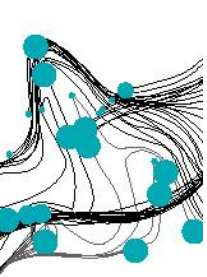


# SCHEDULING SYNCHROMODAL FREIGHT TRANSPORT USING APPROXIMATE DYNAMIC PROGRAMMING

Arturo E. Pérez Rivera & Martijn R.K. Mes

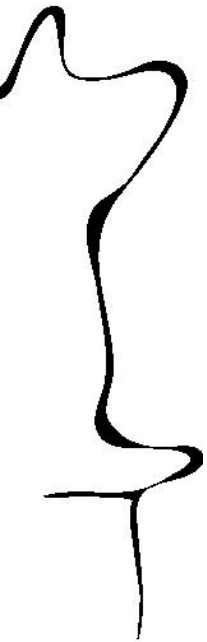
*Department of Industrial Engineering and Business Information Systems  
University of Twente, The Netherlands*





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Synchromodal freight transport



Multi-period scheduling problem:

➤ *Markov Decision Process model*



Heuristic solution:

➤ *Approximate Dynamic Programming algorithm*



Numerical results



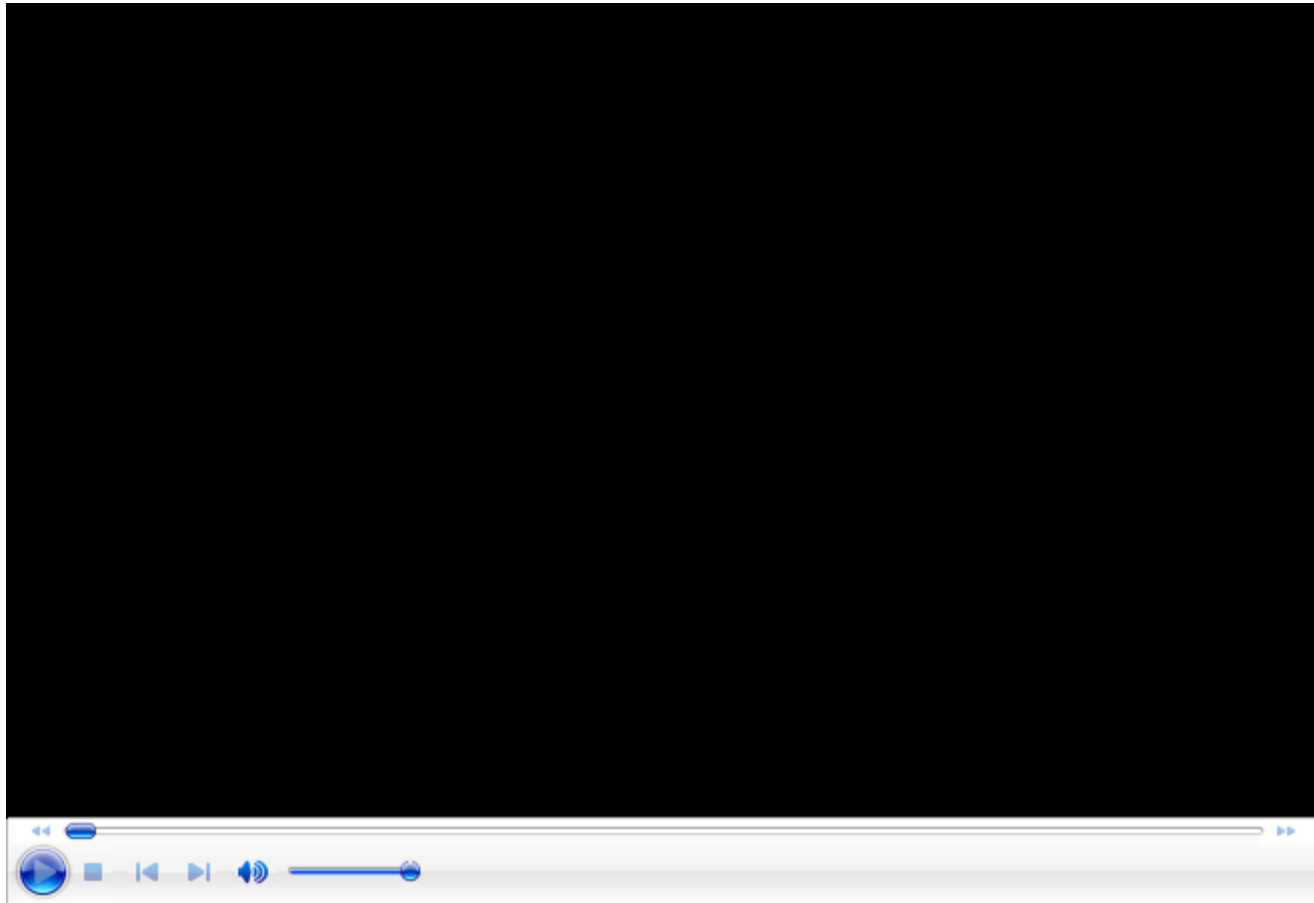
What to remember





# SYNCHROMODAL FREIGHT TRANSPORT

WHAT IS SYNCHROMODALITY?



*\*Source of video: Dutch Institute for Advanced Logistics (DINALOG) [www.dinalog.nl](http://www.dinalog.nl)*

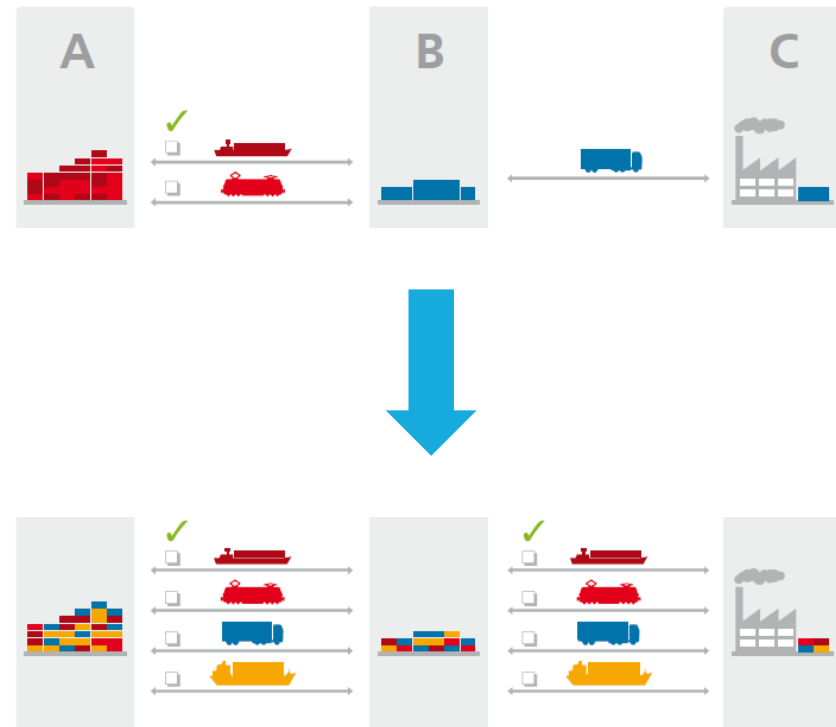
**UNIVERSITY OF TWENTE.**



# SYNCHROMODAL FREIGHT TRANSPORT

WHAT ARE ITS CHARACTERISTICS?

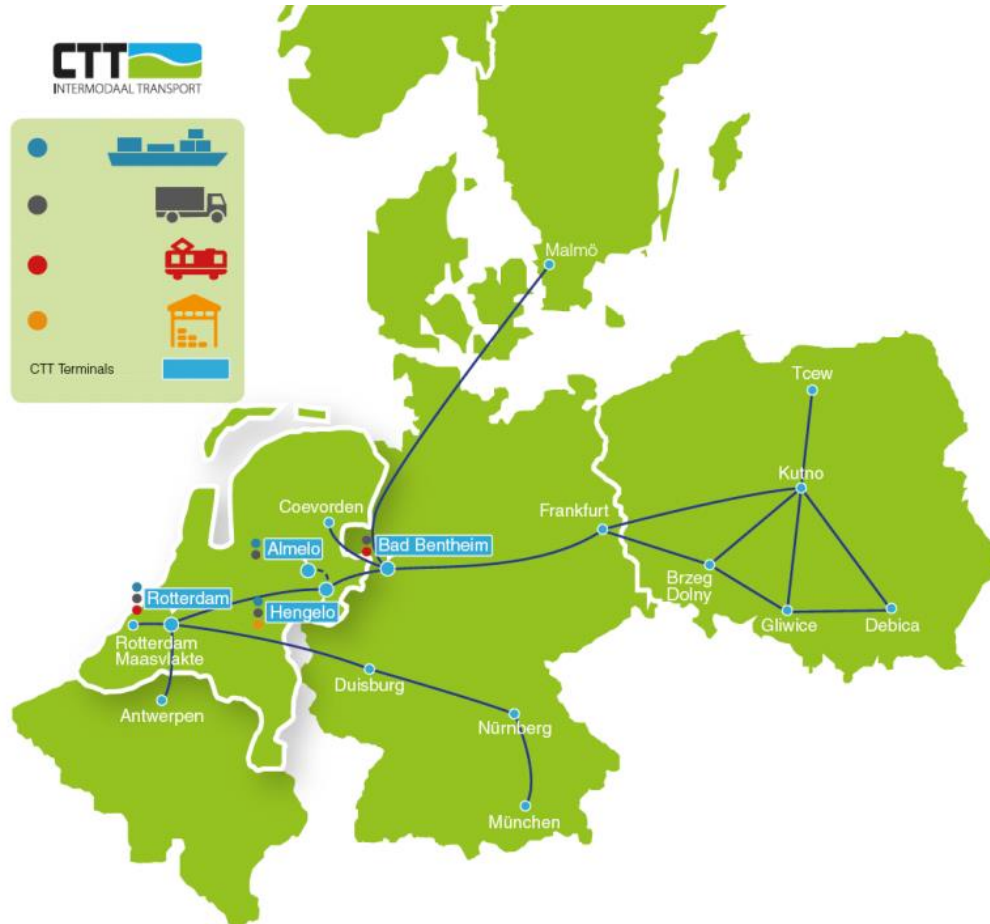
- **Mode-free booking** for all freights.
- **Network-wise scheduling** at any point in time.
- **Real-time information** about the state of the network.
- **Overall performance** in both network and time.



\*Source of artwork: European Container Terminals (ECT) – The future of freight transport (2011).

# SYNCHROMODAL FREIGHT TRANSPORT

## CASE: TRANSPORTATION OF CONTAINERS IN THE HINTERLAND

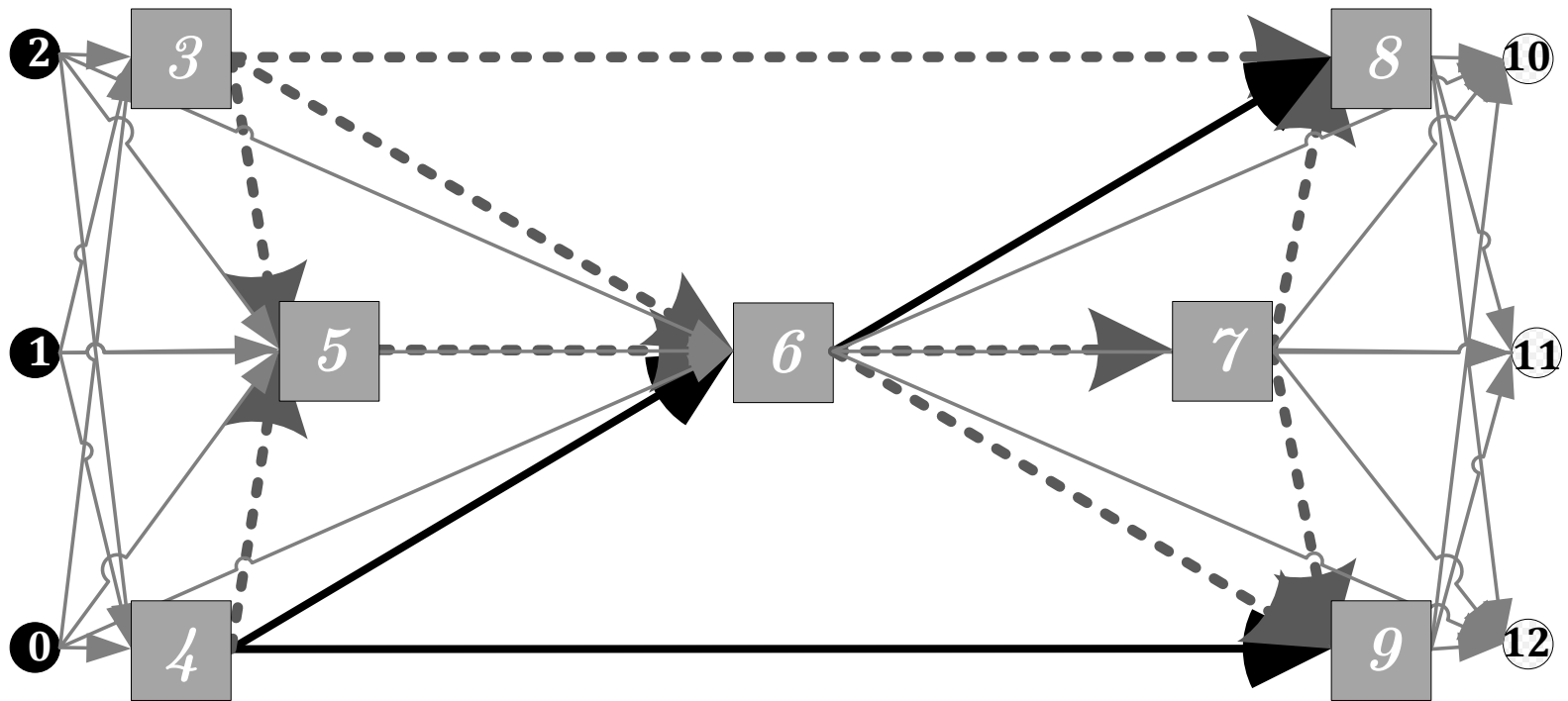


\*Source of artwork: Combi Terminal Twente (CTT) [www.ctt-twente.nl](http://www.ctt-twente.nl)

# MULTI-PERIOD SCHEDULING IN SYNCHROMODALITY

## PROBLEM EXAMPLE

→ Truck    ⋯→ Train    → Barge    ■ Terminal    ● Origin    ○ Destination





# MULTI-PERIOD SCHEDULING IN SYNCHROMODALITY

## PROBLEM DESCRIPTION

---

### ***Input:***

- ***Transport network:*** services, terminals, schedules, durations, capacity, costs, revenues.
- ***Freight demand:*** origin (or location), destination, release-day, due-day, size.
- ***Probability distributions:*** (1) number of freights, (2) their origin, (3) their destination, (4) release-day, and (5) time-window length.

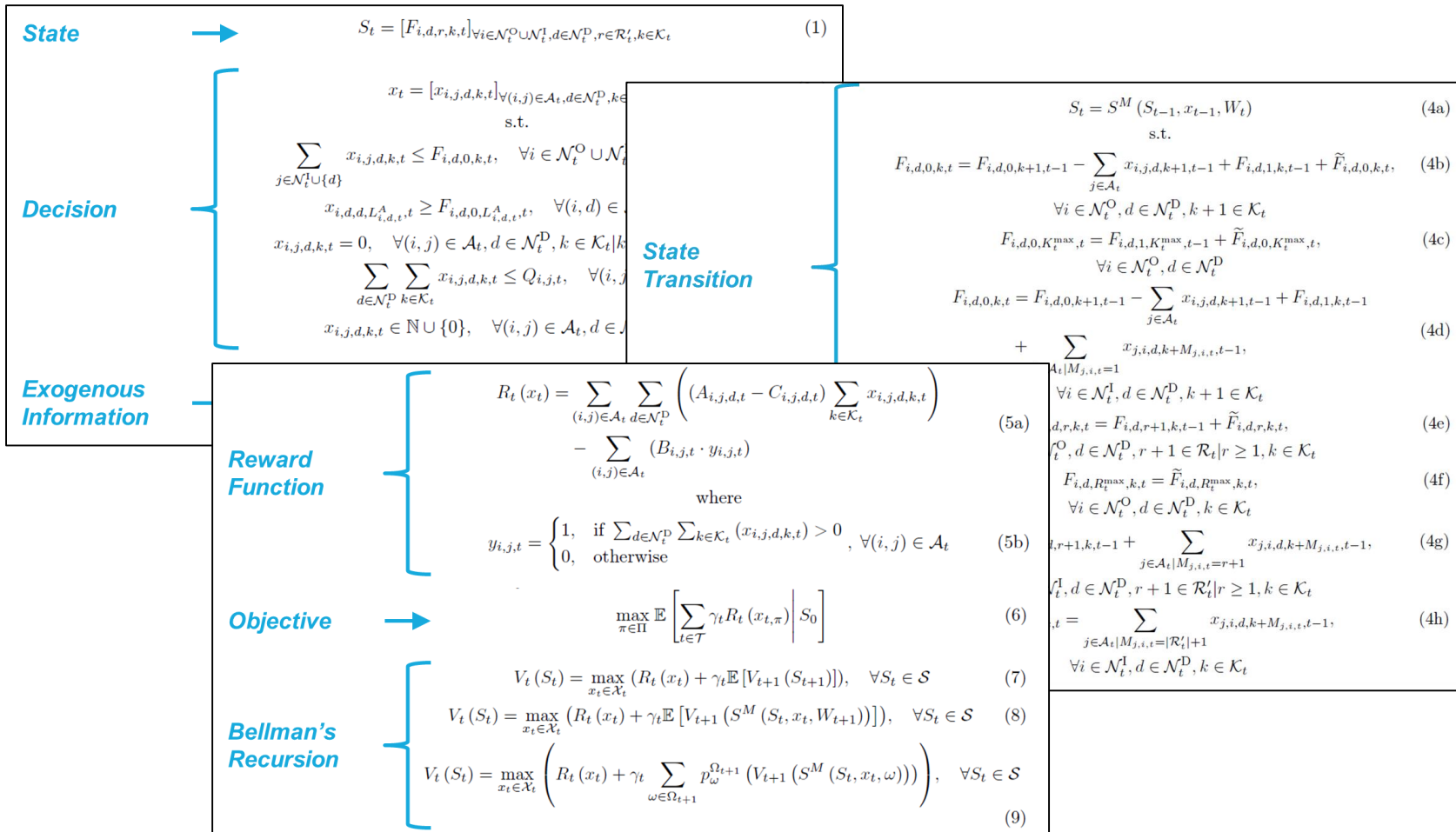
### ***Output:***

- ***Schedule:*** which service to use for each freight, if any.
- ***Performance:*** revenue and costs of the schedule.



# MARKOV DECISION PROCESS (MDP) MODEL

## OPTIMIZATION OF SEQUENTIAL DECISIONS UNDER UNCERTAINTY







# MDP MODEL – NETWORK EVOLUTION

## VIRTUAL TIME-WINDOWS FOR FREIGHT

- The **release-day  $r$**  is relative to the current day  $t$ .
- The **time-window length  $k$**  is relative to the release-day  $r$ .
- Consider  $F_{i,d,r,k,t}$  freights with  $k=4$  sent from terminal  $i$  to terminal  $j$  using a service that lasts 2 days:

	t=7	t=8	t=9	t=10	t=11
	Monday	Tuesday	Wednesday	Thursday	Friday
$i$	$F_{i,d,0,4,7}$				
$j$		$F_{j,d,1,2,8}$	$F_{j,d,0,2,9}$		
$d$					$F_{d,d,0,0,11}$



# MDP MODEL – SOLUTION CHALLENGES

- **Three-curses of dimensionality** restrain the size of networks whose MDP model can be solved to optimality.

$$V_t(S_t) = \max_{x_t \in \mathcal{X}_t} (R_t(x_t) + \gamma_t \mathbb{E}[V_{t+1}(S_{t+1})])$$

- **Multi-period revenues and costs** can make heuristics flounder and get stuck in local-optima.

$$R_t(x_t) = \sum_{(i,j) \in \mathcal{A}_t} \sum_{d \in \mathcal{N}_t^D} \left( (A_{i,j,d,t} - C_{i,j,d,t}) \sum_{k \in \mathcal{K}_t} x_{i,j,d,k,t} \right) - \sum_{(i,j) \in \mathcal{A}_t} (B_{i,j,t} \cdot y_{i,j,t})$$

where

$$y_{i,j,t} = \begin{cases} 1, & \text{if } \sum_{d \in \mathcal{N}_t^D} \sum_{k \in \mathcal{K}_t} (x_{i,j,d,k,t}) > 0, \forall (i,j) \in \mathcal{A}_t \\ 0, & \text{otherwise} \end{cases}$$

# APPROXIMATE DYNAMIC PROGRAMMING (ADP)

## HEURISTIC FRAMEWORK FOR SOLVING LARGE MARKOV MODELS.<sup>1</sup>

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### Algorithm 1 ADP Algorithm

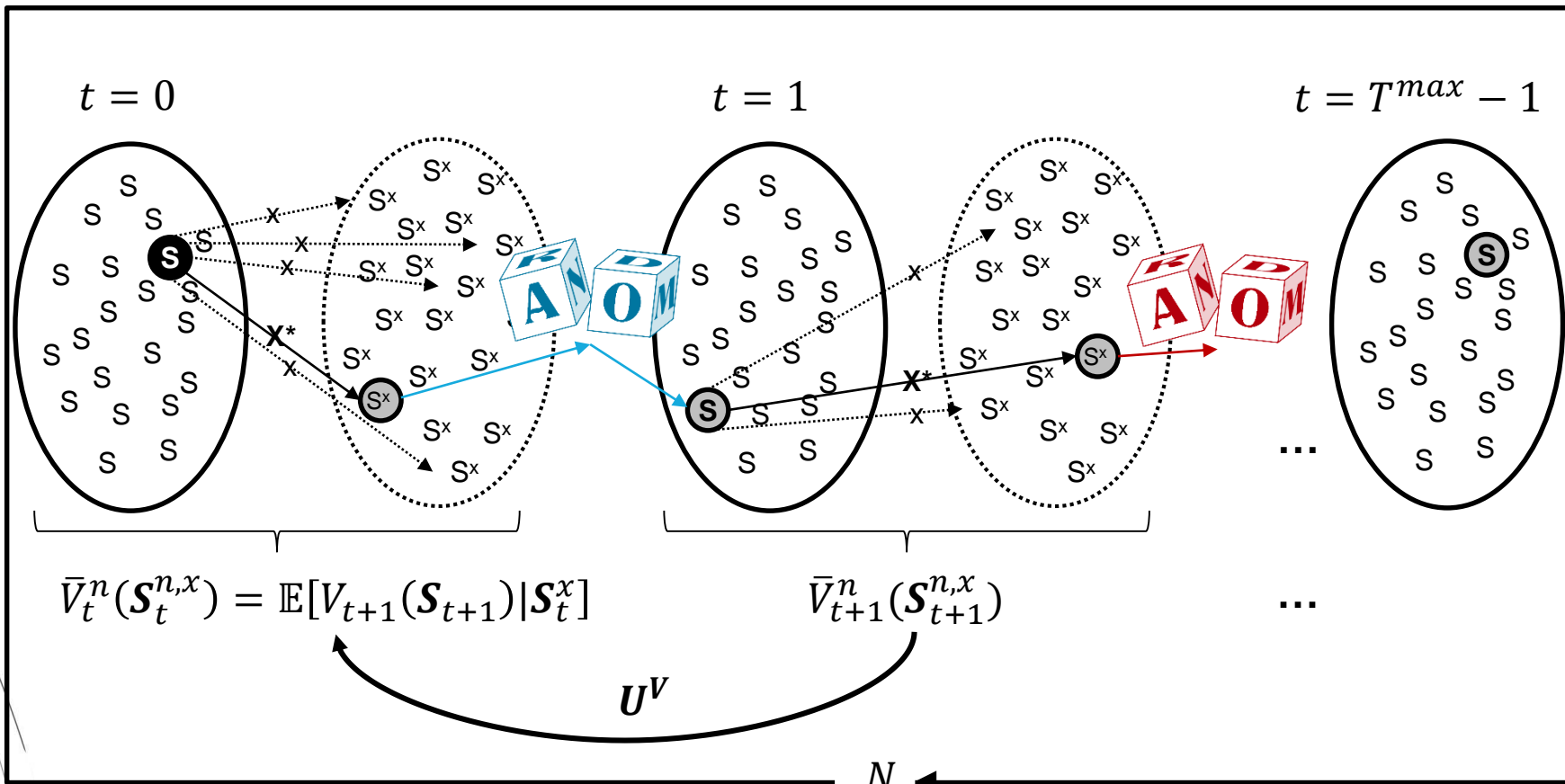
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```
1: Initialize  $[\bar{V}_t^0]_{\forall t \in \mathcal{T}}$ 
2: for  $n = 1$  to  $N$  do
3:    $S_0^n := S_0$ 
4:   for  $t = 0$  to  $T^{max} - 1$  do
5:      $x_t^{n*} := \arg \max_{x_t^n \in \mathcal{X}_t^R} (R_t(x_t^n) + \gamma_t \bar{V}_t^{n-1}(S^{M,x}(S_t^n, x_t^n)))$ 
6:      $S_t^{n,x*} := S^{M,x}(S_t^n, x_t^{n*})$ 
7:      $\hat{v}_t^n := (R_t(x_t^{n*}) + \gamma_t \bar{V}_t^{n-1}(S_t^{n,x*}))$ 
8:      $W_{t+1}^n := \text{Random}(\Omega)$ 
9:      $S_{t+1}^n := S^M(S_t^n, x_t^{n*}, W_{t+1}^n)$ 
10:   end for
11:   for  $t = T^{max} - 1$  to  $0$  do
12:      $\bar{V}_t^n(S_t^{n,x*}) := U_t^n(\bar{V}_t^{n-1}(S_t^{n,x*}), S_t^{n,x*}, [\hat{v}_t^n]_{\forall t \in \mathcal{T}})$ 
13:   end for
14: end for
15: return  $[\bar{V}_t^N]_{\forall t \in \mathcal{T}}$ 
```

---

1. For a comprehensive explanation see Powell (2010) *Approximate Dynamic Programming*.

# ADP – ALGORITHM EXEMPLIFICATION



# ADP – THE VALUE FUNCTION APPROXIMATION (VFA)

## PARAMETRIC APPROXIMATION OF DOWNSTREAM REWARDS

**VFA**



$$\bar{V}_t^n(S_t^{x,n}) = \sum_{b \in \mathcal{B}} \theta_{b,t}^n \phi_{b,t}(S_t^{x,n}) = \phi_t(S_t^{x,n})^T \theta_t^n \quad (11)$$

**Basis functions**

$$\phi_{b(i,d)}(S_t^{x,n}) = \sum_{k \in \mathcal{K}_t | k < \Psi} \sum_{r \in \mathcal{R}'_t} F_{i,d,r,k,t}^{x,n}, \quad \forall i \in \mathcal{N}_t^O \cup \mathcal{N}_t^I, d \in \mathcal{N}_t^D \quad (12a)$$

$$\phi_{b'(i,d)}(S_t^{x,n}) = \sum_{k \in \mathcal{K}_t | k \geq \Psi} \sum_{r \in \mathcal{R}'_t} F_{i,d,r,k,t}^{x,n}, \quad \forall i \in \mathcal{N}_t^O \cup \mathcal{N}_t^I, d \in \mathcal{N}_t^D \quad (12b)$$

$$\phi_{b''(d)}(S_t^{x,n}) = \sum_{i \in \mathcal{N}_t^O \cup \mathcal{N}_t^I} \sum_{k \in \mathcal{K}_t | k \geq \Psi} \sum_{r \in \mathcal{R}'_t} F_{i,d,r,k,t}^{x,n}, \quad \forall d \in \mathcal{N}_t^D \quad (12c)$$

$$\phi_{|\mathcal{B}|}(S_t^{x,n}) = 1 \quad (12d)$$

**Recursive least square method for updating the VFA**

$$\bar{V}_t^n(S_t^{n,x^*}) := U_t^n(\bar{V}_t^{n-1}(S_t^{n,x^*}), S_t^{n,x^*}, [\hat{v}_t^n]_{\forall t \in \mathcal{T}}) \quad (13a)$$

s.t.

$$\theta_t^n = \theta_t^{n-1} - (H_t^n)^{-1} \phi_t(S_t^{x,n}) \left( \bar{V}_{t-1}^{n-1}(S_{t-1}^{x,n}) - \sum_t^{T^{\max}-1} \hat{v}_t^n \right) \quad (13b)$$

$$H_t^n = \lambda^n H_t^{n-1} + \phi_t(S_t^{x,n}) \phi_t(S_t^{x,n})^T \quad (13c)$$

$$\lambda^n = 1 - \frac{\lambda}{n} \quad (13d)$$

# ADP – THE VALUE FUNCTION APPROXIMATION (VFA)

## PARAMETRIC APPROXIMATION OF DOWNSTREAM REWARDS

VFA



$$\bar{V}_t^n(S_t^{x,n}) = \sum_{b \in \mathcal{B}} \theta_{b,t}^n \phi_{b,t}(S_t^{x,n}) = \phi_t(S_t^{x,n})^T \theta_t^n \quad (11)$$

$$\phi_{b(i,d)}(S_t^{x,n}) = \sum_{k \in \mathcal{K}_t | k < \Psi} \sum_{r \in \mathcal{R}'_t} F_{i,d,r,k,t}^{x,n}, \quad \forall i \in \mathcal{N}_t^O \cup \mathcal{N}_t^I, d \in \mathcal{N}_t^D \quad (12a)$$

The features of a post-decision state:

1. *Intermodal-path freights* per location, per destination.
2. *Trucking freights* per location, per destination.
3. *Total freights* per destination.
4. *Constant*.

Recursive least square method for updating the VFA

s.t.

$$\theta_t^n = \theta_t^{n-1} - (H_t^n)^{-1} \phi_t(S_t^{x,n}) \left( \bar{V}_{t-1}^{n-1}(S_{t-1}^{x,n}) - \sum_t^{T^{\max}-1} \hat{v}_t^n \right) \quad (13b)$$

$$H_t^n = \lambda^n H_t^{n-1} + \phi_t(S_t^{x,n}) \phi_t(S_t^{x,n})^T \quad (13c)$$

$$\lambda^n = 1 - \frac{\lambda}{n} \quad (13d)$$



# ADP – EPSILON GREEDY EXPLORATION

## ESCAPING LOCAL OPTIMA

---

### Algorithm 1 ADP Algorithm

---

```

1: Initialize  $[\bar{V}_t^0]_{\forall t \in \mathcal{T}}$ 
2: for  $n = 1$  to  $N$  do
3:    $S_0^n := S_0$ 
4:   for  $t = 0$  to  $T^{max} - 1$  do
5:      $x_t^{n*} := \arg \max_{x_t^n \in \mathcal{X}_t^R} (x_t^n)$ 
6:      $S_t^{n,x*} := S^{M,x}(S_t^n, x_t^{n*})$ 
7:      $\hat{v}_t^n := (R_t(x_t^{n*}) + \gamma_t \bar{V}_t)$ 
8:      $W_{t+1}^n := \text{Random}(\Omega)$ 
9:      $S_{t+1}^n := S^M(S_t^n, x_t^{n*}, W_{t+1}^n)$ 
10:   end for
11:   for  $t = T^{max} - 1$  to  $0$  do
12:      $\bar{V}_t^n(S_t^{n,x*}) := U_t^n(\bar{V}_t^{n-1}(S_t^{n,x*}), S_t^{n,x*}, [\hat{v}_t^n]_{\forall t \in \mathcal{T}})$ 
13:   end for
14: end for
15: return  $[\bar{V}_t^N]_{\forall t \in \mathcal{T}}$ 

```

---

### Algorithm 2 $\epsilon$ -greedy strategy for exploration

---

```

1: if Random  $[0, 1) < \epsilon$  then
2:    $x_t^{n*} := \text{Random}(\mathcal{X}_t^R)$ 
3: else
4:    $x_t^{n*} := \arg \max_{x_t^n \in \mathcal{X}_t^R} (R_t(x_t^n) + \gamma_t \bar{V}_t^{n-1}(S^{M,x}(S_t^n, x_t^n)))$ 
5: end if

```

---

# ADP – VALUE OF PERFECT INFORMATION (VPI)

## EXPLORATION BASED ON A BAYESIAN BELIEF

**Exploration  
decision**



$$x_t^{n*} = \arg \max_{x_t^n \in \mathcal{X}_t^R} (v_t^{E,n}(K_t^n, S_t^n x_t^n)) \quad (14)$$

**Bayesian  
belief**



$$K_t^n = (\bar{V}_t^n, C_t^n) = (\phi_t, \theta_t^n, C_t^n) \quad (15)$$

**Value of  
exploration**



$$v_t^{E,n}(K_t^n, S_t^n, x_t^n) = \sqrt{\sigma_t^{2,n}(K_t^n, S_t^{x,n})} f \left( -\frac{\delta(S_t^{x,n})}{\sqrt{\sigma_t^{2,n}(K_t^n, S_t^{x,n})}} \right) \quad (16a)$$

s.t.

$$\delta(S_t^{x,n}) = \left| \bar{V}_t^{x,n}(S_t^{x,n}) - \max_{y_t^n \in \mathcal{X}_t^R | y_t^n \neq x_t^n} \bar{V}_t^{x,n}(S_t^{y,n}) \right| \quad (16b)$$

$$\sigma_t^{2,n}(K_t^n, S_t^{x,n}) = \phi(S_t^{x,n})^T C_t^n \phi(S_t^{x,n}) \quad (16c)$$

**Update VFA  
and belief**



$$\theta_t^n = \theta_t^{n-1} - \frac{(\theta_t^{n-1})^T \phi(S_t^{x,n}) - \sum_{t=0}^{T^{\max}-1} \hat{v}_t^n}{\sigma^{2,E} + \sigma_t^{2,n-1}(S_t^{x,n})} C_t^n \phi(S_t^{x,n}) \quad (17)$$

$$C_t^n = C_t^{n-1} - \frac{C_t^{n-1} \phi(S_t^{x,n}) \phi(S_t^{x,n})^T C_t^{n-1}}{\sigma^{2,E} + \sigma_t^{2,n}(S_t^{x,n-1})} \quad (18)$$

# ADP – VALUE OF PERFECT INFORMATION (VPI)

## EXPLORATION BASED ON A BAYESIAN BELIEF

*Exploration  
decision*



$$x_t^{n*} = \arg \max_{x_t^n \in \mathcal{X}_t^R} (v_t^{E,n}(K_t^n, S_t^n x_t^n)) \quad (14)$$

*Bayesian  
belief*



$$K_t^n = (\bar{V}_t^n, C_t^n) = (\phi_t, \theta_t^n, C_t^n) \quad (15)$$

*Value of  
exploration*



**Dearden et al., 1999:** the expected improvement in future decision quality arising (through a better VFA) from the information acquired by exploration.

*Update VFA  
and belief*



**Rhyzov et al., 2017:** update is analogous to the recursive least square method with the addition of the current uncertainty knowledge through covariance matrix.

# ADP – VPI MODIFICATIONS

BE MORE CONSERVATIVE IN EXPLORATION AND UPDATING

**1. Exploration decisions** that focus on more than just the value of exploration:

$$x_t^{E2} = \arg \max \left( \bar{V}_t^{x,n}(S_t^{x,n}) + v_t^{E,n}(S_t^n, K_t^n, x_t) \right)$$

$$x_t^{E3} = \arg \max \left( R_t(S_t^n, x_t) + \bar{V}_t^{x,n}(S_t^{x,n}) + v_t^{E,n}(S_t^{x,n}, K_t^n, x_t) \right)$$

$$x_t^{E4} = \arg \max \left( (1 - \alpha^n) \left( R_t(S_t^n, x_t) + \bar{V}_t^{x,n}(S_t^{x,n}) \right) + \alpha^n v_t^{E,n}(S_t^{x,n}, K_t^n, x_t) \right)$$

**2. Update VFA and belief** with stage or post-decision state dependent noise:

$$\sigma_t^{2,E2} = \frac{T^{\max} - t}{T^{\max}} \eta^E$$

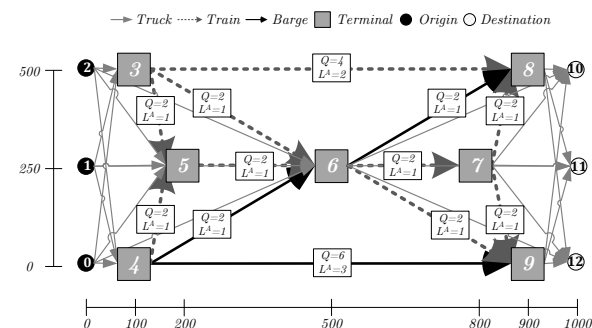
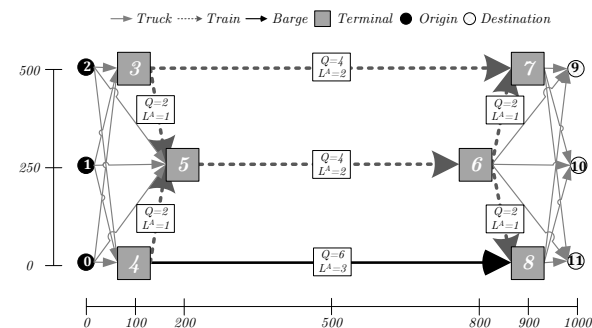
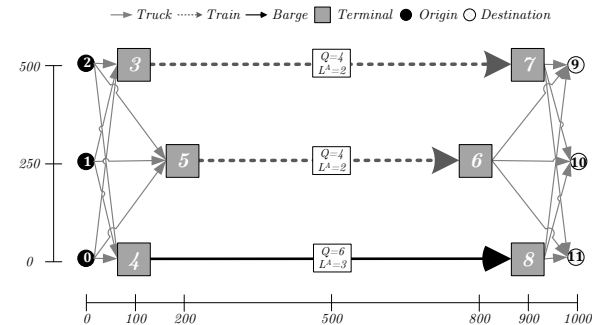
$$\sigma_t^{2,E3} = \sigma_t^{2,n}(S_t^{x,n})$$

$$\sigma_{t,n}^{2,E4} = \frac{T^{\max} - t}{T^{\max}} \eta^E + \sigma_t^{2,n}(S_t^{x,n})$$

# NUMERICAL RESULTS

## PROBLEM INSTANCE SETTINGS

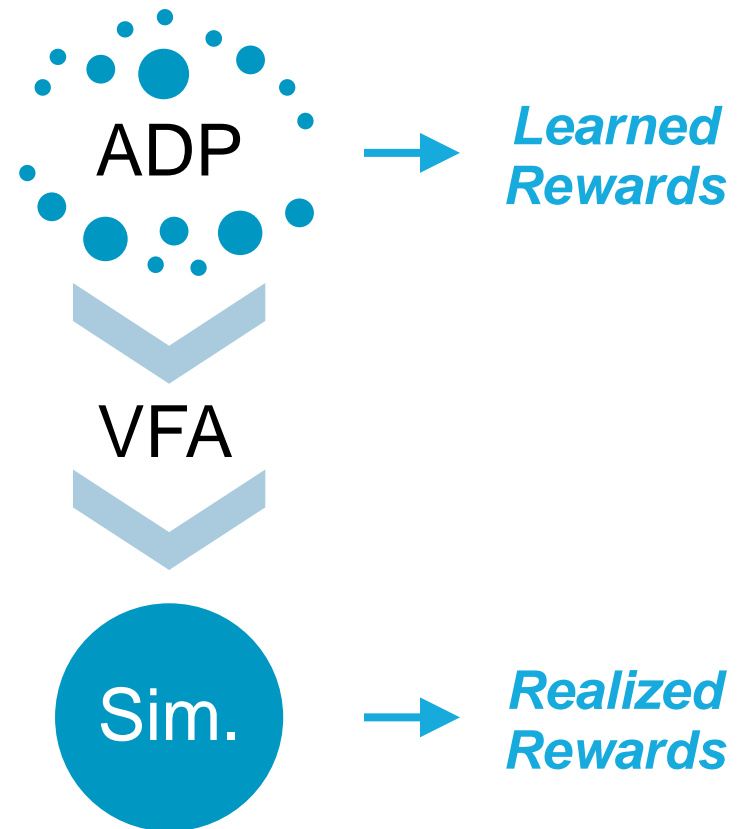
- Cost differs by vehicle, capacity, and distance (Janic, 2007), **revenue received at pick-up.**
- 50 day horizon, at least 14 freight intermodal capacity, at most **three days traveling time.**
- Up to 12 freights per day, different destination probability per origin.
- Freights are immediately released and have a 6 day time-window.



# NUMERICAL RESULTS

## EXPERIMENTAL SETTINGS

- Initial state with six freights.
- **Benchmark heuristic**: Use a service for a freight if the cost difference between the cheapest and second cheapest intermodal path to a freights destination is more than setup cost of the first.
- **Three ADP Designs**: basis functions only, epsilon-greedy, VPI, for 50 iterations.
  - Weights (VFA) initialized to 0, except the constant, which is initialized with the benchmark.





# NUMERICAL RESULTS

## PERFORMANCE OF DIFFERENT ADP DESIGNS

### RP 1:

Aggregated time-windows at each terminals.

Aggregated time-windows, destinations, and origins at each origin.

### RP 2:

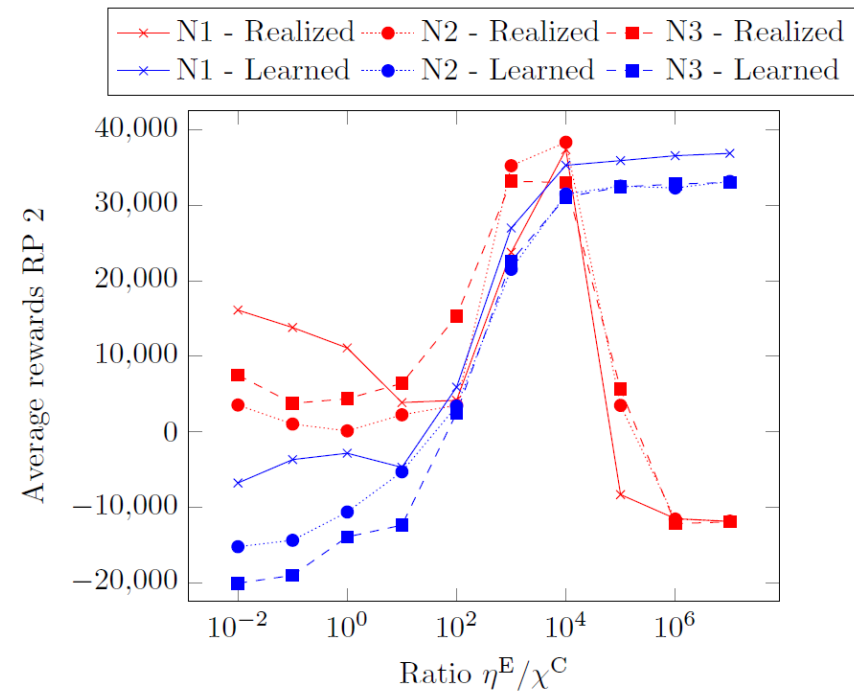
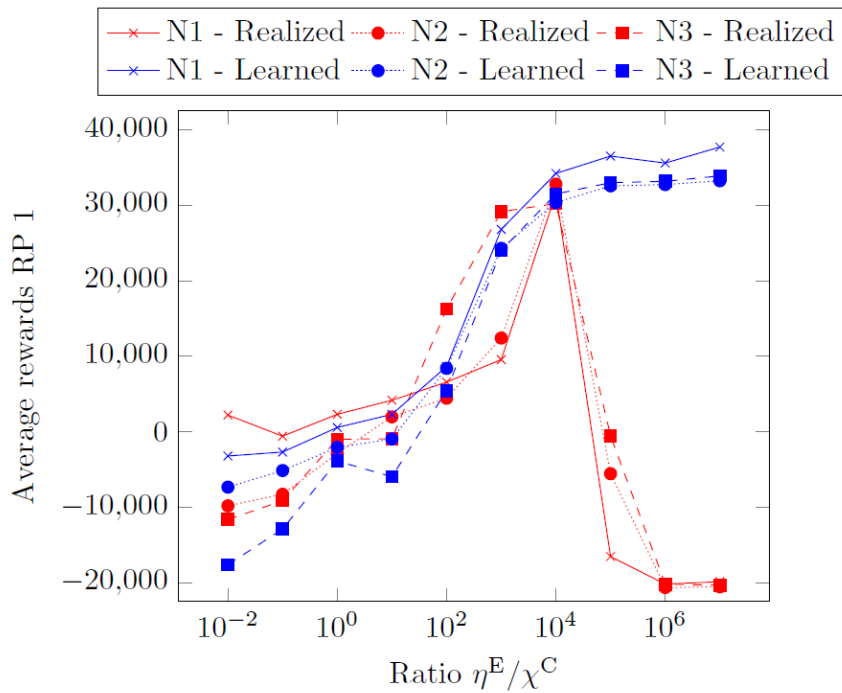
Aggregated time-windows at terminals.

Aggregated time-windows and origins at each origin.

ADP Design	Network 1		Network 2		Network 3		
	Realized	Learned	Realized	Learned	Realized	Learned	
RP 1	BF	-7,994	38,219	-11,247	33,720	-16,548	-17,928
	BF + $\epsilon$ -greedy	-4,628	-6,984	-11,485	33,228	-18,172	-18,507
	BF + VPI	34,044	36,571	34,284	29,493	34,898	23,285
RP 2	BF	-4,912	-3,803	-11,734	34,060	-11,949	34,495
	BF + $\epsilon$ -greedy	880	37,386	-11,450	-12,091	-11,949	33,356
	BF + VPI	40,439	35,407	40,195	31,107	38,314	30,791
Benchmark	38,036	-	33,445	-	33,889	-	

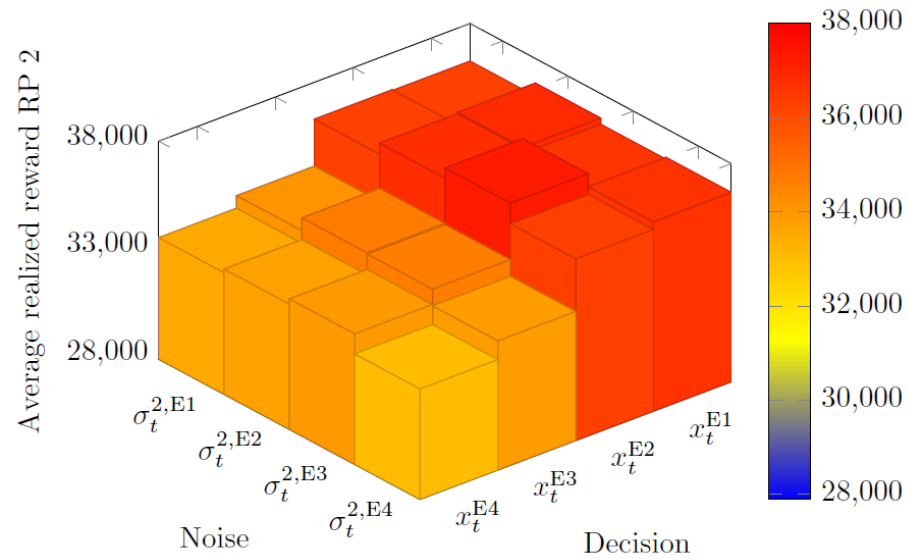
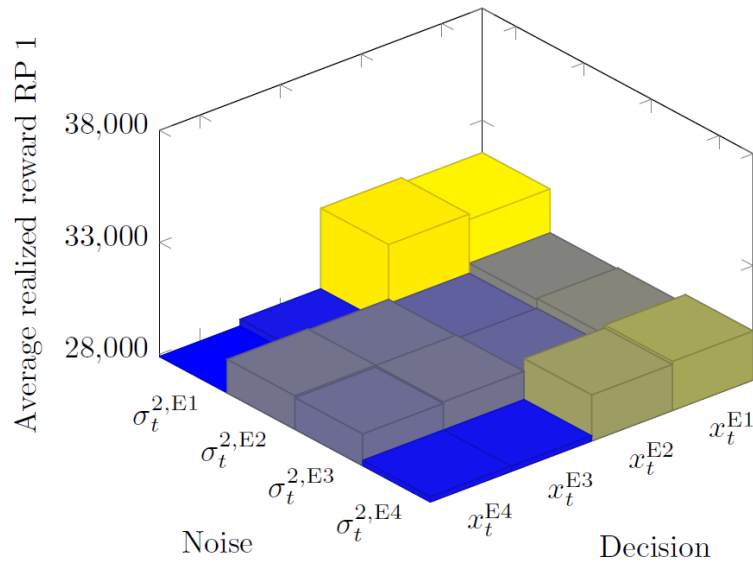
# NUMERICAL RESULTS

## NOISE AND UNCERTAINTY IN VPI



# NUMERICAL RESULTS

THE PROPOSED VPI MODIFICATIONS OVER ALL NETWORKS



# NUMERICAL EXPERIMENTS

## SENSITIVITY ANALYSIS OF TIME-PARAMETER UNCERTAINTY

### New settings:

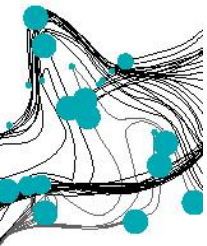
**Release-day** : 0, 1, 2 days

**Time-window length**: 4, 5, 6 days

*Average realized rewards for Network 2*

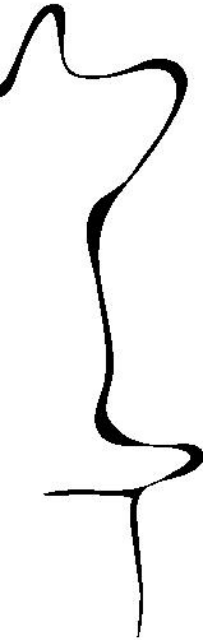
Time-window length	Release-day					
	Short		Medium		Long	
Short	12,339	-9%	12,160	-19%	12,062	-23%
	11,289		9,877		9,281	
Medium	18,232	29%	18,052	27%	17,951	30%
	23,486		23,015		23,422	
Long	25,805	26%	25,420	25%	25,401	28%
	32,524		31,806		32,462	

*Benchmark, ADP with VPI*



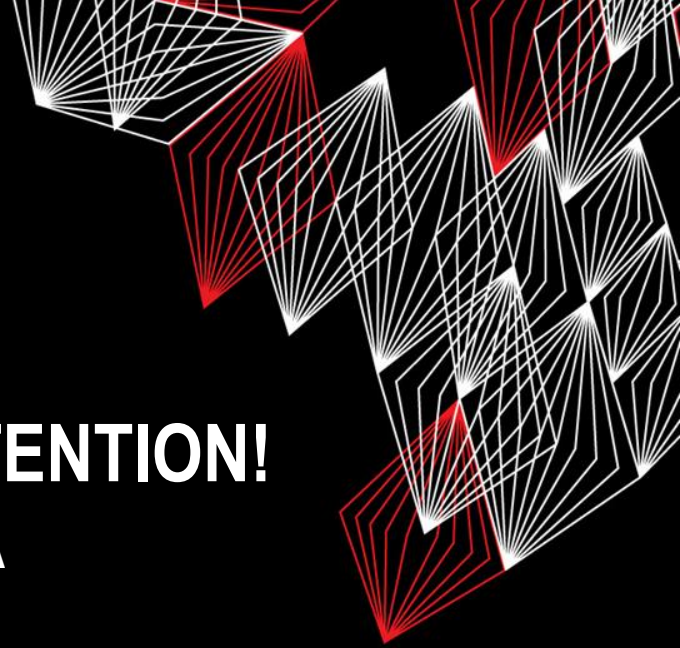
## WHAT TO REMEMBER

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- 🌀 We exemplified how *VPI exploration improves ADP in scheduling synchromodal freight transport* considering uncertainty in the demand and performance over time.
- To apply VPI in a finite-horizon ADP with basis functions, *exploring and updating should be slightly more conservative* than in traditional VPI.
- For larger networks, further research in the *reduction of the decision space and its interaction with the VFA* is necessary for ADP to work properly.





# THANKS FOR YOUR ATTENTION!

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*VeRoLog 2017 - Wednesday, July 12<sup>th</sup>  
Amsterdam, The Netherlands*