DYNAMIC FREIGHT SELECTION
FOR REDUCING LONG-HAUL
ROUND TRIP COSTS

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Motivation

Problem definition

Solution approaches
  - Markov Model
  - Approximate Dynamic Programming

Preliminary numerical results

Conclusions
CASE INTRODUCTION

PLANNING PROBLEM BASED IN COMBI TERMINAL TWENTE (CTT)

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Transportation of **containers to and from Rotterdam**.

Long-haul of the transportation is done **using barges through Dutch waterways**.

More than 150k containers per year (more than 300 per day).

There are around 30 container terminals in Rotterdam.
Barges spend around two days *waiting and sailing between terminals in Rotterdam* due to changes in appointments (e.g., unavailable berths, deep sea vessel arrival, etc.)
PROBLEM DEFINITION
DYNAMIC FREIGHT SELECTION

Today
Delivery  Pickup

Tomorrow
Delivery  Pickup

Day After
Delivery  Pickup

Intermodal Terminal  High-capacity Transp. Mode

Destinations / Origin  Low-capacity Transp. Mode
PROBLEM DEFINITION
DYNAMIC FREIGHT SELECTION

Today

Destination / Origin

Intermodal Terminal

Delivery

Pickup

Tomorrow

Delivery

Pickup

Day After

Delivery

Pickup

High-capacity Transp. Mode

Low-capacity Transp. Mode

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PROBLEM DEFINITION
DYNAMIC FREIGHT SELECTION

Yesterday

Today

Tomorrow

Delivery
Pickup

Delivery
Pickup

Delivery
Pickup

Intermodal Terminal

Destinations / Origin

High-capacity Transp. Mode

Low-capacity Transp. Mode
PROBLEM DEFINITION
DYNAMIC FREIGHT SELECTION

Yesterday  Today

Delivery  Pickup  Delivery  Pickup

Destinations / Origin

Intermodal Terminal

High-capacity Transp. Mode

Low-capacity Transp. Mode
PROBLEM DEFINITION
DYNAMIC FREIGHT SELECTION

Today  |  Tomorrow  |  Day After
--- | --- | ---
Delivery | Delivery | Delivery
Pickup | Pickup | Pickup

Destinations / Origin

Intermodal Terminal

High-capacity Transp. Mode

Low-capacity Transp. Mode

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### Solution Approach

#### The Optimization Problem

<table>
<thead>
<tr>
<th>Main Parameters</th>
<th>Set</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planning horizon</td>
<td>$\mathcal{T} = {0, 1, 2, ..., T^{max} - 1}$</td>
<td>$\quad$</td>
</tr>
<tr>
<td>Number of delivery freights</td>
<td>$\mathcal{F} \subseteq \mathbb{Z}^+$</td>
<td>$p^F_f \ \forall f \in \mathcal{F}$</td>
</tr>
<tr>
<td>Number of pickup freights</td>
<td>$\mathcal{G} \subseteq \mathbb{Z}^+$</td>
<td>$p^G_g \ \forall g \in \mathcal{G}$</td>
</tr>
<tr>
<td>Last-mile destinations</td>
<td>$\mathcal{D}$</td>
<td>$p^F_d, p^D_d \ \forall d \in \mathcal{D}$</td>
</tr>
<tr>
<td>Release-days</td>
<td>$\mathcal{R} = {0, 1, 2, ..., R^{max}}$</td>
<td>$p^F_R, p^G_R \ \forall r \in \mathcal{R}$</td>
</tr>
<tr>
<td>Time-window lengths</td>
<td>$\mathcal{K} = {0, 1, 2, ..., K^{max}}$</td>
<td>$p^K_k, p^K_k \ \forall k \in \mathcal{K}$</td>
</tr>
</tbody>
</table>

**Decision:** Which freights to consolidate in the high-capacity vehicle each period of the horizon?

**Objective:** To reduce the expected total costs over the horizon.
SOLUTION APPROACH
THE MARKOV MODEL

The \textit{state} $S_t$ is the vector of delivery and pickup freights that are known at a given stage:

$$S_t = [(F_{t,d,r,k}, G_{t,d,r,k})]_{\forall d \in D, r \in R, k \in K}, \forall t \in T$$  \hspace{1cm} (1)

The \textit{arriving information} $W_t$ is the vector of delivery and pickup freights that arrived from outside the system between periods $t - 1$ and $t$:

$$W_t = [(\tilde{F}_{t,d,r,k}, \tilde{G}_{t,d,r,k})]_{\forall d \in D, r \in R, k \in K}, \forall t \in T$$  \hspace{1cm} (2)
THE MARKOV MODEL

The decision $x_t$ is the vector of delivery and pickup freights, which have been released, that are consolidated in the high-capacity vehicle without exceeding its capacity $Q$:

$$x_t = \left[ (x_{t,d,k}^F, x_{t,d,k}^G) \right]_{\forall d \in D, k \in K} \mid S_t, \forall t \in T$$ (3a)

s.t.

$$0 \leq x_{t,d,k}^F \leq F_{t,d,0,k}, \forall d \in D, k \in K$$ (3b)

$$0 \leq x_{t,d,k}^G \leq G_{t,d,0,k}, \forall d \in D, k \in K$$ (3c)

$$\sum_{d \in D} \sum_{k \in K} x_{t,d,k}^F \leq Q, \quad (3d)$$

$$\sum_{d \in D} \sum_{k \in K} x_{t,d,k}^G \leq Q, \quad (3e)$$

$$x_{t,d,k}^F, x_{t,d,k}^G \in \mathbb{Z}^+ \quad (3f)$$
SOLUTION APPROACH
THE MARKOV MODEL

The \textit{transition function} $S^M$ captures the evolution of the system from one period of the horizon to the next one:

\begin{align}
S_t &= S^M (S_{t-1}, x_{t-1}, W_t), \quad \forall t \in \mathcal{T} | t > 0 \\
\text{s.t.} \\
F_{t,d,0,k} &= F_{t-1,d,0,k+1} - x_{t-1,d,k+1} + F_{t-1,d,1,k} + \tilde{F}_{t,d,0,k}, \quad k < K^{max} \\
F_{t,d,r,k} &= F_{t-1,d,r+1,k} + \tilde{F}_{t,d,r,k}, \quad r \geq 1 \\
F_{t,d,r,K^{max}} &= \tilde{F}_{t,d,r,K^{max}}, \\
G_{t,d,0,k} &= G_{t-1,d,0,k+1} - x_{t-1,d,k+1} + G_{t-1,d,1,k} + \tilde{G}_{t,d,0,k}, \quad k < K^{max} \\
G_{t,d,r,k} &= G_{t-1,d,r+1,k} + \tilde{G}_{t,d,r,k}, \quad r \geq 1 \\
G_{t,d,r,K^{max}} &= \tilde{G}_{t,d,r,K^{max}}, \\
\forall d \in \mathcal{D}, \ r \in \mathcal{R}, \ r + 1 \in \mathcal{R}, \ k \in \mathcal{K}, \ k + 1 \in \mathcal{K}
\end{align}
SOLUTION APPROACH
THE MARKOV MODEL

The **cost function** $C(S_t, x_t)$ defines the costs at a given period of the horizon as a function of the state and the decision taken:

$$
C(S_t, x_t) = \sum_{D' \subseteq D} \left( C_{D'} \cdot \prod_{d' \in D'} y_{t,d'} \cdot \prod_{d'' \in D \setminus D'} (1 - y_{t,d''}) \right) + \sum_{d \in D} (B_d \cdot z_{t,d})
$$  \hspace{1cm} (5a)

s.t.

$$
y_{t,d} = \begin{cases} 
1, & \text{if } \sum_{k \in K} \left( x_{t,d,k}^F + x_{t,d,k}^G \right) > 0, \forall d \in D \\
0, & \text{otherwise}
\end{cases}
$$  \hspace{1cm} (5b)

$$
z_{t,d} = F_{t,d,0,0} - x_{t,d,0}^F + G_{t,d,0,0} - x_{t,d,0}^G, \forall d \in D
$$  \hspace{1cm} (5c)
SOLUTION APPROACH
THE MARKOV MODEL

The **objective** is to reduce the total expected costs over the horizon, given an initial state:

\[
\min_{\pi \in \Pi} \mathbb{E} \left\{ \sum_{t \in \mathcal{T}} C(S_t, x_t^\pi) \Bigg| S_0 \right\}
\]

(6)

Using Bellman’s principal of optimality, the Markov model can be solved with the backward recursion:

\[
V_t(S_t) = \min_{x_t} (C(S_t, x_t) + \mathbb{E} \{V_{t+1}(S_{t+1})\}), \forall t \in \mathcal{T}
\]

\[
= \min_{x_t} (C(S_t, x_t) + \mathbb{E} \{V_{t+1}(S^M(S_t, x_t, W_{t+1}))\})
\]

(7)

\[
= \min_{x_t} \left( C(S_t, x_t) + \sum_{\omega \in \Omega} (p_\omega \cdot V_{t+1}(S^M(S_t, x_t, \omega))) \right)
\]
Approximate Dynamic Programming (ADP) is an approach that uses algorithmic manipulations to solve large Markov models.¹


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Algorithm 1 Approximate Dynamic Programming Solution Algorithm

Require: \( \mathcal{T}, \mathcal{F}, \mathcal{G}, \mathcal{D}, \mathcal{R}, \mathcal{K}, [C_D] \forall D \subseteq \mathcal{D}, B_d, Q, S_0, N \)

1: Initialize \( \tilde{V}_t^0, \forall t \in \mathcal{T} \)
2: \( n \leftarrow 1 \)
3: while \( n \leq N \) do
4: \( S_0^n \leftarrow S_0 \)
5: for \( t = 0 \) to \( T^{max} - 1 \) do
6: \( \hat{v}_t^n \leftarrow \min_{x^n} \left( C \left( S_t^n, x_t^n \right) + \tilde{V}_{t-1}^{n-1} \left( S^{M,x} (S_t^n, x_t^n) \right) \right) \)
7: if \( t > 0 \) then
8: \( \tilde{V}_{t-1}^{n}(S_{t-1}^{n, x^*}) \leftarrow \Upsilon (\tilde{V}_{t-1}^{n-1}(S_{t-1}^{n, x^*}), S_{t-1}^{n, x^*}, \hat{v}_t^n) \)
9: end if
10: \( x_t^{n, x^*} \leftarrow \arg \min_{x^n} \left( C \left( S_t^n, x_t^n \right) + \tilde{V}_{t}^{n-1} \left( S^{M,x} (S_t^n, x_t^n) \right) \right) \)
11: \( S_t^{n, x^*} \leftarrow S^{M,x} (S_t^n, x_t^{n, x^*}) \)
12: \( W_t^n \leftarrow \text{RandomFrom} (\Omega) \)
13: \( S_{t+1}^n \leftarrow S^{M} (S_t^n, x_t^{n, x^*}, W_t^n) \)
14: end for
15: end while
16: return \( [\tilde{V}_t^N]_{\forall t \in \mathcal{T}} \)
A post-decision state $S_{t}^{n,x}$ is used as a single estimator for all possible realizations of the random variables.

$$S_{t}^{n,x} = S_{t}^{M,x} (S_{t}^{n}, x_{t}^{n}), \forall t \in T$$

A Value Function Approximation (VFA) $\bar{V}_{t}^{n} (S_{t}^{n,x})$ for the post-decision state is used to capture the future costs:

$$\bar{V}_{t}^{n} (S_{t}^{n,x}) = \mathbb{E} \{V_{t+1} (S_{t+1}) | S_{t}^{x}\}$$

The approximation of Bellman’s equations in ADP:

$$\hat{v}_{t}^{n} = \min_{x_{t}^{n}} (C (S_{t}^{n}, x_{t}^{n}) + \bar{V}_{t}^{n-1} (S_{t}^{n,x}))$$

$$= \min_{x_{t}^{n}} (C (S_{t}^{n}, x_{t}^{n}) + \bar{V}_{t}^{n-1} (S_{t}^{M,x} (S_{t}^{n}, x_{t}^{n})))$$
SOLUTION APPROACH
APPROXIMATE DYNAMIC PROGRAMMING

Use a weighted combination of **state-features** for approximating the value of a state (i.e., VFA function).

\[
\hat{V}_t^n(S_t^{n,x}) = \sum_{a \in \mathcal{A}} (\phi_a(S_t^{n,x}) \cdot \theta_a)
\]

Where \(\theta_a\) is a weight for each feature \(a \in \mathcal{A}\), and \(\phi_a(S_t^{n,x})\) is the value of the particular feature given the post-decision state \(S_t^{n,x}\).

**Assumption:** There are specific characteristics of a post-decision state which significantly influence its future costs!
Examples of state-features:

1. Sum of delivery and pickup freights that are not yet released for transport, per destination \((i.e. \text{ future freights})\).

2. Sum of delivery and pickup freights that are released for transport and whose due-day is not immediate, per destination \((i.e., \text{ may-go freights})\).

3. Binary indicator of a destination having urgent delivery or pickup freights \((i.e., \text{ must-visit destination})\).

4. Some power function (e.g., \(^2\)) of each state variable \((i.e., \text{ non-linear components in costs})\).
The VFA must be updated after every iteration $n$ with a function $U^V(\cdot)$.

$$V_{i-1}^n(S_{i-1}^{n,x}) \leftarrow U^V(V_{i-1}^{n-1}(S_{i-1}^{n,x}), S_{i-1}^{n,x}, \hat{w}_t^n), \forall t \in T$$

In our case, the weights are updated through a recursive least squares method for non-stationary data:

$$\theta_a^n = \theta_a^{n-1} - (G^m)^{-1} \phi_a(S_t^{m,x}) (V_{i-1}^{n-1}(S_{i-1}^{m,x}) - \hat{w}_t^n)$$

Two preliminary experiments:

1. **Convergence** Test (one freight - 19,323 states)
2. **Policy-performance** Test (two freights - 8,317,456 states)

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**Table: Input Parameter and Values**

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freights arriving per day ((F, G))</td>
<td>({1, 2})</td>
</tr>
<tr>
<td>Probability (p^F, p^G)</td>
<td>({0.8, 0.2})</td>
</tr>
<tr>
<td>Destinations ((D))</td>
<td>({1, 2, 3})</td>
</tr>
<tr>
<td>Probability (p^FD, p^GD)</td>
<td>({0.1, 0.8, 0.1})</td>
</tr>
<tr>
<td>Release-days ((R))</td>
<td>({0})</td>
</tr>
<tr>
<td>Probability (p^FR, p^GR)</td>
<td>({1})</td>
</tr>
<tr>
<td>Time-window lengths ((K))</td>
<td>({0, 1, 2})</td>
</tr>
<tr>
<td>Probability (p^K, p^GK)</td>
<td>({0.2, 0.3, 0.5})</td>
</tr>
<tr>
<td>Planning horizon ((T_{max}))</td>
<td>5</td>
</tr>
<tr>
<td>Long-haul capacity ((Q))</td>
<td>2</td>
</tr>
</tbody>
</table>
PRELIMINARY NUMERICAL RESULTS
ADP FOR THE DYNAMIC FREIGHT SELECTION IN ROUND-TRIPS

Convergence Test:

![Graph showing convergence test for ADP and DP algorithms](image)

- ADP Algorithm
- y=x

- DP
- ADP

Value of a State (ADP)
Value of a State (DP)

Iteration

Value of a State
PRELIMINARY NUMERICAL RESULTS
ADP FOR THE DYNAMIC FREIGHT SELECTION IN ROUND-TRIPS

Policy-performance Test:

**Smaller Instance**
- Markov Model
- ADP Algorithm
- Benchmark Heuristic

**Larger Instance**
- ADP Algorithm
- Benchmark Heuristic
CONCLUSIONS

- “Looking” into future freight consolidation, through a Markov model, pays off when costs depend on the combination of destinations and the transport capacity is limited.

- Approximate Dynamic Programming (ADP) is an appropriate method for solving large Markov models as long as future costs can be estimated accurately.

- ADP can be used to obtain managerial insights in how destination-combination costs and time-windows influence overall performance.
THANKS FOR YOUR ATTENTION!

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