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A basis for modelling the costs of supplier selection: the economic tender quantity

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In this paper, we consider the decision a purchaser must make regarding the number of suppliers that should be invited to submit a tender for a certain purchase. On the one hand, the costs of sending invitations to tender, the costs of evaluating the tenders received, and the costs of communicating the outcome to the suppliers that were not selected, all increase with the number of suppliers that are invited to tender. On the other hand, with every additional tender, the purchaser might obtain a better bid. We present a theoretical decision model, the *Economic Tender Quantity* (ETQ) model, that under certain conditions may assist the purchaser in finding the number of tenderers that minimises the expected total costs of the tender process, that is the sum of the expected costs of sending, evaluating and communicating, and the expected bid price. We discuss these conditions and possible extensions, as well as future research opportunities in this area.

Keywords: purchasing; tender evaluation; economic tender quantity

Introduction

Traditionally, operations researchers have given substantial attention to the field of purchasing and supply management, but only within a very narrow scope. Their attention has almost exclusively been focused on what we call operational purchasing decisions, that is, decisions regarding the number of items to be ordered from a supplier and the timing and scheduling of these orders. The reader is referred to Graves *et al*[1] for an extensive overview of Operation Research (OR) models for these decisions. However, operational decisions only constitute one of several areas or levels of purchasing decision making. In general, a distinction can be made between:

- initial purchasing, which involves specification, selection and contracting of suppliers;
- operational purchasing, which involves ordering, receipt and inspection of goods.

Accordingly, purchasing *decisions* could be grouped into initial purchasing decisions, for example the specification of what exactly to buy, which supplier to select, the type of contract to use etcetera and the aforementioned operational purchasing decisions.

Also, as any organisational process, purchasing activities need to be initiated, monitored, terminated, facilitated, organised and so on. In other words: purchasing processes need to be managed, which obviously implies making decisions. The latter type of decisions may be called purchasing management decisions. Summarising, we identify the following two basic levels of purchasing decision making: (1) decisions within actual purchasing processes; and (2) decisions about (clusters of) purchasing processes. A more elaborate breakdown of these two basic levels is shown in Table 1.

So far, initial purchasing decisions as well as decisions about purchasing processes, have gained only modest attention from OR, for example, see Degraeve *et al*[2] and Kingsman.[3] This lack of attention is highly unfortunate given the (increasing) importance of these decisions, though explainable if one considers the neglect (of the importance) of purchasing up to the 70s. Following de Boer[4] this paper attempts to contribute to the development of a *toolbox* for supporting initial purchasing decisions, and decisions about purchasing processes. In particular, we consider a frequent and important decision about purchasing processes, that is the decision regarding the number of suppliers that should be invited to submit a tender.

The relevance and complexity of the tender-decision may be highly dependent on such contingencies as the number of potential suppliers, the familiarity with the

Table 1 Levels and areas of purchasing decision making

Decisions about purchasing processes
decisions about purchasing policy, strategy and organisation
decisions about executing, monitoring and changing purchasing processes
Decisions within purchasing processes
initial purchasing decisions
operational purchasing decisions

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purchase, the dependence on the supplier etc. In some cases, the decision may indeed be very straightforward, for example in case of very few available suppliers. In this respect, the vast amount of literature on long term partnerships with suppliers may lead to the conclusion that competitive tendering is not appropriate in a partnership setting.[5] Furthermore, buyers may not perceive the decision as particularly problematic because often both formal and informal working procedures exist in organisations that prescribe how many suppliers should be asked to submit a tender. Still, we hold forth that there are several good reasons for a careful and systematic analysis of the decision on the number of bids to ask for.

Firstly the use of competitive tendering remains a sound purchasing practice, given it is used in appropriate situations and applied to appropriate types of purchased items and services. In terms of Kraljic's[6] purchasing portfolio approach this typically involves leverage items. For these items, competitive bidding is a valid purchasing strategy and makes the number of suppliers to invite an important one because of the direct substantial savings that may be possible. Empirical research by Bensaou[7] in the automotive industry in the USA and Japan shows that typically 30–60% of the buyer–seller relationships allow for competitive tendering strategies. Versteeg[8] reports on the purchasing practices of General Motors and Volkswagen in Germany, USA as well as Brazil which describe some stunning examples of this, for typical leverage items up to 30 quotations were asked from suppliers all over the world. In addition, Versteeg reports how the buying teams were able to develop rules of thumb regarding the relation between the number of quotations and the resulting improvement of the lowest bid. In this way, savings of 10–20% were achieved. The examples illustrate both the potential relevance as well as the technical feasibility of applying a systematic and quantitative approach towards the tender decision.

Secondly, the informal and formal working procedures in many organisations on the one hand may indeed simplify the tender–decision for the individual buyer, they do not necessarily make other things easier for the buyer. For example, buyers in the Dutch Army are forced to ask quotations from three suppliers, regardless of the purchasing situation or the item under consideration. Although the use of organisational rules that pre-fix repetitive decisions provide organisational efficiency and stability,[9,10] the consequences of structurally following poorly designed rules may be undesirable.

Leenders and Fearon[11] state that the bidderslist, that is the list of invited suppliers, should be numerous enough to ensure a truly competitive price and not be more numerous than necessary because of the costs associated with the tender process. As to these costs, we may distinguish between fixed costs and costs that vary with the number of tenderers. The fixed costs mainly consist of the cost of writing the invitation to tender and defining the evaluation procedure. Obviously, these costs do not depend on the number of tenders, and therefore we exclude these costs from our model. The costs that vary with the number of tenderers may be related to (a combination of) the following activities:

- identifying suppliers and obtaining the information required to send an invitation;
- sending invitations to tender;
- handling possible queries from invited suppliers;
- filing the received tenders;
- reading and evaluating the tenders;
- informing the suppliers of the outcome of the evaluation.

In addition to the costs that are directly related to the purchaser's efforts, the suppliers' efforts should also be considered. As Leenders and Fearon[11] points out: 'it should be remembered, however, that a company is put to expense—at times, a very considerable one—when it submits a bid'. As a consequence, suppliers might lose interest in submitting serious and attractive tenders when they are never or only seldomly awarded the contract. Obviously, this effect becomes stronger as more invitations to tender are sent out. In the long run, it may lead to additional costs for the buying firm, resulting from:

- increased difficulty of finding motivated and willing suppliers, or stated otherwise increased search costs;
- a lower degree of competition because suppliers that still respond to the invitation are perhaps less motivated to produce a very competitive bid if they perceive the chance of success as limited.

Although these additional (indirect) costs are difficult to measure, they may play a role in the decision process, and consequently we recognise these costs in our model. In the remainder of this paper, we will refer to the total of these variable costs, direct as well as indirect, as *tendering costs*. When deciding on the number of suppliers to invite, we assume a trade-off between on the one hand tendering costs, and on the other hand the bidprice resulting from the tender-process, see Figure 1, furthermore, we assume that suppliers receiving an invitation to tender are not aware of the total number of invitations that have been sent out. Also, we assume a simultaneous evaluation of all tenders, that is received bids are not, or cannot be used as a negotiation instrument in order to lower other bids. Obviously, these assumptions may all be violated in practice, thereby making direct practical application of this model subject to certain conditions. We address these and other conditions in later sections.

According to Leenders and Fearon,[11] in current purchasing practice the trade-off between tendering costs and the best offered bid is largely a matter of the purchaser's judgment. The purpose of the model presented in this paper is to make this trade-off more explicit and provide

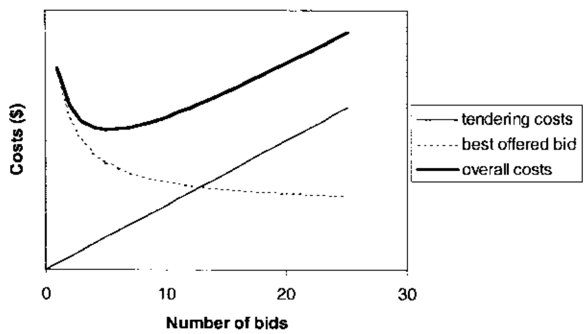

Figure 1 Trade-off between the (direct and indirect) costs of inviting a number of suppliers to submit a tender, and the costs of the best offered bid.

a theoretical basis for developing tools that may enhance decision-making quality and consistency. Based upon a general problem description in the next section, we formally define the ETQ-model in the subsequent section.

Problem description

Consider a list of $N > 0$ potential suppliers or equivalently a set $\mathscr{S} = \{1, \dots, N\}$ of potential suppliers, and let $K_i > 0$ denote the (expected) total tendering costs associated with supplier $i \in \mathscr{S}$. Moreover, let $F_i(\cdot)$ denote the cumulative distribution function of the bidprice offered by supplier $i \in \mathscr{S}$. It is the objective of this paper to determine a set of suppliers $S \subseteq \mathscr{S}$ which should be invited to tender, such that the total (expected) costs are minimised.

As a starting point of our analysis, we denote with X_i the stochastic variable representing the bid offered by supplier $i \in \mathscr{S}$. We assume all variables X_i are independent. By doing so, the best bid $X_S^* = \min\{X_i | i \in S\}$ among a subset of suppliers $S \subseteq \mathscr{S}]$ is also a stochastic variable. Moreover, the corresponding cumulative distribution function $F_s(x) = P\{X_S^* \leqslant x\}$ can be derived in a rather straightforward manner:

$$\begin{aligned} F_s(x) &= P\{\min\{X_i | i \in S\} \leqslant x\} = 1 - P\{\min\{X_i | i \in S\} > x\} \\ &= 1 - P\{\forall i \in S: X_i > x\} = 1 - \prod_{i \in S}(1 - F_i(x)). \end{aligned}$$

here, we assumed for notational convenience—and without loss of generality—that $F_i(\cdot)$ is a continuous function for all $i \in \mathscr{S}$. As a consequence, the expected value of the best bidprice offered by a subset of suppliers $S \subseteq \mathscr{S}$ is given by:

$$E\{X_S^*\} = \int_0^\infty (1 - F_S(x))dx = \int_0^\infty \prod_{i \in S}(1 - F_i(x))dx.$$

Summarising, the total (expected) costs $f(S)$ associated with the selection of a subset $S \subseteq \mathscr{S}$ of suppliers are given by $f(S) = \sum_{i \in S} K_i + \int_0^\infty \prod_{i \in S}(1 - F_i(x))dx$. Since our objective is to determine the set of suppliers that should be invited to submit a tender, such that total (expected) costs are minimised, this leaves us with the following optimisation problem:

$$\min_{S \subseteq S} \left\{ \sum_{i \in S} K_i + \int_0^\infty \prod_{i \in S}(1 - F_i(x))dx \right\}$$

with $S^* = \arg\min\{f(S) | S \subseteq \mathscr{S}\}$, we denote the *economic tender set* (ETS). In the most general situation, the minimisation of $f(S)$ is a complex combinatorial optimisation problem, which is hard to solve to optimality.

In this paper, we will focus on a closely related, but much simpler optimisation problem, in which $K_1 = \cdots = K_N = K$ and $F_1(\cdot) = \cdots = F_N(\cdot) = F(\cdot)$. By doing this, each supplier is fully identical to any other supplier, and it is only the number of selected suppliers n that matters. Moreover, the total (expected) costs associated with the invitation of n suppliers to tender, can be rewritten as $g(n) = K \cdot n + \int_0^\infty (1 - F(x))^n dx$. Hence, the optimisation problem reduces to:

$$\min_{n \in \{1, \dots, N\}} \left\{ K \cdot n + \int_0^\infty (1 - F(x))^n dx \right\}$$

with $n^* = \arg\min\{g(n) | n \in \{1, \dots, N\}\}$, we denote the *economic tender quantity* (ETQ). In the remainder of this paper, we will restrict ourselves to the ETQ problem. Nevertheless, we will return to the ETS problem in a sequel paper, for example see Heijboer and Telgen.[12]

The economic tender quantity (ETQ)

Once again, the objective of the model presented in this paper, is to provide an explicit, theoretical model for the trade-off between tendering costs on the one hand, and the best offered bidprice on the other hand. For small values of N, finding the (ETQ) among the set of possible alternatives $\{1, \dots, N\}$ is a relatively simple problem, which could easily be solved to optimality by using a combination of complete enumeration and numerical integration. Nevertheless, it seems worthwhile to present a more efficient and insightful optimisation procedure, which is based on relaxation of the integer constraint for n:

$$\min_{n > 0} \left\{ K \cdot n + \int_0^\infty (1 - F(x))^n dx \right\}$$

It is easily verified that $g''(n) = \int_0^\infty (1 - F(x))^n \cdot \ln^2(1 - F(x))dx \geqslant 0$ for all $n > 0$, and therefore $g(n)$ is a convex function in n. As a consequence, $g(n)$ has exactly one local minimum $n^* > 0$, and therefore $n^* = \arg\min\{g(n) | n > 0\}$ as well as ETQ $= \arg\min\{g(n) | n \in \{1, \dots, N\} \in \{\lfloor n^* \rfloor, \lceil n^* \rceil\}$ which means that ETQ can easily be determined by calculating n^* and (if that is non-integer) checking the values of $g(n)$ for $\lceil n^* \rceil$ and $\lfloor n^* \rfloor$. Another way to describe ETQ is found by realising that it is optimal to invite n suppliers to tender if and only if $g(n+1) \geqslant g(n)$ and $g(n-1) \geqslant g(n)$, or equivalently:

$$\text{ETQ}\begin{cases} 1 & \text{if } K \geqslant \int_0^\infty F(x)\cdot(1-F(x))dx \\ n \geqslant 2 & \text{if } \int_0^\infty F(x)\cdot(1-F(x))^n\, dx \leqslant K \\ & \leqslant \int_0^\infty F(x)\cdot(1-F(x))^{n-1}\, dx \end{cases}$$

In the remainder of this section, we will investigate the economic tender quantity problem by analysing a selection of well-known distribution functions for the individual bids. The underlying reasoning behind these cases is not only to provide some additional insight into the optimisation problem itself, but also to carry out some sensitivity analysis on each of the (potentially) relevant parameters. Without going into details, we mention here that the following conclusions can be drawn from these examples:

- the economic tender quantity does not depend on the expected value, but *exclusively* on the variance (uncertainty) in the outcomes of the individual bids:
- the economic tender quantity increases *less than proportionally* with the uncertainty in the outcomes of the individual bids.

obviously, these are potentially valuable insights, which are not so well explored in existing literature as well as in purchasing practices. Especially the first of these conclusions may be counter intuitive, and indeed conflicts with some commonly used rules of thumb in practice. After all, many organisations employ guidelines that depend on the expected outcome of the bid, for example, invite three suppliers when the expected outcome is below one million dollars and five suppliers when it is above one million dollars. Our analysis shows that it is the uncertainty in the outcomes of these bids that really matters.

ETQ for uniformly distributed bids

If the outcome of each bid is a uniformly distributed random variable on the interval $[a, b]$, then $F(x) = (x-a)/(b-a)$ for all $a \leqslant x \leqslant b$. Hence, $g(n)$ reduces to the following expression:

$$g(n) = K\cdot n + \int_0^a 1^n\, dx + \int_a^b \left(1 - \frac{x-a}{b-a}\right)^n dx$$
$$= \cdots = K\cdot n + a + \frac{b-a}{n+1}$$

here, the minimum of $g(n)$ is attained at $n^* = \sqrt{(b-a)/K} - 1$. Equivalently, it is optimal to invite n suppliers to tender if and only if $g(n+1) \geqslant g(n)$ and $g(n-1) \geqslant g(n)$, or equivalently:

$$\text{ETQ} = \begin{cases} 1 & \text{if } K \geqslant \dfrac{b-a}{6} \\ n \geqslant 2 & \text{if } \dfrac{b-a}{n\cdot(n+2)} \leqslant K \leqslant \dfrac{b-a}{n\cdot(n+1)} \end{cases}$$

ETQ for Weibull distributed bids

If the outcome of each bid is a Weibull distributed random variable with parameters λ and β, then $F(x) = 1 - e^{-(\lambda\cdot x)^\beta}$ for all $x \geqslant 0$. Hence, $g(n)$ reduces to the following expression:

$$g(n) = K\cdot n + \int_0^\infty e^{-n\cdot(\lambda\cdot x)^\beta}\, dx = \cdots$$
$$= K\cdot n + \frac{1}{n^{1/\beta}\cdot\lambda}\cdot\Gamma\left(\frac{1+\beta}{\beta}\right)$$

In the most general situation, it is impossible to derive an analytical expression for the economic tender quantity n^*. Therefore, we will restrict ourselves to the case $\beta = 1$, or equivalently to exponentially distributed bids. Accordingly, the cost function reduces to $g(n)K\cdot n + (1/n\cdot\lambda)$, and the minimum of $g(n)$ is attained at $n^* = 1/\sqrt{K\cdot\lambda}$. Or, described differently, it is optimal to invite n suppliers to tender if and only if $g(n+1) \geqslant g(n)$ and $g(n-1) \geqslant g(n)$, or equivalently:

$$\text{ETQ} = \begin{cases} 1 & \text{if } K \geqslant \dfrac{1}{2\cdot\lambda} \\ n \geqslant 2 & \text{if } \dfrac{1}{n\cdot(n+1)\cdot\lambda} \leqslant K \leqslant \dfrac{1}{n\cdot(n-1)\cdot\lambda} \end{cases}$$

ETQ for normally distributed bids

If each bid is a normally distributed stochastic variable with mean μ and standard deviation σ, then

$$E\{\min\{X_1,\ldots,X_n\}\} = \mu + \sigma\cdot E\{\min\{(X_1-\mu)/\sigma,\ldots,$$
$$\times (X_n-\mu)/\sigma\}\}$$
$$= \mu + \sigma\cdot E\{\min\{Y_1,\ldots,Y_n\}\},$$

where $Y_i \sim N(0,1)$ is a standard normally distributed random variable. For $n=1$, this yields $E\{\min\{Y_1\}\} = E\{Y_1\} = 0$. For $n=2$, the situation already becomes much more complicated. Using the fact that

$$(\partial/\partial x)P\{\min\{Y_1,Y_2\} \leqslant x\} = (\partial/\partial x)\{1-(1-\Phi(x))^2\}$$
$$= 2\cdot\phi(x)\cdot(1-\Phi(x)),$$

where $\phi(\cdot)$resp. $\Phi(\cdot)$denote the p.d.f. and the c.d.f. of the standard normal distribution, we find:

$$\begin{aligned}
E\{\min\{Y_1, Y_2\}\} &= \int_{-\infty}^{\infty} 2 \cdot x \cdot \phi(x) \cdot (1 - \Phi(x))dx \\
&= 2 \cdot \int_{-\infty}^{\infty} x \cdot \phi(x) \cdot \int_{x}^{\infty} \phi(y)dy\,dx \\
&= 2 \cdot \int_{-\infty}^{\infty} \phi(y) \cdot \int_{-\infty}^{y} x \cdot \phi(x)dx\,dy \\
&= \frac{1}{\pi} \cdot \int_{-\infty}^{\infty} e^{-1/2 \cdot y^2} \cdot \int_{-\infty}^{y} x \cdot e^{-1/2 \cdot x^2}, dx\,dy \\
&= -\frac{1}{\pi} \cdot \int_{-\infty}^{\infty} e^{-y^2} dy = -\frac{1}{\pi} \cdot \sqrt{\pi} \\
&= -\frac{1}{\sqrt{\pi}} \approx -0.564
\end{aligned}$$

For larger values of n, these parameters $\xi(n) = E\{\min\{Y_1, \ldots, Y_n\}\}$ must be determined with the use of numerical integration, or with the use of simulation instead. Moreover, the resulting values can be used over and over again. As an illustrative example, we used Maple to conclude that $\xi(3) \approx -0.864$, $\xi(4) \approx -1.029$, $\xi(5) \approx -1.163$, $\xi(6) \approx -1.267$ and $\xi(7) \approx -1.352$. With this in mind, the cost function $g(n)$ reduces to the following expression:

$$g(n) = K \cdot n + \mu + \sigma \cdot \xi(n)$$

unfortunately, it is impossible to derive an explicit formula for the economic tender quantity n^*. On the other hand, it is optimal to invite n suppliers to tender if and only if $g(n+1) \geqslant g(n)$ and $g(n-1) \geqslant g(n)$, or equivalently:

$$\text{ETQ} = \begin{cases} 1 & \text{if } K \geqslant \dfrac{\sigma}{\sqrt{\pi}} \\ n \geqslant 2 & \text{if } \sigma \cdot \{\xi(n) - \xi(n+1)\} \leqslant K \\ & \leqslant \sigma \cdot \{\xi(n-1) - \xi(n)\} \end{cases}$$

ETQ for triangularly distributed bids

If the outcome of each bid is a triangularly distributed random variable on the interval $[a, b]$ whose peak value is attained at $c \in [a, b]$, see Figure 2, then $F(x) = (x-a)^2/(b-a)/(c-a)$ for all $a \leqslant x \leqslant c$ and $F(x) = 1 - (b-x)^2/(b-a)/(b/c)$ for all $c \leqslant x \leqslant b$. Hence, $g(n)$ reduces to the following expression:

$$\begin{aligned}
g(n) &= K \cdot n + \int_0^a 1^n\,dx + \int_a^c \left(\frac{1-(x-a)^2}{(b-a)\cdot(c-a)} \right)^n dx \\
&\quad + \int_c^b \left(\frac{(b-x)^2}{(b-a)\cdot(b-c)} \right)^n dx \\
&= K \cdot n + a + \int_a^c \sum_{k=0}^{n} \binom{n}{k} \cdot \left(\frac{-(x-a)^2}{(b-a)\cdot(c-a)} \right)^k dx \\
&\quad + \frac{1}{2n+1} \cdot \frac{(b-c)^{n+1}}{(b-a)^n} \\
&= K \cdot n + c + \sum_{k=1}^{n} \binom{n}{k} \cdot \frac{(-1)^k}{2k+1} \cdot \frac{(c-a)^{k+1}}{(b-a)^k} \\
&\quad + \frac{1}{2n+1} \cdot \frac{(b-c)^{n+1}}{(b-a)^n}
\end{aligned}$$

once again, it is not possible to derive an explicit formula for the economic tender quantity n^*, as far as arbitrary values of $c \in [a, b]$ are concerned.

If we will restrict ourselves to the case $c = a + b/2$ we can denote with $\eta(n) = \sum_{k=1}^{n} \binom{n}{k} \cdot (-1)^k / 2k + 1 \cdot 1/2^{k+1} + 1/2n + 1 \cdot 1/2^{n+1}$, and rewrite $g(n)$ as follows:

$$g(n) = K \cdot n + c + (b-a) \cdot \eta(n)$$

as before, these parameters $\eta(n)$ can be determined numerically. Once again, we used Maple to conclude that $\eta(1) = 0, \eta(2) = -\frac{7}{60} \approx -0.117$, $\eta(3) = -\frac{7}{40} \approx -0.175$, $\eta(4) = -\frac{1069}{5040} \approx -0.212$, and $\eta(5) = -\frac{481}{2016} \approx -0.239$. With this in mind, it is optimal to invite n suppliers to

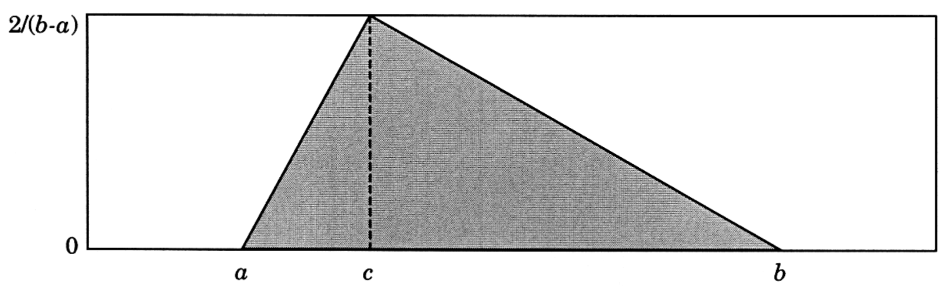

Figure 2 Probability density of a triangular distribution with parameters a, b and c.

tender if and only if $g(n+1) \geqslant g(n)$ and $g(n-1) \geqslant g(n)$, or equivalently:

$$\text{ETQ} = \begin{cases} 1 & \text{if } K \geqslant \frac{7}{60} \cdot (b-a) \\ n \geqslant 2 & \text{if } (b-a) \cdot \{\eta(n) - \eta(n+1)\} \\ & \leqslant K \leqslant (b-a) \cdot \{\eta(n-1) - \eta(n)\} \end{cases}$$

Instead of using the formulas, one could also use a so-called *iso-tender plot*, a graphical instrument that we designed for this purpose. For every one of the bidprice distributions it displays the combinations of the tender costs K and the bid variance that leads to the same ETQ. An example for the triangular distribution is given in Figure 3. Similar plots for other distributions can easily be generated. We suggest to use a log–log scale (as we did) to cover a wider range of possible combinations. Using this plot, a purchaser can easily find the ETQ in any situation as a result of the combination of K and the variance in the bidprices, for example, if variable tender costs (K) are 100 and the difference between the highest and lowest possible bid $(b-a)$ is 1000, the iso-tender plot yields ETQ = 2.

ETQ for generally distributed bids

If the outcome of each bid is a random variable with some general cumulative distribution function $F(\cdot)$, and there is no explicit formula available for $\int_0^\infty (1-F(x))^n\,dx$, one can always use numerical integration or simulation to arrive at approximate results. One might also consider to replace $\int_0^\infty (1-F(x))^n\,dx$ with $F^{-1}(1/n+1)$, an approximation that is frequently used in order statistics, for example, see Arnold and Balakrishnan[13] Galambos[14] and Balakrishnan[15] for details. Here, we only mention that this approximation does not guarantee accurate results, in particular for relatively small values of n. Nevertheless, application of this formula would result in the following approximate expression for $g(n)$:

$$g(n) \approx K \cdot n + F^{-1}\left(\frac{1}{n+1}\right)$$

obviously, it is impossible to derive an analytical expression for the economic tender quantity n^*. On the other hand, it is optimal to invite n suppliers to tender if and only if $g(n+1) \geqslant g(n)$ and $g(n-1) \geqslant g(n)$, or equivalently:

$$\text{ETQ} = \begin{cases} 1 & \text{if } K \geqslant F^{-1}\left(\frac{1}{2}\right) - F^{-1}\left(\frac{1}{3}\right) \\ n \geqslant 2 & \text{if } F^{-1}\left(\frac{1}{n+1}\right) - F^{-1}\left(\frac{1}{n+2}\right) \\ & \leqslant K \leqslant F^{-1}\left(\frac{1}{n}\right) - F^{-1}\left(\frac{1}{n+1}\right) \end{cases}$$

Discussion

The ETQ may serve as a basis for redesign and evaluation of organisational rules and procedures. According to Robert and Mackay,[16] this is of particular relevance in the light of E-procurement as transaction cost structures, including tender costs, are changing dramatically thereby making traditional rules of thumb obsolete.

In this section we identify and address two issues underlying the current ETQ-model. Next, we point out several extensions of the model as well as directions for further research.

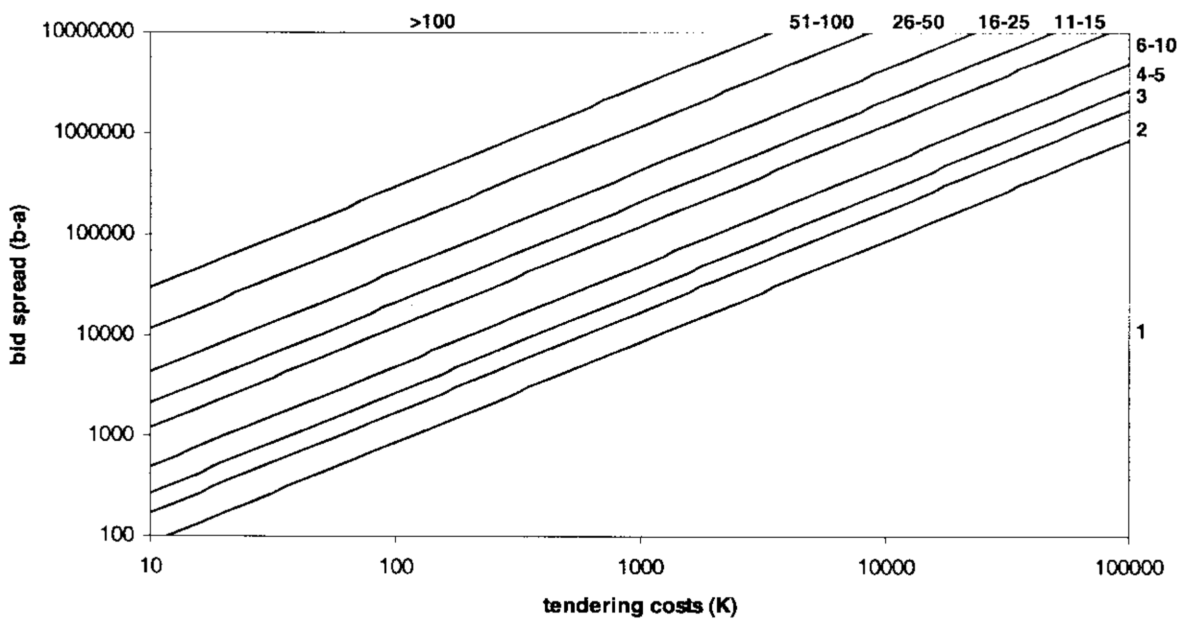

Figure 3 Isotender plot for triangular distributed bids on the interval $[a, b]$ with $c = (a+b)/2$.

The first issue is quite critical as it concerns the assumptions underlying the model. The ETQ-model as presented in this paper uses the following assumptions:

- the received bid-prices X_i are independent of each other;
- buyers are capable of estimating the expected bid-spread;
- the costs of evaluating a bid are equal for all suppliers.

From a perspective of developing tools for direct practical use, these assumptions warrant a closer look at the particular purchasing situations most suitable for applying tools directly derived from the current ETQ-model. We respectively address these assumptions in more detail below.

Assuming independence among bids does not seem very realistic in highly oligopolistic supply markets, for example, the construction industry, where a few suppliers are usually well informed about each other's technologies and bidding-practices. Furthermore, the ability to estimate expected bid-spreads suggests the buyer has some experience with the type of item purchased as well as the supplier market. Hence, in terms of typologies of purchasing situations,[6,17] the first two assumptions are most likely to hold in so-called *modified rebuy* situations that concern the purchase of so-called *leverage-items*. In a modified rebuy the buyer has previous experience with the purchase. *Leverage-items* are characterised by the availability of a high number of suppliers and a prescribed purchasing strategy that is based on frequent bidding-sessions and short-term contracts with suppliers. Under these circumstances, strong interdependence of bids is unlikely and experience for estimating bidspreads will usually be available. In other circumstances, especially in case of *new task* situations and oligopolistic supply markets, the assumptions of independence and availability of bid-spread information will usually not be realistic. In order to support buyers in the latter situations, the basic ETQ-model as presented in this paper will require adaptations that take the specifics of those situations adequately into account. However, we also note that the emergence of Internet and especially its applications in purchasing,[18] could also offer very useful support to the buyer in *new task* and oligopolistic situations. For example, through an internet-consortium a buyer may quite easily obtain very useful information from fellow-buyers and other information sources regarding the expected bidspread for an uncommon purchase. Advertising the call for tender on the web can reduce the impact of local oligopolisric supply markets. The traditional local suppliers can no longer be sure to know all competing firms, let alone direct their bids to them.

The third assumption concerns the costs of evaluating suppliers. In case of *leverage items*, buyers are typically dealing with mature and standard items. Therefore, it seems reasonable to assume that suppliers of these items use production processes and facilities that are rather similar. The latter in turn means that the costs of evaluating suppliers in these respects are probably similar as well, with the possible exception of travel costs. Especially, in case of a global sourcing approach where suppliers may be located all over the world, travel costs may differ substantially. Therefore, when developing practical ETQ-based tools the basic ETQ-model should be extended to take this into account. One possible way of doing this would be by first creating a limited number of supplier categories with each similar characteristics like bidspreads and evaluation costs, for example, local suppliers, European suppliers and suppliers from outside Europe. Obviously, in that ease, a more sophisticated optimisation procedure will be required, still the implications are strictly mathematical.

The second issue concerns the exclusive focus of the current ETQ model on trading off the tendering costs against the price-criterion while supplier selection typically involves a multitude of criteria. However, the logic behind the ETQ model does not imply that price should or is the single criterion in supplier selection. The ETQ model as such does not hinder an evaluation of tenders on multiple criteria (a multi-dimensional variant of the model presented here). In addition, an extended ETQ-model can be developed for sizing the whole supplier selection process from initial screening up to the final selection of one bid. Such an extended ETQ-model would facilitate the trade-off between on the one hand the costs of identifying amid evaluating (potential) additional suppliers (and their ultimate bids) and on the other hand the expected increase in performance on an aggregate of relevant criteria or utility value. This utility value may relate to characteristics of the supplier (distance, turnover, number of employees, width of product range) as well as their tenders (bid price, service level, quality, maintenance). At first glance, the presence of multiple criteria and the possible qualitative (or at least non-financial) nature of some criteria pose a problem in modelling the trade-off between costs and performance on these criteria. However, the areas of goal programming and multiple objective decision making offer many useful techniques especially for these kind of problems. The authors are currently engaged in further research on the application of these techniques in an extended ETQ-model.

Conclusions

In this final section, we draw the following conclusions from our research as presented in this paper.

Firstly, the research offers a new insight in and theoretical modeling of the tender decision: the expected bid-spread rather than the expected bidprice determines the economic tender quantity. As such, this new insight has immediate relevance for today's purchasing practice, which primarily seems to rely on bidprice based rules of thumb for the tender decision. Our research findings suggest these rules of thumb may require careful analysis and should possibly be replaced by bidspread based rules. Obviously, the immediate practical value of the current paper for establishing and actually implementing such new rules depends on the buyer's ability to estimate the bidspread. These and other issues, such as extending the current model to cover multiple criteria indicate useful directions for further development of the ETQ-model.

Secondly, in a broader perspective, the research intends to contribute to opening up a new line of research: a more rigorous mathematical analysis of tactical purchasing decisions and purchasing management decisions. Compared to other (related) functional areas such as operations management and marketing, purchasing still suffers from a lack of such analysis. In that respect, a first step is set towards a decision-theoretic model of purchasing and supply management.

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