Efficient FEM-multigrid solver for granular materials

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1 Project description

This report gives a brief review of the current activities related to the project and provides a few aspects of the ongoing research. In this project we study the behaviour of the granular materials by continuum simulation models. We take the incompressible Navier Stokes equation as our governing equation with different viscosity formulations and then we develop efficient numerical tools to solve it numerically. Our main goal is to study the transient state and consequently the multiscale flow behavior of the granular materials.

2 Publications

S. Mandal, A. Ouazzi, S. Turek (in process)
Finite Element Simulation of Quasi-Newtonian Flow Problems with Application to Granular Materials, 2014

3 Ongoing efforts

3.1 Different models

As the physical properties and mechanisms of the transient state are still not completely understood, many different approaches based on the Navier-Stokes equation have been proposed to simulate the flow of granular materials and the most popular ones are the following, where the viscosity is defined as

1. Poliquen model: $\eta(\gamma_{II}, p) = \frac{\sqrt{2}}{2}\left(\frac{\alpha p}{|\dot{\gamma}|} + \frac{\beta d p}{\delta\sqrt{\frac{p}{\rho}} + |\dot{\gamma}| d}\right)$;
2. Schaeffer model: $\eta(\gamma_{II}, p) = \sqrt{2} p \sin\phi \frac{1}{|\dot{\gamma}|}$;
3. Schaeffer-Tardos model: $\eta(\gamma_{II}, p) = \sqrt{2} p (\sin\phi + b\cos\phi |\dot{\gamma}|^n) \frac{1}{|\dot{\gamma}|}$

Currently we are working with the Poliquen model and we are in the process of validating the results for benchmark problems.

3.2 FEM solver

Featflow2 is a highly efficient object oriented fast solver specifically built for solving incompressible flow problems. We solve the generalized Navier-Stokes equation by a Multi-Grid solver, where we use a VANKA smoother and BiCGStab (with Vanka preconditioning) as coarse grid solver. We use Q2/P1 finite elements for discretizing the problem domain and use Newton iteration (implemented for the convective term and Frechet derivative of the nonlinear viscosity) scheme for each nonlinear loop. Stabilization techniques can also be used while performing the multigrid techniques if the solution does not converge without it.

3.3 Benchmark results

As the viscosity model for the granular materials involves a pressure term (which makes it more difficult to solve), first we take Newtonian fluid and the fluid obeying power law as our pre-studies for Featflow2. While validating the results for the above mentioned models, we take Hogen's work [3] as our reference results and here we represent our results achieved in the new version of Featflow2 in compact form.

Fluid	Level	Drag		Lift		NL/MG	
		Hogen	Featflow2	Hogen	Featflow2	Hogen	Featflow2
Newtonian	2	5.540999	5.555030E+00	9.447473e-03	9.497590E-03	5/2	5/3
	3	5.566928	5.572228E+00	1.046885e-02	1.060061E-02	5/2	5/2
	4	5.576088	5.577628E+00	1.056787e-02	1.061564E-02	5/2	5/2
	5	5.578652	5.579065E+00	1.060398e-02	1.061780E-02	5/2	5/2
Shear Thinning	2	3.19922	3.089445E+00	-0.01238	-1.212569E-02	9/3	8/3
	3	3.26246	3.229266E+00	-0.01334	-1.313954E-02	4/2	4/2
	4	3.27553	3.266381E+00	-0.01335	-1.334060E-02	3/2	3/2.3
	5	3.27781	3.275369E+00	-0.01332	-1.330639E-02	2/2	3/2.3
Shear Thickening	2	13.66925	1.397544E+01	0.34424	3.474860E-01	7/2	6/3
	3	13.78575	1.386550E+01	0.35232	3.535111E-01	3/2	3/2
	4	13.81625	1.383644E+01	0.35148	3.519312E-01	3/2	3/1.7
	5	13.82490	1.382989E+01	0.35248	3.526509E-01	3/1	3/1.3

Table 1: Benchmark quantities for the 'Flow around cylinder' configuration

4 Outlook

Once we validate our results for the Poliquen model, we would like to investigate the same for the simulation for the Tardos model. At first glance, the shear banding phenomenon gives the impression to be treated mathematically as a discontinuity, but this would cause severe problems for numerical algorithms. On the other hand, shear bands might not be a true physical discontinuity, rather than a change in the involved physical system which could be captured with a compressible model. The flow for the generalized compressible Navier-Stokes-like equation with a mass conservation equation look like:

$$\rho \frac{Du}{Dt} = -\nabla p + \nabla \cdot \left[\frac{q(p,\rho)}{||D - \frac{1}{n}\nabla \cdot uI||} \left(D - \frac{1}{n}\nabla \cdot uI \right) \right] + \rho g; \qquad n = 2, 3$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

However, this is not enough as the density ρ is now a dependent variable instead of a constant and to complete the system, an additional equation is required in the form of the so-called normality condition:

$$\nabla \cdot u = \frac{\partial q(p,\rho)}{\partial p} ||D - \frac{1}{n}\nabla \cdot uI||$$

The mathematical and computational methodology of the FEM solver can be naturally extended to these compressible granular and powder flow models. After validating each model for the benchmark problems, we will apply them in more practical geometries associated with the realistic problems.

References

[1] M. Kheiripour Langroudi, S. Turek, A. Ouazzi, G.I. Tardos, 2010. *An investigation of frictional and collisional powder flows using a unified constitutive equation*, Powder Technology Vol. 197, No. 1-2.

[2] Pierre Jop, Yoel Forterre, Olivier Pouliquen, 2007. *Initiation of granular surface flows in a narrow channel*, Nature 441,727.

[3] H. Damanik, J. Hron, A. Ouazzi, S. Turek, 2011. *Monolithic Newton-multigrid solution techniques for incompressible nonlinear flow models*, International Journal for Numerical Methods in Fluids, Volume 71, Issue 2, pages 208222.

[4] http://www.featflow.de/en/index.html