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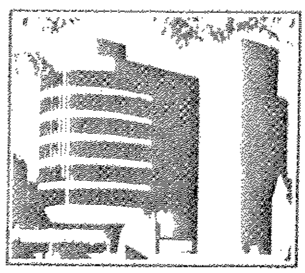

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A NOTE ON A LINEAR PROGRAMMING PROBLEM THAT CYCLED

Jan [illegible] Telgen
Management Science Program
College of Business Administration
The University of Tennessee

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A NOTE ON A LINEAR PROGRAMMING PROBLEM THAT CYCLED[1]

Jan Telgen
Management Science Program[2]
University of Tennessee
Knoxville, TN 37916 U.S.A.

Abstract

Kotiah and Steinberg [1977; 1978] reported on an instance of cycling in a practical linear programming problem using IBM's MPS package. We solved their problem a number of times with the MPSX package, using two different formulations and sometimes removing redundant constraints. Results indicate that both the problem formulation and the removal of redundant constraints may improve considerably the performance of the package and preserve the problem from cycling.

[1]Work performed at Erasmus University Rotterdam, The Netherlands

[2]Visiting NATO Science Fellow for the Netherlands Organization for the Advancement of Pure Research (ZWO)

1. Introduction

Kotiah and Steinberg [1977; 1978] reported on a practical linear programming problem exhibiting the phenomenon known as cycling, when solved with IBM's MPS package. Recently Gass [1979] clarified the situation by distinguishing between classical cycling and computer cycling. Whereas the former is the kind of cycling described in most textbooks, the latter is caused by finite precision computations on real computers. We prefer to use the term package cycling instead of computer cycling, since this kind of cycling is a function of the package used. Gass [1979] already mentioned the fact that the problem of Kotiah and Steinberg [1977] did not cycle when solved with the FMPS/UNIVAC 1108 package. We have similar experience with the MPSX/370 package, but the problem did cycle when solved by an unsophisticated tableau form simplex code.

This note can be seen as an extension of the remarks in Gass [1979]. We report some computational experience with the problem given in Kotiah and Steinberg [1977]. More specifically we solved that problem using a number of different problem formulations involving both equalities and inequalities and removing none, all or some of the redundant constraints. The problem formulations are described in section 2, computational results are given in section 3 and some concluding remarks are made in section 4.

2. Problem Formulation

The problem as given by Kotiah and Steinberg [1979] is stated in the form

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

As Dantzig [1963] noted, an equivalent formulation is

$$\max\ c^T x$$
$$\text{s.t.}\quad Ax \le b$$
$$i^T A x \ge i^T b$$
$$x \ge 0$$

where $i^T = (1,\ldots,1)$. In our experiments we used both the equality (EQ) and the inequality (INEQ) formulation.

The first 11 constraints of the problem have the form

$$\sum_{\substack{j=1\\ j\ne i}}^{20} a_{ij}x_j - a_{ii}x_i = 0 \qquad i = 1,\ldots,10$$

$$\sum_{j=1}^{10} a_{11,j}x_j \qquad = b_{11}$$

with $a_{ij} \ge 0$, $i = 1,\ldots,10$, $j = 1,\ldots,20$; $a_{11,j} > 0$, $j = 1,\ldots,10$ and $b_{11} > 0$. Since $x_j \ge 0$, $j = 1,\ldots,20$, the first 10 constraints imply that if one of the first 10 x_j's is zero, then all of them are. But this is impossible because of the 11th constraint. Therefore, the nonnegativity constraints on x_j, $j = 1,\ldots,10$ are redundant.

In fact using the REDINQ algorithm (Telgen [1979]) it turns out that these are the only redundant constraints in the problem. Removing the redundant constraints (freeing the corresponding variables) simplifies the problem considerably. The reduced problem is denoted as either RED-EQ or RED-INEQ. Finally we applied the REDUCE option of the MPSX problem to both problem formulations. In both cases 7 redundant nonnegativity constraints were removed in 2 matrix passes. Experiments with this problem formulation are indicated by either REDUCE-EQ or REDUCE-INEQ.

3. Experimental Results

All experiments were performed on the IBM 370/168 computer of Technical University Delft, The Netherlands, using the PRIMAL option of the MPSX package. Table 1 summarizes our results.

	# it phase I	# it phase II	# it total	solution
EQ	12	4	16	a
INEQ	33	4	37	b
RED-EQ	11	7	18	a
RED-INEQ	10	7	17	b
REDUCE-EQ	15	6	21	a
REDUCE-INEQ	11	8	19	b

Table 1: Computational Results

The solutions as found in our experiments and those reported by Kotiah and Steinberg [1977] can be compared in table 2.

		a	b	K&S
Objective function value		30.693939	30.693970	30.69392
COL01	BS	2.5069958 -1	2.5069952 -1	2.5070020 -1
COL02	BS	8.1723952 -4	8.1723928 -4	8.1724092 -4
COL03	BS	2.8075077 -2	2.8075036 -2	2.8075056 -2
COL04	BS	1.0003197 -1	1.0003197 -1	1.0003298 -1
COL05	BS	0.0000000 +0	0.0000000 +0	3.9764259 -6
COL06	BS	1.3596838 -4	1.3596818 -4	1.3596827 -4
COL07	BS	4.8447028 -4	4.8447051 -4	4.8447529 -4
COL08	BS	5.2973395 -4	5.2971765 -4	5.2971828 -4
COL09	BS	1.0265466 -3	1.0264998 -3	1.0265075 -3
COL10	BS	1.9547403 -1	1.9547427 -1	1.9547282 -1
COL11	LL	0.0000000 +0	0.0000000 +0	-
COL12	LL	0.0000000 +0	0.0000000 +0	-
COL13	LL	0.0000000 +0	0.0000000 +0	-
COL14	LL	0.0000000 +0	0.0000000 +0	-
COL15	LL	0.0000000 +0	0.0000000 +0	-
COL16	BS	1.9905402 -3	1.9905602 -3	1.9905265 -3
COL17	LL	0.0000000 +0	0.0000000 +0	-
COL18	BS	0.0000000 +0	000000000 +0	-1.0010399 -9
COL19	BS	3.9164126 -3	3.9164536 -3	3.9163120 -3
COL20	BS	1.1977060 -3	1.1977181 -3	1.1977078 -3

Table 2: Solutions; K&S indicates the solution as reported by Kotiah and Steinberg [1977]

It is interesting to note that in both solutions a and b x_5 is equal to 0.0. By the argument in section 2 this is impossible, since the solution would be infeasible. However, inspection of the values of the other variables learns that the value of x_5 is approximately 4×10^{-6}, which may be too small to be printed by MPSX. A similar argument applied to x_{18}.

To conclude the experiments T. C. T. Kotiah and D. I. Steinberg were so friendly to rerun the problem with MPS but without the nonnegativity constraints on the first 10 variables. The problem that cycled first, now was solved in 19 iterations: 14 in phase I, 5 in phase II). The same result was experienced with the unsophisticated code mentioned in the Introduction.

4. Concluding Remarks

The fact that a practical linear programming problem cycled is a rare event, but with the problem of Kotiah and Steinberg [1977] it did not have to come totally unexpected. As Gass [1979] noted, the problem is ill-conditioned and bound to run into numerical problems. This fact was accentuated by the fact that the dual option of MPSX stopped without finding a feasible solution.

Several ways are open to prevent cycling and/or numerical problems. Apart from proper scaling, anti-cycling devices and switching to another mathematical programming package, the one we paid special attention to is the problem formulation. Removing redundant constraints generally reduces the computational effort to solve a problem. In particular removing weakly redundant constraints (Telgen [1979]) reduces the degree of degeneracy of a solution. Therefore in the latter case both classical and package cycling are less probable. It is probably not true that the phenomenon of cycling exists only by virtue of the presence of redundancy. However, I believe that cycling is not possible if the constraint system is a minimal representation of the feasible region (see Telgen [1979]).

Acknowledgement

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