ANTICIPATORY FREIGHT CONSOLIDATION IN INTERMODAL NETWORKS USING APPROXIMATE DYNAMIC PROGRAMMING

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Motivation

Freight consolidation problem

Our solution approach:

- Markov Decision Process model
- Approximate Dynamic Programming

Numerical results:

- 1-way, single terminal, one high-capacity mode
- 2-way, single terminal, one high-capacity mode
- 1-way, multi-terminal, multiple high-capacity modes

What to remember
TRANSPORTATION OF CONTAINERS FROM THE HINTERLAND TO THE DEEP-SEA PORT

*Source of artwork: Combi Terminal Twente B.V. www.ctt-twente.nl

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FREIGHT CONSOLIDATION IN INTERMODAL NETWORKS


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- Freights have different
  - Destination
  - Release day
  - Time-window

**Challenge:** To balance daily and future costs when freights become known gradually over time.
FREIGHT CONSOLIDATION IN INTERMODAL NETWORKS

Today

High-capacity mode
Truck

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DYNAMIC MULTI-PERIOD PLANNING

Today

Tomorrow

Day-after

High-capacity mode
Truck

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DYNAMIC MULTI-PERIOD PLANNING

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Tomorrow

Day-after

High-capacity mode

Truck

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DYNAMIC MULTI-PERIOD PLANNING

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Truck

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**MARKOV DECISION PROCESS (MDP) MODEL**

**STOCHASTIC PROCESS UNDER CONTROL**

**Stochasticity:** Arrival of freights and their characteristics:
- Number of freights \( \mathcal{F} \subseteq \mathbb{Z}^+ \)
- Destinations \( \mathcal{D} \)
- Release day \( \mathcal{R} = \{0, 1, 2, ..., R_{max}\} \)
- Time-window length \( \mathcal{K} = \{0, 1, 2, ..., K_{max}\} \)

**Control:** Freights to consolidate/postpone every day.

**Objective:** Minimize the costs over the planning horizon.
MARKOV DECISION PROCESS (MDP) MODEL
STATE, EXOGENOUS INFORMATION, DECISION, AND STAGES

\[ W_t = \left[ \tilde{F}_{t,d,r,k} \right]_{\forall d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \]

\[ \mathcal{T} = \{0, 1, 2, ..., T_{\text{max}} - 1\} \]

\[ S_t = [F_{t,d,r,k}]_{\forall d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \]

\[ x_t = [x_{t,d,k}]_{\forall d \in \mathcal{D}, k \in \mathcal{K}} \]

\[ \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} x_{t,d,k} \leq Q, \]

High-capacity mode
Truck

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**Transition:** Today’s state depends on (1) yesterday’s state, (2) yesterday’s decision, and (3) the realizations of the random variables:

\[ S_t = S^M (S_{t-1}, x_{t-1}, W_t), \quad \forall t \in \mathcal{T} | t > 0 \]

s.t.

\[ F_{t,d,0,k} = F_{t-1,d,0,k+1} - x_{t-1,d,k+1} + F_{t-1,d,1,k} + \tilde{F}_{t,d,0,k}, \quad |k| < K_{max} \]

\[ F_{t,d,r,k} = F_{t-1,d,r+1,k} + \tilde{F}_{t,d,r,k}, \quad |r| \geq 1 \]

\[ F_{t,d,r,K_{max}} = \tilde{F}_{t,d,r,K_{max}}, \quad \forall d \in D, \ r \in R, \ r + 1 \in R, \ k \in K, \ k + 1 \in K \]

*Time-window length decreases once a freight is released.*
**MARKOV DECISION PROCESS (MDP) MODEL**

**COST DEFINITION AND OBJECTIVE**

**Costs:** Visiting a subset of destinations with the high-capacity mode and using trucks:

\[
C(S_t, x_t) = \sum_{D' \subseteq D} \left( C_{D'} \cdot \prod_{d' \in D'} y_{t,d'} \cdot \prod_{d'' \in D \setminus D'} (1 - y_{t,d''}) \right) + \sum_{d \in D} (B_d \cdot z_{t,d})
\]

s.t.

\[
y_{t,d} = \begin{cases} 
1, & \text{if } \sum_{k \in K} x_{t,d,k} > 0, \forall d \in D \\
0, & \text{otherwise}
\end{cases}
\]

\[
z_{t,d} = F_{t,d,0,0} - x_{t,d,0}, \forall d \in D
\]

**Objective:** Find the policy \( \pi : S_t \rightarrow x_{t}^{\pi} \) that minimizes the expected costs over the horizon.

\[
\min_{\pi \in \Pi} \mathbb{E} \left\{ \sum_{t \in T} C(S_t, x_t^{\pi}) \mid S_0 \right\}
\]
Using Bellman’s recursion (dynamic programming), which balance daily and future costs:

\[
V_t(S_t) = \min_{x_t} \left( C(S_t, x_t) + \mathbb{E} \left[ V_t(S^M_t(S_t, x_t, W_{t+1})) \right] \right)
\]

\[
V_t(S_t) = \min_{x_t} \left( C(S_t, x_t) + \sum_{\omega \in \Omega} (P(W_{t+1} = \omega) \cdot V_t(S^M_t(S_t, x_t, \omega))) \right)
\]
MARKOV DECISION PROCESS (MDP) MODEL

**PROS:** The MDP model outputs a dynamic decision making function that achieves the lowest expected costs over the horizon.

\[ \pi : S_t \rightarrow x_t^\pi \]

**CONS:** The MDP model can only be solved (e.g., using the Bellman’s recursion) for small instances of the problem.

- **However:** the building blocks of the MDP model can be used within the approximate dynamic programming framework to solve the MDP model heuristically for large instances.
APPROXIMATE DYNAMIC PROGRAMMING (ADP)
FRAMEWORK FOR SOLVING LARGE MDP MODELS. ¹

Algorithm 1 Approximate Dynamic Programming Solution Algorithm

Require: \( \mathcal{T}, \mathcal{F}, \mathcal{D}, \mathcal{R}, \mathcal{K}, [C_{D'}]_{D \subseteq D}, B_d, Q, S_0, N \)
1: Initialize \( \hat{V}_t^0, \forall t \in \mathcal{T} \)
2: \( n \leftarrow 1 \)
3: while \( n \leq N \) do
4: \( S_0^n \leftarrow S_0 \)
5: for \( t = 0 \) to \( T^{max} - 1 \) do
6: \( \hat{v}_t^n \leftarrow \min_{x_t^n} \left( C \left( S_t^n, x_t^n \right) + \hat{V}_{t-1}^{n-1} \left( S_{t}^{M,x} \left( S_t^n, x_t^n \right) \right) \right) \)
7: if \( t > 0 \) then
8: \( \hat{V}_{t-1}^n(S_{t-1}^{n,x*}) \leftarrow UV \left( \hat{V}_{t-1}^{n-1}(S_{t-1}^{n,x*}), S_{t-1}^{n,x*}, \hat{v}_t^n \right) \)
9: end if
10: \( x_t^{n*} \leftarrow \arg \min_{x_t^n} \left( C \left( S_t^n, x_t^n \right) + \hat{V}_{t-1}^{n-1} \left( S_{t}^{M,x} \left( S_t^n, x_t^n \right) \right) \right) \)
11: \( S_t^{n,x*} \leftarrow S_{t}^{M,x} \left( S_t^n, x_t^{n*} \right) \)
12: \( W_t^n \leftarrow \text{RandomFrom} \left( \Omega \right) \)
13: \( S_{t+1}^n \leftarrow S^{M} \left( S_t^n, x_t^{n*}, W_t^n \right) \)
14: end for
15: end while
16: return \( \left[ \hat{V}_t^n \right]_{\forall t \in \mathcal{T}} \)

¹ For a comprehensive explanation see Powell (2010) Approximate Dynamic Programming.
The post-decision state $S_{t}^{n,x}$ describes the system “estimating” all possible realizations of the random variables.

$$S_{t}^{n,x} = S_{t+1}^{M,x} (S_{t+1}^{n}, x_{t}^{n}), \forall t \in T$$

The Value Function Approximation (VFA) $\bar{V}_{t}^{n}(S_{t}^{n,x})$ approximates the future costs of the post-decision state:

$$\bar{V}_{t}^{n}(S_{t}^{n,x}) = \mathbb{E} \{ V_{t+1} (S_{t+1}) | S_{t}^{x} \}$$

RESULT: It is not necessary to consider all realizations of the random variables in the new Bellman’s recursion:

$$\hat{v}_{t}^{n} = \min_{x_{t}^{n}} (C (S_{t}^{n}, x_{t}^{n}) + \bar{V}_{t}^{n-1}(S_{t}^{n,x}))$$

$$= \min_{x_{t}^{n}} (C (S_{t}^{n}, x_{t}^{n}) + \bar{V}_{t}^{n-1}(S_{t}^{M,x} (S_{t+1}^{n}, x_{t}^{n})))$$
We use the concept of *basis functions*, or post-decision characteristics, where the value of a post-decision state is a weighted combination of its characteristics:

\[
\hat{V}_t^n(S_t^{n,x}) = \sum_{a \in \mathcal{A}} \left( \phi_a(S_t^{n,x}) \cdot \theta_a \right)
\]

**RESULT:** It is not necessary to consider all post-decision states (and hence states), since there is a function \( \phi_a(S_t^{n,x}) \) that returns its characteristic \( a \in \mathcal{A} \) and the weights \( \theta_a \) depend only on the characteristic considered.
Examples of basis functions or post-decision characteristics:

1. Number of freights that are not yet released for transport, per destination *(i.e. future freights)*.
2. Number of freights that are released for transport and whose due-day is not immediate, per destination *(i.e., may-go freights)*.
3. Binary indicator of a destination having urgent freights *(i.e., must-visit destination)*.
4. Some power function (e.g., \(^2\)) of each state variable *(i.e., non-linear components in costs)*.
After every iteration $n$, we have observed the costs we estimated in the previous, and thus we can improve our approximation:

$$V_{t-1}^n(S_{t-1}^{n,x}) \leftarrow U^V(V_{t-1}^{n-1}(S_{t-1}^{n,x}), S_{t-1}^{n,x}, \hat{v}_t^n), \quad \forall t \in T$$

In our case, $U^V(\cdot)$ updates the weights $\theta^a_n$ using a recursive least squares (LSQ) method for non-stationary data:

$$\theta^a_n = \theta^a_{n-1} - (G^n)^{-1} \phi_a(S_{t}^{n,x}) (V_{t-1}^{n-1}(S_{t-1}^{n,x}) - \hat{v}_t^n)$$

APPROXIMATE DYNAMIC PROGRAMMING (ADP)
A GRAPHICAL REPRESENTATION OF THE CONSTRUCTS AND THE ALGORITHM

\[ V_t^n(S_t^n, x^n) = \mathbb{E}[V_{t+1}(S_{t+1}) | S_t^n, x^n] \]

\[ V_{t+1}(S_{t+1}) \]

\[ x_t^\pi = \arg \min \left( C(S_t, x_t^\pi) + \sum_{\alpha \in A} \theta_{\alpha, t} \phi_\alpha(S_t^n) \right) \]
NUMERICAL EXPERIMENTS
1-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE

Two types of experiments:

A. Convergence of the ADP approach

Convergence of the resulting ADP policy costs to the optimal costs obtained via the Markov model, for different initial states, in small instances. (≈ 3000 states)

B. Performance of the resulting ADP policy

Comparison of the resulting ADP policy costs against the costs of a benchmark heuristic (myopic optimization), for different initial states, in larger instances. (> 8 x 10^{18} states)

For the experimental settings:

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NUMERICAL EXPERIMENTS

CONVERGENCE: 1-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE

Table 2: Performance of the different VFAs

<table>
<thead>
<tr>
<th>Test indicator</th>
<th>Lookup-table</th>
<th>VFA 1</th>
<th>VFA 2</th>
<th>VFA 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>--</td>
<td>0.8897</td>
<td>0.8915</td>
<td>0.8897</td>
</tr>
<tr>
<td>Average difference</td>
<td>7.50%</td>
<td>2.67%</td>
<td>2.45%</td>
<td>2.36%</td>
</tr>
</tbody>
</table>

Table 3: Various sets of features (basis functions of a post-decision state)

<table>
<thead>
<tr>
<th>Feature type</th>
<th>VFA 1</th>
<th>VFA 2</th>
<th>VFA 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>All post-decision state variables (9)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>All post-decision state variables squared (9)</td>
<td>*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Count of MustGo destinations (1)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Number of MustGo freights (1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Product of MustGo destinations and MustGo freights (1)</td>
<td>*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Count of MayGo destinations (1)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Number of MayGo freights (1)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Product of MayGo destinations and MayGo freights (1)</td>
<td>*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Count of Future destinations (1)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Number of Future freights (1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Product of Future destinations and Future freights (1)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Indicator MustGo freights per destination (3)</td>
<td>-</td>
<td>*</td>
<td>-</td>
</tr>
<tr>
<td>Indicator MayGo freights per destination (3)</td>
<td>-</td>
<td>*</td>
<td>-</td>
</tr>
<tr>
<td>Indicator Future freights per destination (3)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Number of all freights (1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Constant (1)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>
NUMERICAL EXPERIMENTS
CONVERGENCE: 1-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE

State 1:
F_{0,2,0,2} = 1

State 2:
F_{0,2,0,0} = 1
F_{0,3,0,0} = 1
F_{0,2,0,1} = 3
F_{0,2,0,2} = 1
## NUMERICAL EXPERIMENTS

**PERFORMANCE: 1-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE**

<table>
<thead>
<tr>
<th>State</th>
<th>Normal Capacity</th>
<th>Large Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heuristic</td>
<td>ADP</td>
</tr>
<tr>
<td>Large A</td>
<td>2962.9</td>
<td>2579.4</td>
</tr>
<tr>
<td>Large B</td>
<td>9687.9</td>
<td>8729.4</td>
</tr>
<tr>
<td>Large C</td>
<td>5937.9</td>
<td>5579.4</td>
</tr>
<tr>
<td>Large D</td>
<td>1737.9</td>
<td>1754.4</td>
</tr>
<tr>
<td>Large E</td>
<td>2162.9</td>
<td>1804.4</td>
</tr>
<tr>
<td>Large F</td>
<td>1362.9</td>
<td>1254.4</td>
</tr>
<tr>
<td>Large G</td>
<td>1362.9</td>
<td>1254.4</td>
</tr>
<tr>
<td>Large H</td>
<td>2187.9</td>
<td>2079.4</td>
</tr>
<tr>
<td>Large I</td>
<td>3585.5</td>
<td>3550.0</td>
</tr>
<tr>
<td>Large J</td>
<td>2537.9</td>
<td>2179.4</td>
</tr>
<tr>
<td>Large K</td>
<td>3462.9</td>
<td>2979.4</td>
</tr>
<tr>
<td>Large L</td>
<td>1778.1</td>
<td>1677.1</td>
</tr>
</tbody>
</table>

Average: -8.3%  
Average: -0.6%

*State A has no urgent freights (F_{0,d,0,0}) and State L has only urgent freights.*
### NUMERICAL EXPERIMENTS

**PERFORMANCE: 1-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE**

<table>
<thead>
<tr>
<th>State</th>
<th># of Freights</th>
<th># of Destinations</th>
<th>Myopic (ILP)</th>
<th>ADP</th>
<th>%Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Low Low Low Low Low Low</td>
<td>2978.85</td>
<td>2608.10</td>
<td>-12.4%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Low Low Medium Low Medium High</td>
<td>5194.60</td>
<td>5146.40</td>
<td>-0.9%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Low Medium Low High High Medium</td>
<td>5396.90</td>
<td>2148.10</td>
<td>-60.2%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Low Medium High High Low High</td>
<td>7941.40</td>
<td>6365.10</td>
<td>-19.8%</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Low High Medium Medium High Low</td>
<td>14730.35</td>
<td>7301.40</td>
<td>-50.4%</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Low High High Medium Medium Medium</td>
<td>12069.95</td>
<td>10206.45</td>
<td>-15.4%</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Medium Low Medium High High Medium</td>
<td>5868.20</td>
<td>5740.30</td>
<td>-2.2%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Medium Low High High Medium Low</td>
<td>13070.95</td>
<td>8839.30</td>
<td>-32.4%</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Medium Medium Low Medium Medium High</td>
<td>6443.05</td>
<td>6348.10</td>
<td>-1.5%</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Medium Medium Medium Medium Low Low</td>
<td>9895.95</td>
<td>8432.55</td>
<td>-14.8%</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Medium High Low Low Low Medium</td>
<td>14567.95</td>
<td>14534.15</td>
<td>-0.2%</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Medium High High Low High High</td>
<td>13764.55</td>
<td>13636.65</td>
<td>-0.9%</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>High Low Low Medium High High</td>
<td>10173.15</td>
<td>10045.25</td>
<td>-1.3%</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>High Low High Medium Low Medium</td>
<td>10429.00</td>
<td>10286.90</td>
<td>-1.4%</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>High Medium Medium Low Medium Medium</td>
<td>10111.50</td>
<td>10033.90</td>
<td>-0.8%</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>High High Low High Medium Low</td>
<td>9680.75</td>
<td>9667.55</td>
<td>-0.1%</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>High High Medium High Low High</td>
<td>9881.80</td>
<td>9872.05</td>
<td>-0.1%</td>
<td></td>
</tr>
</tbody>
</table>

**Average Diff.** -12.6%
NUMERICAL EXPERIMENTS

2-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE

**Extension:** The high-capacity mode travels in round-trips, delivering some freights and picking-up some others:

$$S_t = [(F_{t,d,r,k}, G_{t,d,r,k})]_{\forall d \in D, r \in R, k \in K}$$

$$C(S_t, x_t) = \sum_{D' \subseteq D} \left( C_{D'} \cdot \prod_{d' \in D'} y_{t,d'} \cdot \prod_{d'' \in D \backslash D'} (1 - y_{t,d''}) \right)$$

$$+ \sum_{d \in D} (A_d \cdot z_{t,d})$$

$$+ \sum_{d \in D} \sum_{k \in K} (B_d \cdot (x_{t,d,k}^F + x_{t,d,k}^G))$$

s.t.

$$y_{t,d} = \begin{cases} 1, & \text{if } \sum_{k \in K} (x_{t,d,k}^F + x_{t,d,k}^G) > 0, \forall d \in D \\ 0, & \text{otherwise} \end{cases}$$

$$z_{t,d} = F_{t,d,0,0} - x_{t,d,0}^F + G_{t,d,0,0} - x_{t,d,0}^G, \forall d \in D$$

where

$$F_{t,d,0,k} = F_{t-1,d,0,k+1} - x_{t-1,d,k+1}^F + F_{t-1,d,1,k} + F_{t,d,0,k},$$

$$G_{t,d,0,k} = G_{t-1,d,0,k+1} - x_{t-1,d,k+1}^G + G_{t-1,d,1,k} + G_{t,d,0,k},$$

$$\forall d \in D, \text{ and } k \in K \mid k < K^{max}.$$
NUMERICAL EXPERIMENTS
2-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE

Two types of experiments:

A. *Convergence of the ADP approach* \(^3\)

Convergence of the resulting ADP policy costs to the optimal costs obtained via the Markov model, for different initial states, in small instances. (≈19000 states)

B. *Performance of the resulting ADP policy* \(^3\)

Comparison of the resulting ADP policy costs against the costs of a benchmark heuristic (myopic optimization), for different initial states, in large instances. (>> 8 x 10\(^2\)\(\) states)

For the experimental settings:
NUMERICAL EXPERIMENTS
CONVERGENCE: 2-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE

Figure 2: Accuracy of VFA 3 in Instance $I_2$

Table 4: Performance of the different VFAs in instances $I_1$ and $I_2$

<table>
<thead>
<tr>
<th>Instance</th>
<th>VFA 1 $R^2$</th>
<th>VFA 1 Diff.</th>
<th>VFA 2 $R^2$</th>
<th>VFA 2 Diff.</th>
<th>VFA 3 $R^2$</th>
<th>VFA 3 Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>0.63</td>
<td>5.6%</td>
<td>0.69</td>
<td>5.9%</td>
<td>0.55</td>
<td>5.6%</td>
</tr>
<tr>
<td>$I_2$</td>
<td>0.64</td>
<td>6.6%</td>
<td>0.68</td>
<td>7.7%</td>
<td>0.55</td>
<td>6.8%</td>
</tr>
<tr>
<td>$I_1$-delivery</td>
<td>0.89</td>
<td>16%</td>
<td>0.89</td>
<td>14%</td>
<td>0.89</td>
<td>8%</td>
</tr>
<tr>
<td>$I_2$-delivery</td>
<td>0.89</td>
<td>8%</td>
<td>0.90</td>
<td>7%</td>
<td>0.90</td>
<td>7%</td>
</tr>
</tbody>
</table>
NUMERICAL EXPERIMENTS
CONVERGENCE: 2-WAY, SINGLE TERMINAL, ONE HIGH-CAPACITY MODE

Instances differ in their **distribution of the random variables.**

- C7
- C8
- C9
- C4
- C5
- C6
- C1
- C2
- C3

**Number of released freights**

**Number of destinations**

- \(I_3\) Balanced
- \(I_4\) Unbalanced
- \(I_5\) Least in-advance freights
- \(I_6\) Most in-advance freights
- \(I_7\) Most urgent freights
- \(I_8\) Least urgent freights

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**Table 5: Average cost difference between the ADP policy and the competing policy**

<table>
<thead>
<tr>
<th>Category</th>
<th>I₃</th>
<th>I₄</th>
<th>I₅</th>
<th>I₆</th>
<th>I₇</th>
<th>I₈</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>-5.9%</td>
<td>-8.6%</td>
<td>-9.4%</td>
<td>-5.5%</td>
<td>-0.6%</td>
<td>-5.2%</td>
<td>-5.9%</td>
</tr>
<tr>
<td>C₂</td>
<td>-9.1%</td>
<td>-12.3%</td>
<td>-4.0%</td>
<td>-2.7%</td>
<td>-0.6%</td>
<td>-11.0%</td>
<td>-6.6%</td>
</tr>
<tr>
<td>C₃</td>
<td>-1.9%</td>
<td>-6.7%</td>
<td>-8.2%</td>
<td>-3.1%</td>
<td>1.1%</td>
<td>-7.2%</td>
<td>-4.3%</td>
</tr>
<tr>
<td><strong>C₄</strong></td>
<td><strong>-14.9%</strong></td>
<td><strong>-25.5%</strong></td>
<td><strong>-5.2%</strong></td>
<td><strong>-11.8%</strong></td>
<td><strong>-1.5%</strong></td>
<td><strong>-8.0%</strong></td>
<td><strong>-11.2%</strong></td>
</tr>
<tr>
<td>C₅</td>
<td>-13.1%</td>
<td>-1.9%</td>
<td>-9.7%</td>
<td>-25.9%</td>
<td>-0.4%</td>
<td>-9.7%</td>
<td>-10.4%</td>
</tr>
<tr>
<td>C₆</td>
<td>1.3%</td>
<td>-4.5%</td>
<td>-3.8%</td>
<td>-10.6%</td>
<td>-2.0%</td>
<td>-7.8%</td>
<td>-4.6%</td>
</tr>
<tr>
<td>C₇</td>
<td>-4.4%</td>
<td>-3.7%</td>
<td>-24.2%</td>
<td>-0.1%</td>
<td>-11.0%</td>
<td>-7.3%</td>
<td>-8.4%</td>
</tr>
<tr>
<td>C₈</td>
<td>3.3%</td>
<td>16.7%</td>
<td>2.1%</td>
<td>7.1%</td>
<td>0.6%</td>
<td>3.2%</td>
<td>5.2%</td>
</tr>
<tr>
<td><strong>C₉</strong></td>
<td><strong>-0.9%</strong></td>
<td><strong>2.3%</strong></td>
<td><strong>-4.4%</strong></td>
<td><strong>-11.0%</strong></td>
<td><strong>4.7%</strong></td>
<td><strong>-7.6%</strong></td>
<td><strong>-2.8%</strong></td>
</tr>
<tr>
<td>Average</td>
<td><strong>-5.9%</strong></td>
<td><strong>-8.6%</strong></td>
<td><strong>-7.9%</strong></td>
<td><strong>-8.6%</strong></td>
<td><strong>-1.2%</strong></td>
<td><strong>-7.5%</strong></td>
<td><strong>-6.6%</strong></td>
</tr>
<tr>
<td>Weighted Average</td>
<td><strong>-7.0%</strong></td>
<td><strong>-8.7%</strong></td>
<td><strong>-7.6%</strong></td>
<td><strong>-10.1%</strong></td>
<td><strong>0.3%</strong></td>
<td><strong>-8.0%</strong></td>
<td><strong>-6.9%</strong></td>
</tr>
</tbody>
</table>
NUMERICAL EXPERIMENTS
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Extension: There are multiple terminals with freight, and multiple high-capacity modes:

$$S_t = \left[ F_{i,d,r,k,t} \right]_{\forall i \in \mathcal{N}_t^O \cup \mathcal{N}_t^I, d \in \mathcal{N}_t^D, r \in \mathcal{R}_t, k \in \mathcal{K}_t}$$

Decision becomes more complex due to the dynamic number of intermediate stops:

$$x_t = \left[ x_{i,j,d,k,t} \right]_{\forall (i,j) \in \mathcal{A}_t, d \in \mathcal{N}_t^D, k \in \mathcal{K}_t}$$

s.t.

$$\sum_{j \in \mathcal{N}_t^I \cup \{d\}} x_{i,j,d,k,t} = F_{i,d,0,k,t}, \quad \forall i \in \mathcal{N}_t^O \cup \mathcal{N}_t^I, d \in \mathcal{N}_t^D, k \in \mathcal{K}_t$$

$$x_{i,d,d,L_i,d,t}^A \geq F_{i,d,0,L_i,d,t}, \quad \forall (i,d) \in \mathcal{A}_t^D, k \in \mathcal{K}_t$$

$$x_{i,j,d,k,t} = 0, \quad \forall (i,j) \in \mathcal{A}_t, d \in \mathcal{N}_t^D, k \in \mathcal{K}_t | k < M_{i,j,t} + M_{j,d,t}$$

$$\sum_{d \in \mathcal{N}_t^D} \sum_{k \in \mathcal{K}_t} x_{i,j,d,k,t} \leq Q_{i,j,t}, \quad \forall (i,j) \in \mathcal{A}_t^I$$
**NUMERICAL EXPERIMENTS**

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**Transition:** Multi-period traveling times and stops are captured in the state variables:

\[
S_t = S^M(S_{t-1}, x_{t-1}, W_t)
\]

s.t.

\[
F_{t,i,d,0,k} = F_{t-1,i,d,0,k-1} - \sum_{j \in A_t} x_{t-1,i,j,d,k} + F_{t-1,i,d,1,k} + \tilde{F}_{t,i,d,0,k} \\
\forall i \in N^O_t, d \in N^D_t, k + 1 \in \mathcal{K}_t
\]

\[
F_{t,i,d,0,k} = F_{t-1,i,d,0,k+1} - \sum_{j \in A_t} x_{t-1,i,j,d,k+1} + F_{t-1,i,d,1,k} \\
+ \sum_{j \in A_t | M_{j,i,t} = 1} x_{t-1,j,i,d,k+M_{j,i,t}} \\
\forall i \in N^I_t, d \in N^D_t, k + 1 \in \mathcal{K}_t
\]

\[
F_{t,i,d,0,K^\text{max}_t} = F_{t-1,i,d,1,K^\text{max}_t} + \tilde{F}_{t,i,d,0,K^\text{max}_t} \\
\forall i \in N^O_t, d \in N^D_t
\]

\[
F_{t,i,d,r,k} = F_{t-1,i,d,r+1,k} + \tilde{F}_{t,i,d,r,k} \\
\forall i \in N^O_t, d \in N^D_t, r + 1 \in \mathcal{R}_t | r \geq 1, k \in \mathcal{K}_t
\]

\[
F_{t,i,d,r,k} = F_{t-1,i,d,r+1,k} + \sum_{j \in A_t | M_{j,i,t} = r+1} x_{t-1,j,i,d,k+M_{j,i,t}} \\
\forall i \in N^I_t, d \in N^D_t, r + 1 \in \mathcal{R}'_t | r \geq 1, k \in \mathcal{K}_t
\]

\[
F_{t,i,d,|\mathcal{R}'_t|,k} = \sum_{j \in A_t | M_{j,i,t} = |\mathcal{R}'_t| + 1} x_{t-1,j,i,d,k+M_{j,i,t}} \\
\forall i \in N^I_t, d \in N^D_t, k \in \mathcal{K}_t
\]

\[
F_{t,i,d,R^\text{max}_t,k} = \tilde{F}_{t,i,d,R^\text{max}_t,k} \\
\forall i \in N^O_t, d \in N^D_t, k \in \mathcal{K}_t
\]
NUMERICAL EXPERIMENTS
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One type of experiments:

A. Performance of the resulting ADP policy

Comparison of cost resulting from two different ADP policies against the costs of (1) myopic optimization and (2) sampling, for different initial states, in small instances.

ADP 1: \[ \overline{V}_{t}^{x,n} (S_t^{x,n}) = \sum_{a \in A} \theta^m_{a,t} \phi_a (S_t^{x,n}) \]

ADP 2: \[ \overline{V}_{t}^{x,n} (S_t^{x,n}) = \alpha \sum_{a \in A} \theta^m_{a,t} \phi_a (S_t^{x,n}) + (1 - \alpha) \overline{C}_t^n (S_t^{x,n}) \]

ADP 1 and 2: \[ x_t^\pi = \arg \min \left( C (S_t, x_t^\pi) + \sum_{a \in A} \theta^N_{a,t} \phi_a (S_t^x) \right) \]

For the experimental settings:
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Instances differ in their *distribution of the time-window length.*
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Table 1: Results for Instance $I_1$

<table>
<thead>
<tr>
<th>State</th>
<th>Freights</th>
<th>Benchmark</th>
<th>ADP 1</th>
<th>ADP 2</th>
<th>Sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>$k &lt; 3$</td>
<td>$k \geq 3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>12221</td>
<td>-13.6%</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>14684</td>
<td>-12.8%</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>13042</td>
<td>-13.1%</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>13863</td>
<td>-12.3%</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>13863</td>
<td>-12.0%</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>13863</td>
<td>-10.4%</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>13042</td>
<td>-12.6%</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>12221</td>
<td>-14.7%</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>10579</td>
<td>-14.9%</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>13042</td>
<td>-11.2%</td>
</tr>
</tbody>
</table>

Table 2: Average results for Instance $I_2$ and $I_3$

<table>
<thead>
<tr>
<th>Instance</th>
<th>Benchmark</th>
<th>ADP 1</th>
<th>ADP 2</th>
<th>Sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_2$</td>
<td>11078</td>
<td>-5.2%</td>
<td>-9.8%</td>
<td>-31.2%</td>
</tr>
<tr>
<td>$I_3$</td>
<td>12874</td>
<td>2.9%</td>
<td>0.4%</td>
<td>-3.3%</td>
</tr>
</tbody>
</table>
We propose an MDP model and ADP approach for dynamic and anticipatory decision making in intermodal transportation of freight.

- Through various VFA designs and problem structures, we show that the gap between the ADP and the optimal MDP (or other benchmark heuristics) solutions for is heavily instance/state dependent.

- In all different intermodal settings considered, the ADP approach seemed to perform better with more in-advance freight information and more complex transport networks.
THANKS FOR YOUR ATTENTION!

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