LONG-HAUL FREIGHT SELECTION FOR LAST-MILE COST REDUCTION

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Case introduction

The long-haul freight selection problem

Solution approaches

- Mixed-Integer Linear Programming
- Dynamic Programming
- Approximate Dynamic Programming

Our approach

What to remember
Core business is the transportation of *containers to and from Rotterdam.*

Long-haul of the transportation is done *using barges through Dutch waterways.*

More than 150k containers per year (more than 300 per day).

There are 30 terminals regularly visited in Rotterdam.
Barges spend around two days *waiting and sailing between terminals in Rotterdam* due to changes in appointments (e.g., unavailable berths, deep sea vessel arrival, etc.)
THE LONG-HAUL FREIGHT SELECTION PROBLEM

Today
THE LONG-HAUL FREIGHT SELECTION PROBLEM

Today | Tomorrow | Day after

- Grids with colored squares

Diagram with nodes labeled 'a' and 'b'

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THE LONG-HAUL FREIGHT SELECTION PROBLEM

Today

Tomorrow

Day after

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THE LONG-HAUL FREIGHT SELECTION PROBLEM

Today

Tomorrow

Day after

\[ a \rightarrow b \]

\[ \text{Diagram showing connections between nodes} \]
THE LONG-HAUL FREIGHT SELECTION PROBLEM

Today

Tomorrow

Day after

a

b

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THE LONG-HAUL FREIGHT SELECTION PROBLEM

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Tomorrow

Day after

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What are our problem characteristics?

- Discrete and finite planning horizon $t \in \mathcal{T}$
- Set of freights $f \in \mathcal{F}$
  - Release-date $r \in \mathcal{R}$
  - Due-date $k \in \mathcal{K}$
  - Destination $d \in \mathcal{D}$
- Cost per subset of destinations via barge $C_{\mathcal{D}'} \in \mathbb{R}^+, \forall \mathcal{D}' \subseteq \mathcal{D}$
- Cost of direct transport via truck $B_d \in \mathbb{R}^+, \forall d \in \mathcal{D}$
- Capacity of the barge $Q \in \mathbb{N}$
Assumptions and constraints:

- One barge sails per time unit (decision moment)
- Barge has a maximum capacity.
- Each freight consists of one unit (i.e., container).
- Each freight must be transported after its release-date and before its due-date.
- There is an unlimited number of trucks for the direct option.
MIXED-INTEGER LINEAR PROGRAMMING MODEL

\[
\begin{align*}
\text{min } & \quad Z \\
\text{s.t.} & \quad Z = \sum_{t \in T} \sum_{D' \subseteq D} \left( C_{D'} \cdot \prod_{d' \in D'} y_{d',t} \cdot \prod_{d'' \in D_t \setminus D'} (1 - y_{d'',t}) \right) + \sum_{t \in T} \sum_{f \in F} \sum_{d \in D} (B_d \cdot L_{f,d} \cdot v_{f,t}) \\
& \quad \sum_{f \in F} x_{f,t} \leq Q, \forall t \in T \\
& \quad \sum_{t \in T} (x_{f,t} + v_{f,t}) = 0, \forall f \in F \\
& \quad \sum_{t \in T} (x_{f,t} + v_{f,t}) = 1, \forall f \in F \\
& \quad \sum_{f \in F} (L_{f,d} \cdot x_{f,t}) - M_d \cdot y_{d,t} \leq 0, \forall d \in D_t, t \in T \\
& \quad D_t = \{d : L_{f,d} = 1 \text{ and } R_f \leq t, \forall d \in D, f \in F\}, \forall t \in T \\
& \quad x_{f,t} \in \{0, 1\}, \forall f \in F, t \in T \\
& \quad y_{d,t} \in \{0, 1\}, \forall d \in D_t, t \in T \\
& \quad v_{f,t} \in \{0, 1\}, \forall f \in F, t \in T
\end{align*}
\]

Non-linear!

Freight goes by barge ➔
Destination is visited ➔
Freight goes by truck ➔
MIXED-INTEGER LINEAR PROGRAMMING MODEL

- The objective can be linearized as follows:

\[
 Z = \sum_{t \in T} \sum_{D' \subseteq D} \left( C_{D'} \cdot w_{D',t} \right) + \sum_{t \in T} \sum_{f \in F} \sum_{d \in D} \left( B_d \cdot L_{f,d} \cdot v_{f,t} \right) \quad (10)
\]

\[
 w_{D',t} - y_{d',t} \leq 0, \quad \forall D' \subseteq D_t, d' \in D'_t, t \in T \quad (11)
\]

\[
 w_{D',t} + y_{d',t} \leq 1, \quad \forall D' \subseteq D_t, d' \in D_t \setminus D', t \in T \quad (12)
\]

\[
 w_{D',t} + (|D'| - 1) - \sum_{d' \in D'} y_{d',t} + \sum_{d'' \in D' \setminus D_t} y_{d'',t} \geq 0, \quad \forall D' \subseteq D_t, t \in T \quad (13)
\]

\[
 w_{D',t} \in [0, 1], \quad \forall D' \subseteq D_t, t \in T \quad (14)
\]

Subset of destinations is visited

All subsets of the set of destinations!

- MILP does not include uncertainty in arrival of freights!
DYNAMIC PROGRAMMING MODEL

- One **stage** for each time period $t \in \mathcal{T}$.

*Model’s Uncertainty in arrivals between stages:*
- Number of freights $F : P(F = f), \ f \in \mathcal{F}$
- Release-day of each freight $R : P(R = r), \ r \in \mathcal{R}$
- Due-day of each freight $K : P(K = k), \ k \in \mathcal{K}$
- Destination of each freight $D : P(D = d), \ d \in \mathcal{D}$

All random variables are captured in an **exogenous information vector** $W_t$:

$$W_t = \left[ \widetilde{F}_{t,d,r,k} \right]_{\forall d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}}, \ \forall t \in \mathcal{T}$$
Model’s states and decisions:

- A state $S_t$ is the collection of freights, and their characteristics, that are known at a given stage:

  $$S_t = [F_{t,d,r,k}]_{\forall d \in D, r \in R, k \in K}, \forall t \in T.$$

- A decision $x_t$ is the collection of freights, which have been released, that we are going to transport via barge at a given stage:

  $$x_t = [x_{t,d,k}]_{\forall d \in D, k \in K}, \forall t \in T$$

  s.t.

  $$0 \leq x_{t,d,k} \leq F_{t,d,0,k}, \forall t \in T, d \in D, k \in K$$

  $$\sum_{d \in D} \sum_{k \in K} x_{t,d,k} \leq Q, \forall t \in T$$
Model’s state transition between stages:

- A transition function $S^M$ captures the evolution of the system over the stages as a result of the decisions and the stochastic arrivals.

\[
S_t = S^M (S_{t-1}, x_{t-1}, W_t), \ \forall t \in \mathcal{T}
\]

s.t.

\[
F_{t,d,0,0} = F_{t-1,d,1,1} + F_{t,d,0,0},
\]

\[
\forall t \in \mathcal{T}, d \in \mathcal{D}
\]

\[
F_{t,d,0,k} = F_{t-1,d,0,k+1} - x_{t-1,d,k+1} + F_{t-1,d,1,k+1} + F_{t,d,0,k},
\]

\[
\forall t \in \mathcal{T}, d \in \mathcal{D}, k \in \mathcal{K} \setminus \{0, |\mathcal{K}|\}
\]

\[
F_{t,d,r,k} = F_{t-1,d,r+1,k+1} + F_{t,d,r,k},
\]

\[
\forall t \in \mathcal{T}, d \in \mathcal{D}, r \in \mathcal{R} \setminus \{0\}, k \in \mathcal{K} \setminus \{0, |\mathcal{K}|\}
\]
DYNAMIC PROGRAMMING MODEL

- A small example on how the transition function works:

<table>
<thead>
<tr>
<th>$S_{t-1,d}$</th>
<th>$F_{t-1,d,0,0}$</th>
<th>$F_{t-1,d,0,1}$</th>
<th>$F_{t-1,d,0,2}$</th>
<th>$F_{t-1,d,1,2}$</th>
<th>$F_{t-1,d,1,3}$</th>
<th>$F_{t-1,d,2,3}$</th>
<th>$F_{t-1,d,3,4}$</th>
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<tr>
<th>$x_{t-1,d}$</th>
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<th>$F_{t-1,d,0,1}$</th>
<th>$F_{t-1,d,0,2}$</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S^x_{t-1,d}$</th>
<th>$F_{t-1,d,0,0}$</th>
<th>$F_{t-1,d,0,1}$</th>
<th>$F_{t-1,d,0,2}$</th>
<th>$F_{t-1,d,1,2}$</th>
<th>$F_{t-1,d,1,3}$</th>
<th>$F_{t-1,d,2,3}$</th>
<th>$F_{t-1,d,2,4}$</th>
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<tbody>
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<td>0</td>
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<td></td>
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</table>

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<th>$F_{t,d,0,1}$</th>
<th>$F_{t,d,0,2}$</th>
<th>$F_{t,d,1,2}$</th>
<th>$F_{t,d,1,3}$</th>
<th>$F_{t,d,2,3}$</th>
<th>$F_{t,d,2,4}$</th>
</tr>
</thead>
<tbody>
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<td>2</td>
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<table>
<thead>
<tr>
<th>$S_{t,d}$</th>
<th>$F_{t-1,d,0,0}$</th>
<th>$F_{t-1,d,0,1}$</th>
<th>$F_{t-1,d,0,2}$</th>
<th>$F_{t-1,d,1,2}$</th>
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<td>3</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
DYNAMIC PROGRAMMING MODEL

Model’s costs and objective:

\[
C(S_t, x_t) = \sum_{D' \subseteq D} \left( C_{D'} \cdot \prod_{d' \in D'} y_{d',t} \cdot \prod_{d'' \in D \setminus D'} (1 - y_{d'',t}) \right) \\
+ \sum_{d \in D} (B_d \cdot (F_{t,d,0,0} - x_{t,d,0}))
\]

s.t.

\[
y_{d,t} = \begin{cases} 
1, & \text{if } \sum_{k \in K} x_{t,d,k} > 0 \\
0, & \text{otherwise}
\end{cases}, \forall t \in T, d \in D
\]

- The objective is to find a policy \( \pi \) that minimizes the expected costs over the planning horizon given an initial state.

\[
\min_{\pi \in \Pi} \mathbb{E} \left\{ \sum_{t \in T} C(S_t, x^\pi_t) | S_0 \right\}
\]
How to find this policy?
Using Bellman’s principle of optimality and backward induction:

\[ V_t(S_t) = \min_{x_t} \left( C(S_t, x_t) + \mathbb{E} \left\{ V_t(S^M_t, x_t, W_{t+1}) \right\} \right) \]

\[ V_t(S_t) = \min_{x_t} \left( C(S_t, x_t) + \sum_{\omega \in \Omega} \left( P(W_{t+1} = \omega) \cdot V_t(S^M_t, x_t, \omega) \right) \right) \]

\[ \omega = \left[ \hat{F}_{t,d,r,k} \right]_{\forall d \in D, r \in R, k \in K} \]

\[ P(W_t = \omega) = \frac{\left( \sum_{d \in D, r \in R, k \in K} \hat{F}_{t,d,r,k} \right)!}{\prod_{d \in D, r \in R, k \in K} (\hat{F}_{t,d,r,k}!)} \]

All possible realizations of the random variables!
All possible decisions in a state!
All possible states!

\[ \prod_{d \in D, r \in R, k \in K} \left( P(F = \sum_{d \in D, r \in R, k \in K} \hat{F}_{t,d,r,k}) \cdot [P(D = d) \cdot P(R = r) \cdot P(K = k)]^{\hat{F}_{t,d,r,k}} \right) \]
Approximate Dynamic Programming Model

Same cost and transition function as the DP model, however:

- A **post-decision state** \( S_t^x \) is used as a single estimator for all possible realization of the random variables.

\[
S_t^x = S_{t-1}^M (S_{t-1}, x_{t-1})
\]

- An **approximated value function** \( V_t^x (S_t^x) \) for the post-decision state to capture the future costs:

\[
V_t (S_t) = \min_{x_t} (C (S_t, x_t) + \mathbb{E} \{ V_t (S_{t+1}^M (S_t, x_t, W_{t+1})) \})
\]

\[
V_t (S_t) = \min_{x_t} (C (S_t, x_t) + V_t^x (S_t^x))
\]
How to find the best decision for an initial state?¹

- By stepping forward in time:
  1. Find best decision for current state with current estimated value function of post-decision states.
  2. Update the estimated value of the previous post-decision state.
  3. Sample all exogenous information (in a Monte Carlo fashion), and get the new state.
- Repeat for a number of iterations until convergence.

¹ For the comprehensive algorithm see Powell (2010) Approximate Dynamic Programming.
Comparison between the DP and ADP (with lookup tables) models, for a small example with 7k states.

In 6% of the states the ADP decisions differ from optimal.
OUR APPROACH

- Based on the ADP model with post-decision state approximation.
- Use **basis functions** for approximating the value of a state.

*Basis functions are specific features of a state which have a significant impact on its value.*

\[
\bar{V}_t^n (S^x_t) = \sum_{f \in F} \theta_f^n \phi_f (S^x_t), \quad \forall t \in T.
\]

- Where \( \theta_f^n \) is a weight for each feature \( f \in F \), and \( \phi_f (S^x_t) \) is the value of the particular feature given the post-decision state \( S^x_t \).
With *regression analysis* we investigate which features have a significant impact on the value of a state.

In an example instance (with approx. 78k states) the following choice of basis functions explain a large part of the variance in the computed values with the DP model ($R^2 = 0.94$):

- All state variables.
- Number of different destinations of all freights that have the same release-day (for each release-day).
- Sum of all freights that that have the same release-day (for each release-day).
WHAT TO REMEMBER

- Selecting which freights to consolidate today while considering consolidation of freights in future days is important when costs depend on the combination of freights consolidated.

- The DP model can easily handle costs as a function of the combination of freights and uncertainty in the arrival of freights, but solving it means facing “the curses of dimensionality”.

- The ADP model overcomes the DP model dimensionality issues through the use of a post-decision state and basis functions.
THANKS FOR YOUR ATTENTION!

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