchase price, but does not know the underlying function that was used by the selling organization to determine the quantity discount schedule. In this paper, we provide an analytical and empirical basis for one general quantity discount function (QDF) that can be used to describe the underlying function of almost all different quantity discount types. We first develop such a QDF analytically. Among other things, this QDF enables buying organizations to calculate detailed prices for a large number of quantities. We subsequently show that the QDF fits very well with 66 discount schedules found in practice. We discuss that the QDF and related indicators can be a useful tool in supplier selection and negotiation processes. It can also be used for competitive analyses, multiple sourcing decisions, and allocating savings for purchasing groups. Additionally, the QDF can be included in research models incorporating quantity discounts. We conclude the paper with an outlook on further QDF research regarding the characterization of commodity markets from a demand elasticity point of view.

Keywords: Bidding; Decision support systems; Forecasting; Heuristics; Operations management

1. Introduction

Quantity discount schedules have been used widely for many centuries [1]. These days, quantity discount schedules come in all shapes and sizes. A typical example of a discount schedule is shown in Table 1 [2].This discrete schedule provides the price per item for a limited number of quantities and prices. It does not provide the assumed function which the supplier used to calculate the price per item. Therefore, it is difficult for a buying organization to calculate negotiable prices for the many possible quantities for which no specific prices are provided by the supplier. For instance, if a buying organization needs a quantity of 9500 items, then the question often is what price—in between 40.9 and 45.4—the buying organization can start negotiating with. Another question is whether the buying organization should start negotiations with supplier A (see Table 1) or supplier B (see Table 2) for a quantity of 9500 items.

As a result of this information deficiency, it is difficult for buying organizations to compare quotes of different suppliers and to determine negotiating spaces.

☆ This manuscript was processed by Associate Editor Teunter.

* Corresponding author. Tel.: +31 6 1209 4702; fax: +31 53 489 2159.
E-mail addresses: f.schotanus@utwente.nl (F. Schotanus), j.telgen@utwente.nl (J. Telgen), luitzen.de.boer@iot.ntnu.no (L. de Boer).

0305-0483/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.omega.2007.09.002

Table 1
Example of a discount schedule of supplier A [2]

Quantity	Price per item
1000–4999	50.0
5000–9999	45.4
10,000–29,999	40.9
30,000–49,999	38.1
50,000–199,999	37.1
200,000 and more	33.5

Table 2
Example of a discount schedule of fictional supplier B

Quantity	Price per item
500–999	51.0
1000–1999	49.0
2000–3999	47.0
4000–7999	45.0
8000–15,999	43.0
16,000 and more	41.0

In this paper, one of our aims is to tackle this information deficiency problem. We do this by deriving the quantity discount function (QDF) that the supplier used to calculate the price per item. With this QDF, a buying organization can calculate prices for all possible quantities. But more importantly, the QDF derived can be used as a basic ingredient in research models incorporating quantity discounts. Before we discuss more applications and the academic and practical relevance of the QDF in more detail (Sections 1.2 and 1.3), we shortly introduce some important insights into quantity discounts in Section 1.1.

1.1. Relevant knowledge base on quantity discounts

First of all, calculating prices for all possible quantities is only a useful practice if the purchase price is negotiable. According to Munson and Rosenblatt [3], this is true for most situations. They argue in their study that purchase prices and lot sizes are mostly determined through negotiations. Munson and Rosenblatt also note that quantity discount schedules may have different characteristics:

- The number of price breaks in a discount schedule may be one, two, multiple or infinite [2,4].
- The form may be all-items or incremental. An all-items form means that all items receive the same discount [5]. An incremental form means that only the items within a price break interval receive that interval's discount [5].
- Time aggregation may be individual or multiple. This describes whether the discount applies to individual or multiple purchases over a certain time period [3].
- And finally, discounts may apply to one or multiple items [3].

The economic rationales of quantity discounts are mainly three-fold:

- Achieving perfect price discrimination against a single customer or a set of homogenous customers or achieving partial price discrimination against heterogeneous customers [6].
- Influencing the buying organization's ordering pattern to coordinate the supply chain [7] or to increase the logistics system efficiency in a distribution channel [8].
- And often, buying organizations are simply expecting a certain quantity discount for purchasing large amounts [9].

The body of knowledge on different types of quantity discounts is large, both from the seller's perspective and from the buyer's perspective. From the seller's perspective, a great deal has been written about when suppliers should offer quantity discounts, and if so, what type of quantity discount schedule they should offer to maximize profits (e.g., [10]). From the buyer's perspective, a great deal has been written about the application of quantity discounts in economic order quantity (EOQ) models (e.g., [11]) and inventory ownership problems (e.g., [12]).

Nearly all the previous research on quantity discounts is focused on creating quantity discount schedules or applying discount schedules in new or existing models. We use a different perspective. Given a quantity discount schedule, we are interested in deriving the supplier's original function that was used to create the schedule. Up to now, very little is known on deriving a QDF from a quantity discount schedule.

A given discount schedule (see Table 1 for an example) provides an indication of potential price discounts. For instance, Table 1 provides an indication of potential price discounts for five price breakpoints. For buying organizations, this in itself is useful information for supplier comparison and negotiations. A QDF, however, can provide more information. In the next sections, we discuss the academic and practical relevance of a QDF.

1.2. Academic relevance

In this section, we discuss the academic relevance of the paper. First, a QDF is an important ingredient in many research models incorporating quantity discounts. It can be implemented in EOQ models (e.g., [13,14]), inventory ownership problems (e.g., [15,16]), and cooperative purchasing-games (e.g., [17,18]). Surprisingly, very little is available to provide a sound analytical and empirical base for a QDF. In this paper, we aim at filling that void.

Second, it seems that there is a large research gap between price elasticity and demand elasticity. Price elasticity of demand is a concept that is used throughout economics. The concept is based on people doing less of what they want to do as the price of doing it rises [19]. Almost all textbooks discussing the principles of economics or marketing include this topic. In addition, several academic papers describe the price elasticity of different commodities (e.g., [20,21]). Demand elasticity of price is rarely discussed in the literature [22]. Demand elasticity posits that the price of a product or service may drop as the demand for it rises. These price drops can be explained by increased economies of scale and/or decreased transaction costs. A QDF is a measure for the demand elasticity of different commodities.

1.3. Practical relevance

Apart from the academic relevance of a QDF, we claim a QDF has some direct practical relevance as well. First, a QDF provides an indication of potential price discounts on a detailed level. For instance, for Table 1, it provides indications of potential price discounts for every quantity between 1000 and 200,000. A potential price per item for a quantity of 3000 could be 45.7. A potential price per item for a quantity of 125,000 could be 34.6. Thus, a QDF can be used for calculating theoretical discounts for all possible quantities. As discussed earlier, this reduces information deficiencies for buying organizations and enables buying organizations to compare supplier quotes of different suppliers and determine theoretical negotiating spaces.

Second, retrieving information about quantity discounts may also be a useful step in a competitive analysis. One of the steps in a competitive analysis is to compare and analyze products and services from competitors. A QDF enables organizations to easily compare and analyze different quantity discount schedules from competitors.

Third, calculating purchase prices for all possible quantities is also useful in multiple sourcing decisions (e.g., [23,24]). In multiple sourcing decisions, one needs to decide how to allocate a certain quantity between different suppliers. For all possible quantities of the feasible allocations, the quantity discounts need to be calculated and weighed up against other important factors, such as spreading risks.

Fourth, a QDF should not be used to convince suppliers that some sort of curve represents its cost function. But as we demonstrate in this paper, a QDF can be used to calculate theoretical minimum prices. If the theoretical minimum price for a certain supplier is relatively low, then this may indicate a large negotiating space. Thus, a QDF can provide additional information about the prices of a supplier. This additional information about purchase prices is useful for buying organizations, as the purchase price is often an important criterion for purchasing decisions [25].

Finally, the accuracy of the estimation of price savings due to the pooling of demand can be improved by a QDF as well. For instance, typical problems of group purchasing are related to the calculation and allocation of prices and price savings [26]. It is difficult to calculate these savings, because it is often unknown what the group members would have paid individually if they were not involved in the purchasing group. A QDF can be used to solve such problems.

1.4. Research objectives

In this paper, we focus on the buyer's perspective on quantity discounts and we assume that prices and lot sizes are negotiable in most situations. Our analytical objective is to describe a general QDF defined by a limited number of parameters. It should be uncomplicated and practical to derive these parameters from different types of quantity discount schedules (see Section 2). Our empirical objective is to test how well the QDF represents different types of quantity discount schedules found in practice. In addition, we develop and test related hypotheses (see Sections 3 and 4). Our conceptual objective is to develop several practical QDF indicators (see Section 5 for some examples). Based on the QDF indicators, we aim to build a basis for more research to demand elasticity of price. This research line could explore QDF parameters and indicator values of typical products and services. These values could serve as guidelines in purchasing processes (see Section 5 for some examples). As mentioned in Section 1.2, a similar line of research already exists in the price elasticity literature.

2. Modeling quantity discounts

In this section, we focus on our first objective. We first provide the rationale for using a continuous QDF (Section 2.1). Next, we develop a general QDF (Section 2.2 to Section 2.4). Finally, we discuss some issues related to minimum prices (Section 2.5).

2.1. Rationale for a continuous function

From an operations management perspective, quantity discount schedules are developed by a supplier to maximize profits. To maximize profits, suppliers commonly adopt discrete stepwise quantity discount schedules. According to Wang [27], a continuous quantity discount schedule could reduce the supplier's discount benefits.

With a continuous discount schedule, a buying organization obtains more price information, which it can use in negotiations and supplier comparison. In other words, a continuous schedule suits better with the wishes and needs of a buying organization. As such, deriving a continuous quantity discount function based on a discrete quantity discount schedule could be an interesting operation for a buying organization. To achieve our first objective, we therefore choose to develop a continuous QDF (see Fig. 1 for an example of a continuous and a discrete schedule).

As we show in Section 4, a continuous QDF is capable of fitting all discount schedule types mentioned in Section 1.1. Still, it may be a subject for debate whether it is possible to fit discrete stepwise discount schedules with a continuous function. For instance, in a stepwise schedule, a price of 400 could apply to 50–99 items and a price of 390 could apply to 100–199 items. A simple continuous function cannot fit such a stepwise schedule well. However, as most prices are determined through negotiations, such a stepwise schedule usually does not exist in practice (based on [3]). For instance, if a buying organization needs 95 items, then it could order 100 items or it could negotiate a lower price than 400.

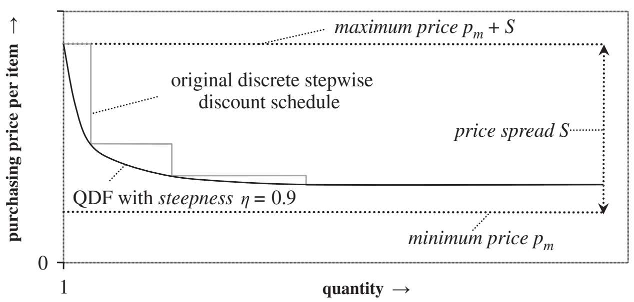

Fig. 1. Example of a quantity discount schedule with a positive steepness.

In supplier selection and during negotiations, a QDF can help a buying organization by reducing the information deficiency regarding the purchase price of 95 items. First, a QDF enables the buying organization to compare different schedules of different suppliers for 95 items. Next, the buying organization can start negotiations with the most competitive supplier. Finally, during negotiations, the buying organization has more information regarding the purchase price due to a QDF. This reduced price information deficiency is valuable to buying organizations [28], as it could lead to lower purchase prices and/or better quality.

An additional disadvantage of stepwise discount schedule functions is that stepwise functions could lead to an anomaly [29,30]. This anomaly concerns the possibility that it can save money by purchasing more items than needed and throwing the surplus away to obtain a certain quantity discount.

For all the reasons above, we use a continuous price function instead of a discrete stepwise function. We do note that lot sizes are sometimes fixed and nonnegotiable. This may be the case when logistical aspects, such as truck capacities, are a limiting condition. In this paper, such discount schedules are not our focus of interest.

We also acknowledge that Munson and Rosenblatt [3] showed some buyers and/or sellers in their study a simple linear schedule. Many of their respondents opposed continuous schedules. Indicated reasons for this are the administrative challenges that such schedules would create. Using the QDF involves some extra operations. Throughout the paper, we argue that these are well worth the effort.

2.2. Defining a QDF

Quantity discount schedules are specific to the item sold and hence, the cost structure of the supplier and the competition from other suppliers. As the competition and cost structures may differ a lot over items, one could argue that a general QDF that can be applied to different items cannot exist. However, even if the discount schedule foundations themselves are quite different, the shape of the schedules might be similar. This would imply that only some parameters vary between quantity discount schedules.

We use a continuous function described by Heijboer [17] to build a QDF with a limited number of parameters. The function Heijboer used can be rewritten

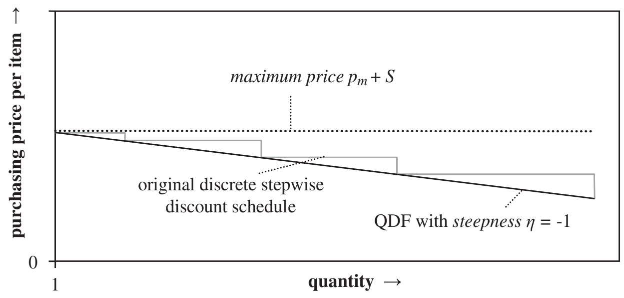

Fig. 2. Example of a quantity discount schedule with a negative steepness.

as $p(q) = p_m + S/\sqrt{q}$. Here p_m is the theoretical minimum price and S scales the function $p(q)$ for quantity q. We build on this function for two reasons. First, it is assumed that a strictly decreasing discount is given with more items being purchased. Second, it is assumed that the total price is increasing with the number of items being purchased. These assumptions hold for most practical situations [2,17]. Still, the function described by Heijboer has one main disadvantage. The function does not fit well with discount schedules with a high incremental curve or an almost linear-like curve. For instance, the function is not well suited to fit discount schedules as shown in Figs. 1 and 2. Therefore, we weaken the first assumption and introduce an extra parameter η. We allow that a non-decreasing discount is given with more items being purchased within a limited range of the discount schedule (see Fig. 2 for an example). The introduced parameter η represents the steepness of a quantity discount function:

$$QDF(q) = \text{fixed amount} \pm \text{variable amount} = p(q) = p_m + S/q^{\eta}, \quad q \geqslant 1. \tag{1}$$

The total purchase cost function is then defined as

$$TC(q) = q \cdot p_m + S/q^{\eta-1}, \quad q \geqslant 1. \tag{2}$$

We introduce the steepness parameter η to be able to find a better fit with different types of quantity discount schedules as mentioned in Section 1.1. The other parameters in the function are p_m and S. Overall, the following restriction applies: $p_m > 0$. The QDF can be scaled and shaped into two categories: positive steepness and negative steepness. In the next two sections, we discuss both categories in more detail.

2.3. A QDF with a positive steepness

Fig. 1 shows an example of a quantity discount schedule with a positive steepness. If a QDF has a positive steepness η, then the parameter p_m represents the theoretical minimum price of the function (for $q = \infty$). The scaling parameter S represents the price spread of a function with a positive steepness. For instance, if the steepness of a function is positive and the price spread is 100, then the difference between the price per item for purchasing 1 item (maximum price) and the price per item for purchasing an infinite number of items (minimum price) is 100. Thus, $p(1) - p(\infty) = p_m + S/1^{\eta} - (p_m + S/\infty^{\eta}) = S$. For a QDF with a positive steepness ($\eta > 0$), it implies $S > 0$.

2.4. A QDF with a negative steepness

Fig. 2 shows an example of a quantity discount schedule with a negative steepness. If a QDF has a negative steepness η, then $p(q)' < 0$ implies $S < 0$. Note that while p_m represents a theoretical $p(\infty)$ for a QDF with a positive steepness, it represents a theoretical $p(0)$ for a QDF with a negative steepness. Again, the maximum price (for $q = 1$) is $p(1) = p_m + S/1^{\eta} = p_m + S$. For a QDF with a negative steepness, it implies $0 > \eta \geqslant -1$ and $S < 0$. Note that $\eta < 1$ would lead to an increasingly decreasing price.

Discount schedules with $\eta \geqslant -1$ are still somewhat peculiar, because extrapolating such functions eventually leads to negative purchase prices. Nevertheless, discount schedules with a negative steepness occur in practice for limited ranges as we show in Section 3.2. For instance, we argue that a supplier may use a linear schedule ($\eta = -1$) for a limited range because of marketing reasons or because of the simplicity of such schedules.

2.5. Defining the range for a QDF with a negative steepness

A theoretical minimum price for a negative steepness QDF does not follow directly from the QDF, as p_m does not represent the minimum price for negative steepness functions. Still, we can calculate the point where the purchase price becomes zero (see also Fig. 3).

$QDF(q) = p_m + S/q^{\eta} = 0$, thus

$$q^* = (-S/p_m)^{1/\eta}. \tag{3}$$

The point q^{**} after which the total purchase costs $q \cdot p(q)$ decrease can be calculated by differentiating Eq. (2). This gives $TC(q)' = -\eta \cdot S/q^{\eta} + S/q^{\eta} + p_m = 0$ (see also Fig. 3). This can be rewritten as $TC(q)' = (1-\eta) \cdot S/q^{\eta} + p_m = 0$ and as $TC(q)' = q^{\eta} + (1-\eta) \cdot S/p_m = 0$. This finally gives

$$q^{**} = ((-1+\eta) \cdot S/p_m)^{1/\eta}. \tag{4}$$

Note that $q^{**} = (1-\eta)^{1/\eta} \cdot q^* \leqslant q^*$.

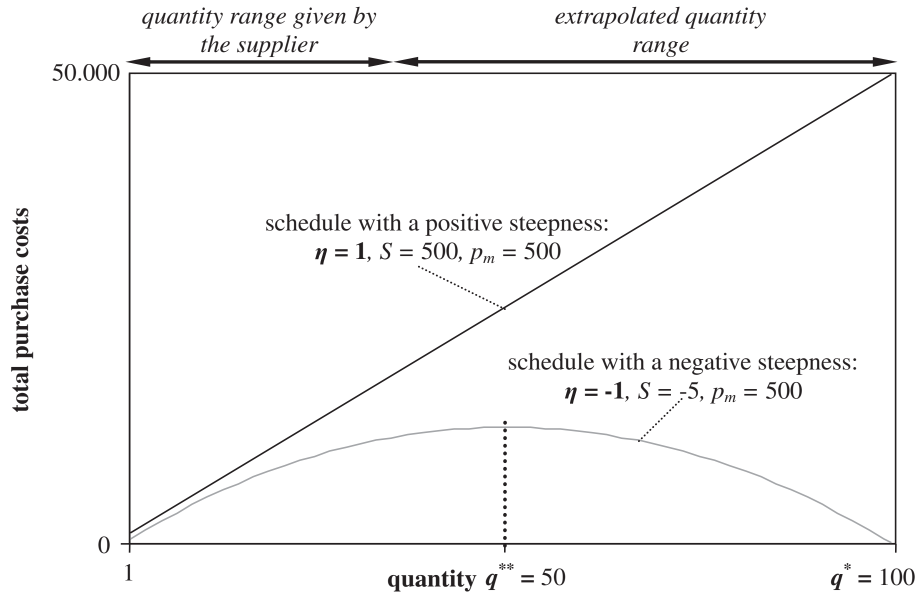

Fig. 3. Total purchase cost analysis examples.

Eq. (4) marks the point until which extrapolation is theoretically possible. We use this equation to calculate a purely theoretical minimum price for a QDF with a negative steepness $p(q^{**}) = p_{\mathrm{m}} + S/((-1+\eta)\cdot S/p_{\mathrm{m}})$. This can be rewritten as $p(q^{**}) = p_{\mathrm{m}} + S \cdot p_{\mathrm{m}}/((-1+\eta)\cdot S)$ and as $p(q^{**}) = p_{\mathrm{m}} + p_{\mathrm{m}}/(-1+\eta)$. This finally gives

$$p(q^{**}) = (\eta \cdot p_{\mathrm{m}})/(-1+\eta). \tag{5}$$

Now, we have the basic elements to develop indicators for supplier selection and negotiation processes in Section 5. But before we do that, we first test the QDF empirically in Sections 3 and 4.

3. Method

In this section, we discuss the methodology which we used to achieve the second objective. First, we describe the data collection and the data set itself. Next, we discuss the procedures used to test how well the QDF represents different quantity discount schedule types.

3.1. Collection of the data set

We tested the fit of the QDF to quantity discount schedules found in academic papers [2,31], actual offers provided to purchasing groups, and internet stores. We found the internet price schedules by search engine searches on the keyword "quantity discounts". None of the products analyzed had exceptional discounts for marketing or logistical reasons. Some product groups occurred more often in our selection than others. But as the properties of discount schedules within product groups can differ significantly, we did not correct for product groups.

All the different discount schedule types mentioned in Section 1 were incorporated in our analysis. After we found all different types mentioned in Section 1, we stopped collecting and analyzing new discount schedules, leaving a total number of 66 quantity discount schedules. We converted all these discount schedules to the same form as shown in Tables 1 and 2. The data collection was carried out at the end of 2004.

3.2. Description of the data set

The basic properties of the data set are shown in Table 3 . The first two columns of the table show properties regarding the number of price breaks. The table shows that there is quite some variety in the number of price breaks. The last two columns of the table show properties regarding the difference between the maximum and minimum price in the discount schedules. The difference in terms of percentage is formulated as the difference between the maximum and minimum price divided by the maximum price. Both the mean difference and maximum difference are high. Thus, quantity discounts can have a major impact on the total purchase costs.

The steepness of the discount schedules ranges from −1.00 to 1.60. The schedules with a positive steepness (40% of the total number of observations) have a mean steepness of 0.58. Schedules with a negative steepness

Table 3
Number of price breaks and differences between maximum and minimum prices

Number of price breaks[a,b]	Value	Difference between the maximum and minimum price given by the supplier	Value
Mean number	4.0	Mean difference	31.3%
Median number	4	Median difference	N/A
Minimum number	2	Minimum difference	1.8%
Maximum number	10	Maximum difference	90.1%
Standard deviation	1.7	Standard deviation	21.6%
Skewness of distribution	1.2	Skewness of distribution	0.8

Note: $n = 66$; total number of prices =327.
[a]Here the break measures are corrected by removing two schedules with an infinite number of breaks.
[b]We did not take schedules into account with only one price break.

Table 4
Example of a quantity discount analysis (see also Fig. 4)

Quantity given by the supplier	Quantity used for the QDF	Price per item given by the supplier [2]	Estimated price per item given the QDF	Difference in percent[a]
1000–4999	1000	50.0	50.0	0.0
5000–9999	5000	45.4	43.9	−0.2
10,000–29,999	10,000	40.9	41.6	0.3
30,000–49,999	30,000	38.1	38.3	−0.5
50,000–199,999	50,000	37.1	36.9	−0.2
200,000 and more	200,000	33.5	33.5	0.0

[a]The R^2 of the estimation is 0.991, the adjusted R^2 is 0.986, and the root mean squared error is 0.718.

(60% of the observations) have a mean steepness of −0.50. We found five linear schedules (8%) with a steepness of exactly minus one and one schedule (2%) with a steepness of exactly one.

3.3. Procedure

We analyzed the discount schedules under the following assumption. We assume that a purchase price for a certain range applies to the lowest quantity in this range. For instance, if a price of 400 applies to 50–99 items and a price of 300 applies to 100–199 items, then we assume that a price of 400 applies to 50 items and a price of 300 applies to 100 items. As mentioned before, the supplier does not quote prices for 51–99 items, but we assume that a lower price than 400 can be obtained through negotiations.

We estimated the three parameters of the QDF with an exact algorithm and several nonlinear least squares algorithms, which are commonly used in curve fitting. We tested the performance of the Gauss–Newton [32], Levenberg–Marquardt [33–35] or trusted region algorithm [36–39] in combination with no method, the bisquare [40] or the least absolute residuals robust fitting method [41].

We found very small differences in the accuracy of the algorithms. All algorithms performed very well. As an exact method exceeds an acceptable calculation time, we chose to use the popular Levenberg–Marquardt curve fitting algorithm in combination with the least absolute residuals method. Without going into detail, we found that this combination most frequently gives the best fit of the QDF with the actual schedules.

The Levenberg–Marquardt algorithm found a theoretical minimum price somewhat smaller than zero (for an infinite quantity) in five exceptional schedules. Here we applied the trusted region algorithm with lower bounds for the theoretical minimum price. See Table 4 and Fig. 4 for an example of how we estimated the QDF parameters of an actual quantity discount schedule with five price breaks [2].

In Table 4, the R^2 used in the quality measurement is calculated based on the breakpoints of the discount schedule. Here we note that the measurement of the quality of approximation is not rigorous for discount schedules with only one breakpoint. For instance, this

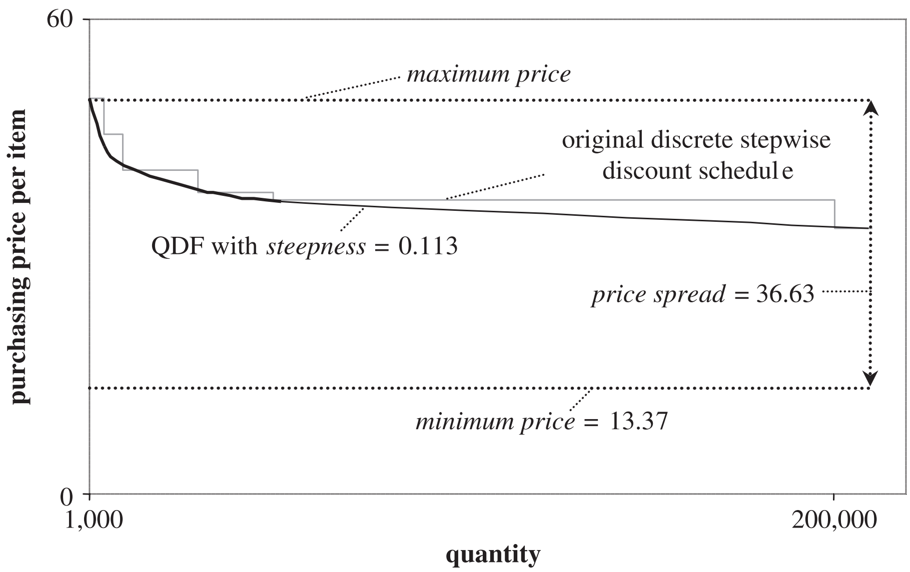

Fig. 4. Example of a quantity discount analysis (see also Table 4).

Table 5
Fit of the QDF with the data set

Measure	Restricted QDF	Less restricted QDF
Average R^2	0.944	0.995
Average adjusted R^2	0.908	0.988
Average root mean squared error	1.602	0.642
Minimum R^2	0.701	0.961
Minimum adjusted R^2	0,403	0.913
Maximum root mean squared error	25.550	4.628

Note: $n = 65$.

is the case if a price of 400 applies to 1–99 items and a price of 390 applies to 100 items and more. If we only look at the breakpoints, then using a steepness of, for instance, minus one will lead to a perfect fit. However, a large error might exist everywhere except at the breakpoint. This is a modeling problem that cannot be resolved by using better fitting algorithms. For this reason, we only consider discount schedules with two or more breakpoints.

4. Results

In this section, we aim to achieve the second objective. We test how well the general QDF represents 66 discount schedules. Hence, we test the following hypothesis:

H_1. *The general QDF fits well with all different types of quantity discount schedules as mentioned in Section 1.1.*

Table 5 shows the fit of the QDF with the data set. The second column shows the fit of a 'restricted' QDF in which negative steepness is not allowed. The third column shows the fit of a 'less restricted' QDF in which both positive and negative steepness are allowed. In other words, for the second column, we fitted the data under the additional restrictions $\eta > 0$ and consequently $S > 0$ (in addition to the general restrictions). For the third column, we fitted the data without these additional restrictions.

The table shows that the less restricted QDF fits very well with different types of discount schedules as mentioned in Section 1.1. The minimum R^2, the average R^2, and the adjusted R^2 are very high. The adjusted R^2 is not significantly lower than the R^2, which normally means that no explanatory variable(s) are missing. Only three schedules analyzed had relatively large differences between the R^2 and the adjusted R^2. These relatively large differences were caused by outlying points. For the restricted QDF, the minimum R^2 and the minimum adjusted R^2 are relatively low. These minima are observed while fitting an almost linear quantity discount schedule. A restricted QDF (with a positive steepness) does not fit very well with such a schedule.

On first sight, H_1 seems to be supported by the data set. However, we have some marginal notes. For one discrete stepwise schedule, we found a QDF with a q^{**} smaller than the maximum quantity given by the supplier. This would mean that the total purchase costs decrease after a certain point within the quantity range given by the supplier. So, the QDF is not a reliable approximate for this schedule. We explain this issue as follows. The discrete stepwise schedule had a clear twist after a certain point. After this point, the stepwise schedule changed form. In other words, the supplier apparently used two different quantity discount functions: one function for the first part of the schedule and another function for the other part of the schedule. Although we did find a high R^2 for the schedule, we removed this exceptional schedule from the analysis.

We also have some notes concerning the extrapolated quantity range (see Fig. 3 for an example of an extrapolated range). While not using lower bounds for the theoretical minimum price, we found five positive steepness schedules in which the theoretical minimum price was somewhat smaller than zero. Calculating all possible quantities within the quantity range given by the supplier is possible for such schedules. However, extrapolating prices for much higher quantities should not be done.

Note that the remarkably high values of R^2 can partly be explained because there were several discount schedules with only two price breaks (i.e., three data points). Nevertheless, for three or more price breaks, we found a very good fit as well. Indeed, the minimum R^2 is very high for the less restricted QDF (see Table 5). Our explanation for the goodness of fit is that most of the discount schedules seem to have a fairly simple underlying basis. In other words, the goodness of fit can be explained by the fact that most quantity discount schedules show a similar decreasing behavior. In addition, the discount schedules usually have no outliers, but follow a more or less logical line.

As discussed in Section 2, discount schedules with a negative steepness are somewhat peculiar. To explain negative steepness, one could argue that there is a relationship between steepness and the difference between the minimum and maximum price of discount schedules provided by the supplier. It could be that negative steepness only exists in discount schedules with a small range regarding the minimum and maximum price. This is because if the range would be larger, then eventually negative prices would occur. So, for large ranges concerning the minimum and maximum price, a positive steepness would normally be found. Thus, we hypothesize:

H_2 *Quantity discount schedules with a positive steepness have a higher difference between minimum and maximum prices given by the supplier than schedules with a negative steepness.*

With an independent samples *t*-test, we tested the correlation between the difference between minimum and maximum prices and negative or positive steepness. We assumed the variances of both groups being unequal (Levene's test $p = 0.002$) and found a significant correlation ($t = -2.173$, $df = 37.060$, $p = 0.036$, 2-tailed), supporting H_2. Price schedules with a positive steepness have a significantly higher difference between minimum and maximum prices (mean difference is 38.0%) than schedules with a negative steepness (mean difference is 25.8%). Therefore, we assume that discount schedules with a negative steepness only provide prices for relatively small quantities. So, discount schedules with a negative steepness should not be used for extrapolating and calculating prices for much larger quantities than given by the supplier.

5. Discussion

In this section, we discuss several implications of the QDF. By doing so, we aim to achieve the final objective. Here our main assumption is that if the QDF fits very well with a quantity discount schedule, then related indicators provide useful insights in the schedule. We describe several QDF indicators and parameters in Table 6. As shown in the final column of the table, the indicators and parameters have several applications (see Sections 1.2 and 1.3 for more discussions of the academic and practical relevance of the QDF and its indicators and parameters).

Regardless of the simple form of quantity discount schedules, there are many differences between QDF parameters and indicators for different supplier offers. A further extension of the possible use of the QDF is related to group purchasing. The concept of group purchasing becomes more interesting if items have a large difference between maximum (4(a)) and minimum prices (4(b)). Group purchasing could have a large impact on the purchase prices of these items. Of course, before purchasing such items in a group, factors such as mutual trust, similar purchasing needs, and

Table 6
Descriptions and applications of QDF indicators and parameters

QDF indicator/ parameter description	Measure	Estimated QDF value	Value given by supplier A [2][a]	QDF application
1. Price for a all possible quantities	$p(q) = p_m + S/q^\eta$ (e.g., $q = 4000$)	44.72	50.00	Calculate prices for decisions related to comparing suppliers and indicating negotiating spaces
2. For instance, a purchasing group needs to calculate price savings. The group has two members. Member A needs 200 items. Member B needs 800 items. They pay a price of 50.00 per item to one supplier for 1000 items:				Calculate and allocate price savings for all possible quantities in multiple sourcing and purchasing group decisions
• Savings of member A: $200 \cdot (p(200) - 50.00)$		1460	N/A	
• Savings of member B: $800 \cdot (p(800) - 50.00)$		748	N/A	
3. Steepness of the QDF	η	0.113	N/A	Characterization of quantity discounts
4(a). Maximum price given the minimum order quantity q_{min}	$p_{max} = p(q_{min})$ (e.g., $q = 1000$)	50.00	50.00	Compare suppliers and indicate negotiating spaces; in this example ($q = 1000$), there is a theoretical negotiable discount range for the price per item between 13.37 and 50.00
4(b). Minimum price	$p_{min} = \begin{cases} p_m = p(\infty), & \eta > 0 \\ p(q^{**}), & \eta < 0 \end{cases}$	13.37	33.50	
5(a). Maximum quantity discount percent	$(p_{max} - p_{min})/p_{max}$	73%	33%	Compare suppliers and indicate negotiating spaces; in this example ($q = 4000$), there is a theoretical negotiable discount range between 11% and 73% for 4000 items
5(b). Minimum quantity discount percent for a certain quantity	$(p_{max} - p(q))/p_{max}$ (e.g., $q = 4000$)	11%	0%	

[a]The values given by supplier A are also shown in Table 1. The original discount schedule in Table 1 does not provide information as shown here in this table.

commitment have to be taken into account as well [42]. Still, knowing which items have large differences between maximum and minimum prices could be useful for purchasing groups.

Another possible application is related to supplier selection and negotiation processes. In some markets, suppliers could have similar methods to create discount schedules. In these markets, most of the schedules of different suppliers are alike, that is, p_m, η, and S are alike. Other markets could show a different behavior. Here suppliers differentiate by offering schedules different from their competitors. In markets with a large price spread between suppliers, it has been shown that it is interesting for buying organizations to consider a large number of suppliers in the selection process [43]. There might be more negotiating space as well in such markets. For these reasons, it would be interesting for buying organizations to know which markets have a large price spread between suppliers.

6. Limitations

Due to the general character of this paper, there are some assumptions and limitations regarding the interpretation of our empirical results. Our main preference is to use a continuous QDF instead of a discrete stepwise QDF. Further case study research among suppliers and buying organizations could be carried out to empirically test this preference.

Although the QDF fits very well with almost all quantity discount schedule types that we found, analytical limitations concern the fact that we only applied a limited number of fitting algorithms. We did this as the focus of this paper is not on finding the best algorithm to fit discount schedules. In further research, even better fitting results could be obtained by using other estimation methods, such as semi-parametric or nonparametric methods. Another limitation concerns the fact that we only considered one QDF type. Other types could be formulated as exponential functions (e.g., $p(q) = p_{\rm m} + S \cdot \exp(-\eta \cdot q)$), functions with more parameters or spline functions.

7. Conclusions

Previous research on quantity discounts has focused on creating discount schedules and on applying discount schedules in new or existing models. We propose a different perspective. We consider the situation in which a buying organization has to deal with a negotiable discrete quantity discount schedule, but does not know the underlying function that was used by the selling organization to determine the schedule. In this paper, we provide an analytical and empirical basis for one continuous quantity discount function (QDF) that can be used to describe this underlying function. The QDF consists of only three parameters, which can be derived easily from almost all kinds of different types of quantity discounts as mentioned in Section 1.1.

In the paper, we show that the QDF fits very well with almost all quantity discount schedule types that we found. The QDF information can be used in supplier selection and negotiation processes. Specific QDF applications range from calculating and allocating savings for purchasing groups to justifying multiple sourcing decisions. In addition, the QDF can be used in research models incorporating quantity discounts. To summarize, we argue that the QDF reduces the price information deficiency for organizations regarding quantity discount schedules provided by suppliers. This reduced information deficiency could lead to lower purchase prices and/or better quality for buying organizations.

Still, using the QDF involves some extra operations. Throughout the paper, we argue that these are well worth the effort. For instance, according to Wang [27], a continuous discount schedule could reduce the supplier's discount benefits. In addition, extra information about purchase prices is useful for buying organizations, as prices are often the main basis for purchasing decisions [25]. Finally, our data set shows that quantity discounts can have a major impact on the total purchase costs. We found a maximum discount percent of 90.1 and a mean discount percent of 31.3.

The discount schedule behavior of suppliers may differ per market and may also develop during time, following the product life cycle. Further research to QDF indicators and parameters could characterize commodity markets and provide several applications. For instance, for some commodity markets, it could be worthwhile for buying organizations to purchase in groups or to consider a large number of suppliers in selection processes. To be able to utilize such QDF applications, a promising market research line to demand elasticity of price could be set up, following the research line to price elasticity of demand.

Acknowledgments

We thank the editor, the associate editor, and three anonymous referees for their constructive and valuable comments. We also thank the referees of WION 2005 for their useful comments and contributions to this paper. Finally, we acknowledge partial financial support from the Dutch Association for Purchasing Management (NEVI).

References

[1] Elmaghraby W, Keskinocak P. Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions. Management Science 2003;49(10):1287–309.

[2] Dolan RJ. Quantity discounts: managerial issues and research opportunities. Marketing Science 1987;6(1):1–22.

[3] Munson C, Rosenblatt MJ. Theories and realities of quantity discounts: an exploratory study. Production and Operations Management 1998;7(4):352–69.

[4] Martin GE. A buyer-independent quantity discount pricing alternative. Omega 1993;21(5):567–72.

[5] Xiao T, Qi X. Price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers. Omega, in press, doi: 10.1016/j.omega.2006.02.008.

[6] Buchanen JM. A theory of monopolistic quantity discounts. Review of Economic Studies 1953;20(3):199–208.

[7] Qi X, Bard JF, Yu G. Supply chain coordination with demand disruptions. Omega 2004;32(4):301–12.

[8] Crowther JF. Rationale for quantity discounts. Harvard Business Review 1967;42(2):121–7.

[9] Nason RW, Della Bitta AJ. The incidence and consumer perceptions of quantity surcharges. Journal of Marketing 1983;59(2):40–53.

[10] Lee HL, Rosenblatt MJ. A generalized quantity discount pricing model to increase suppliers profits. Management Science 1986;32(9):1177–85.

[11] Viswanthan S, Wang Q. Discount pricing decisions in distribution channels with price sensitive demand. European Journal of Operational Research 2003;149(3):571–87.

[12] Boyaci T, Gallego G. Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. International Journal of Production Economics 2002;77:95–111.

[13] Dye CY. Joint pricing and ordering policy for a deteriorating inventory with partial backlogging. Omega 2007;35(2):184–9.

[14] Teunter R. A note on Khouja and Park, optimal lot sizing under continuous price decrease. Omega 2003;33(6):467–71.

[15] Chandra C, Grabis J. Inventory management with variable lead-time dependent procurement cost. Omega, in press, doi: 10.1016/j.omega.2006.04.009.

[16] Day JM, Wright PD, Schoenherr T, Venkataramanan M, Gaudette K. Improving routing and scheduling decisions at a distributor of industrial gasses. Omega, in press, doi: 10.1016/j.omega.2006.11.007.

[17] Heijboer G. Mathematical and statistical analysis of initial purchasing decisions. Ph.D. dissertation, University of Twente, Enschede; 2003.

[18] Schotanus F, Telgen J, Boer L de. Unfair allocation of gains under the equal price allocation method in purchasing groups. European Journal of Operational Research, in press.

[19] Case KE, Fair RC. Principles of economics. 7th ed., Englewood Cliffs, NJ: Prentice-Hall; 2003.

[20] Babbel DF. The price elasticity of demand for whole life insurance. The Journal of Finance 1985;15(1):225–39.

[21] Loderer CL, Cooney JW, Van Drunen LD. The price elasticity of demand for common stock. The Journal of Finance 1991;46(2):621–51.

[22] Ramsay J. Demand competition. Purchasing and Supply Management 1981.

[23] Berger PD, Gerstenfeld A, Zeng AZ. How many suppliers are best? A decision-analysis approach. Omega 2004;32(1): 9–15.

[24] Xia W, Wu Z. Supplier selection with multiple criteria in volume discount environments. Omega 2007;35(5):494–504.

[25] Lehmann DR, O'Shaughnessy J. Differences in attribute importance for different industrial products. Journal of Marketing 1974;38(2):36–42.

[26] Schotanus, F. Horizontal cooperative purchasing. Ph.D. dissertation, University of Twente, Enschede; 2007.

[27] Wang Q. Determination of suppliers' optimal quantity discount schedules with heterogeneous buyers. Naval Research Logistics 2002;49:47–59.

[28] Stigler G. The economics of information. Journal of Political Economy 1961;69(3):213–25.

[29] Arcelus FJ, Rowcroft JE. All-units quantity-freight discounts with disposals. European Journal of Operational Research 1992;57(1):77–88.

[30] Sethi SP. A quantity discount lot size model with disposals. International Journal of Production Research 1984;22(1): 31–9.

[31] Lal R, Staelin R. An approach for developing an optimal discount pricing policy. Management Science 1984;30(12): 1524–39.

[32] Dennis Jr. JE, Schnabel RB. Numerical methods for unconstrained optimization and nonlinear equations. Englewood Cliffs, NJ: Prentice-Hall; 1983.

[33] Levenberg K. A method for the solution of certain problems in least squares. Quarterly Applied Mathematics 1944;2l:164–8.

[34] Marquardt D. An algorithm for least squares estimation of nonlinear parameters. SIAM Journal of Applied Mathematics 1963;11:431–41.

[35] Moré JJ, Sorensen DC. Computing a trust region step. SIAM Journal on Scientific and Statistical Computing 1983;3:553–72.

[36] Branch MA, Coleman TF, Li Y. A subspace, interior, and conjugate gradient method for large-scale bound-constrained minimization problems. SIAM Journal on Scientific Computing 1999;21(1):1–23.

[37] Byrd RH, Schnabel RB, Shultz GA. Approximate solution of the trust region problem by minimization over two-dimensional subspaces. Mathematical Programming 1988;40:247–63.

[38] Coleman TF, Verma A. A preconditioned conjugate gradient approach to linear equality constrained minimization. Computational Optimization and Applications 2001;20(1):61–72.

[39] Steihaug T. The conjugate gradient method and trust regions in large scale optimization. SIAM Journal on Numerical Analysis 1983;20:626–37.

[40] DuMouchel W, O'Brien F. Integrating a robust option into a multiple regression computing environment. In: American Statistical Association Conference Proceedings, Alexandria, VA; 1989.p. 297–301.

[41] Meyer JR, Glauber RR. Investment decisions, economic forecasting and public policy. Cambridge, MA: Harvard Business School Press; 1964.

[42] Schotanus F, Telgen J. Developing a typology of organisational forms of cooperative purchasing. Journal of Purchasing and Supply Management 2007;13(1):53–68.

[43] de Boer L, van Dijkhuizen G, Telgen J. A basis for modelling the costs of supplier selection: the economic tender quantity. The Journal of the Operational Research Society 2000;51(10):1128–35.