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Accommodating individual preferences in nurse scheduling via auctions and optimization

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Abstract This paper describes a two-stage approach to nurse scheduling that considers both nurse preferences and hospital constraints. In the auction stage, nurses bid for their preferred working shifts and rest days using "points". An optimization model awards shifts to the highest bidders insofar as possible while maintaining hospital requirements. In the schedule completion stage, an optimization model allocates the unfilled shifts to nurses who have not yet met their minimum required hours. The approach is demonstrated via a case study in the emergency department at York Hospital. A schedule with a high percentage of awarded bids was generated in a few minutes of computer time. Further experimentation suggests that the approach works well under a variety of conditions.

Keywords Nurse scheduling · Health care applications · Optimization · Auction

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1 Introduction

Hospital care units must provide 24-h nursing coverage at levels to match patient demand while adhering to organizational policies designed to protect the health and welfare of patients and staff. The already difficult scheduling problem is further compounded by a shortage of nurses. The United States is currently experiencing a substantial nursing shortage which is projected to increase over the next two decades [1]. Because of the shortage, nurse retention is of great concern to most healthcare organizations. High turnover rates of nursing staff are a result of significant levels of job dissatisfaction, and inflexible work schedules are a contributing factor to dissatisfaction [2]. Many hospitals have instituted self-scheduling in an attempt to provide flexibility and increase job satisfaction for nurses.

In the self-scheduling approach, each nurse submits an individual schedule or request. A nurse manager then creates the base schedule for the care unit. In some hospitals, the schedule is created manually after reviewing the requests, while other hospitals allow direct signups subject to certain rules, with final approval and conflict resolution performed by the nurse manager. Manual scheduling or conflict resolution can take hours or even days to complete. Individual preferences may not be reflected in the resulting base schedule due to the difficulties inherent in manual scheduling or to direct conflicts.

Many computerized nurse scheduling heuristics and optimization algorithms have been proposed; Kellogg and Walczak [3] found that few such approaches have been accepted by practitioners. Some reasons cited are that the models often narrow the scheduling-problem focus, research objectives do not always match practitioners' objectives, and there exists a general lack of trust by nurses of computerized models and the time required to learn new software. Kellog and Walczak [3] argue that there is a need for algorithms which accommodate the self-scheduling approach that currently predominates in practice.

We have developed a new method for scheduling base hours for a nursing unit. The method starts by obtaining nurses' preferences for specific days and shifts, and works to build a schedule that accommodates those preferences while maintaining important hospital constraints. An auction is used to obtain preferences: nurses bid on work shifts and rest days using "points," and shifts are awarded to the highest bidders insofar as possible.

The concept of an auction is not new in nurse scheduling. Auctions are currently used to fill vacant shifts after the base schedule is completed. Nurses bid an hourly rate for an available shift, and the lowest bid wins [4–6]. To the best of our knowledge, auctions have not been used to create the base schedule, because there is no built-in mechanism to ensure that hospital requirements are met. Such requirements include minimum time off between shifts, minimum and maximum hours worked per week, and coverage for each shift. Because of these constraints, a bid for a particular shift cannot be evaluated in isolation. A bid could be invalidated because it is inconsistent with other bids submitted by that individual, or because honoring the bid could prevent generation of a feasible schedule for other nurses in the unit.

Since the nurse scheduling problem cannot be solved via auction alone or by scheduling packages alone, we use a hybrid of the two by beginning with an auction stage and then supplementing the auction with an schedule completion stage. The objectives of the method are (1) to honor as many individual bids as possible, while maintaining hospital requirements, and (2) to reduce the time and effort spent by the nurse manager in scheduling work.

The remainder of this paper is organized as follows. Section 2 reviews previous approaches to nurse scheduling. Section 3 presents the general methodology. Section 4 describes a case study which applied the method in the Emergency Department at York Hospital. Section 5 reports on a simulation study to further characterize bidding patterns and their effects. Finally, practical challenges and suggestions for future research are presented.

2 Literature review

The nurse scheduling problem has been researched for over 30 years [7, 8]. The earliest approaches used cyclic (rotational) scheduling because it could be performed manually or with little computational effort [9]. In cyclic scheduling, a fixed set of schedules is generated and nurses are rotated among the schedules in consecutive time periods. Though easy to use, many nurses feel that cyclic scheduling does not provide enough flexibility [10]. Thus, subsequent research focused on non-cyclic scheduling, which generates unique schedules for each period based on specific requirements. Optimization methods are commonly used to solve non-cyclic scheduling problems, including integer programming [11], stochastic programming [12], non-linear programming [13], and goal programming [14–16].

The literature reveals that some researchers have ignored nurses' preferences completely [17, 18]. Others have developed a group preference rating for each shift by using perceived preferences [12], preferences for a particular shift pattern in a cyclic schedule [19], or aggregate preferences from survey [20]. A few researchers have attempted to incorporate individual nurse preferences, but they treat them as soft constraints which can be violated [21, 22].

In an attempt to provide nurses with more flexibility, "self-scheduling" was introduced [23–26]. This approach allows nurses to sign up for the shifts they want to work during each scheduling period, given pre-determined coverage needs and rules defining acceptable schedules. Self-scheduling may involve negotiation among nurses in the unit. The nurse manager coordinates the scheduling, resolves conflicts, and produces the final schedule. Nurses' perceived control of scheduling is positively associated with job satisfaction [2]; thus, allowing nurses to schedule themselves can improve retention. While self-scheduling provides more flexibility, it is time-consuming. In addition, it can be very difficult to guarantee fairness, especially in cases where signing up for shifts is done on a first-come, first-served basis.

Recently, auctions have emerged as a method for scheduling overtime hours. Nurses bid on open shifts, with bids starting at a predefined maximum pay rate; shifts are awarded to the lowest bidder [4–6]. User reports indicate that the average winning bid is higher than the nurses' base rates [6, 27]. Though some hospitals are concerned about this type of system [4], many hospitals have benefited from overtime shift auctions. Hospitals have experienced significant savings by reducing their use of temporary nurses from outside agencies (agency nurses are generally more expensive, and may be less productive if they are not familiar with the hospital's routines). Nurses are more satisfied because they control how much overtime they work and their overtime pay rate. In some hospitals, nurses are permitted to bid for overtime work in other departments, and thus can gain additional experience helpful for career advancement.

The overtime auctions seen in practice cannot be used for scheduling regular hours. Typically, nurses are contracted to work a specified number of base hours at a predetermined compensation rate; contract changes would be needed before any pay-based bidding system could be used. In addition, an auction alone cannot guarantee a schedule that meets coverage requirements and other hospital constraints. In the following section, we describe a combination of auctions and optimization to create a base schedule for a nursing unit.

3 General methodology

The scheduling process consists of two stages: an auction stage, in which nurses bid for their preferred shifts and the winners are selected, and a schedule completion stage, which assigns nurses to any vacant shifts. An auction can be set up in a variety of ways; we chose to use a sealed bid auction. A sealed bid auction is characterized by a single bidding round in which bidders do not see other individuals' bids and cannot update bids once they submit them [28]. A sealed bid auction fits well in this research because it is a simple and practical choice for nurses; they can submit bids once and not have to worry about updating them. Furthermore, an approach based on sealed bid auction provides an easy transition and improvement upon the self-scheduling methodology that nurses and hospitals currently favor [3].

Once bidding is complete, winners are selected using an optimization model which seeks to award shifts to the highest bidders while simultaneously meeting hospital requirements. After winners have been determined, the schedule completion stage uses a second optimization model to allocate the unfilled shifts to nurses who have not yet met their minimum hours. The stages are now explained in more detail.

3.1 Auction stage

The schedule period for the nursing unit (e.g. 1 week, 1 month), the number of shifts per day, and the required coverage for each shift are determined before the auction takes place. Nurses are allocated a predetermined number of points to use for bidding. In the present work, it is assumed that each nurse will have the same number of points.

The auction stage consists of a bidding step followed by an award step. The bidding is flexible; in other words, nurses may split up their points for shifts however they like. Nurses make their selections and submit a bid package with their "on" and "off" shifts and the number of points they have allocated to each. Each nurse submits a complete bid package before any awards are made. This bid package could be submitted on paper or using a web interface, depending on preferences of the particular hospital. Because we use a sealed bid auction, only one bidding round takes place.

In the award step, bids are sorted in descending order with respect to the point value, and the highest bidders are selected as candidate winners. The number of candidate winners for "on" shifts is dictated by the staffing requirement for the particular shift; thus, if three nurses are needed, then there may be a first, second, and third place candidate winner. For "off" bids, the number of candidate winners is determined by the total number of nurses less the upper limit of requirements for a particular shift. For example, if there are eight nurses available and each shift can have no more than five working at a time, then the total "off" candidate winners in the auction will be three.

The award step selects winners using an optimization model. The optimization model's constraints are specific to the particular hospital, since the schedule it generates must meet that hospital's requirements. In Section 4, we show the optimization model used for our case study.

The award step first checks the candidate winning bids to determine if a feasible schedule can be constructed if those candidates are selected as winners (awarded the shift). This is accomplished with the optimization model. If the candidate winning bids are feasible, the award step is complete and all candidates are awarded their shifts. If not, the model selects winners by maximizing the point value of all awarded bids. Any candidate winning bids which either violate hospital constraints or prevent construction of a feasible schedule will not be awarded. For example, if a nurse is a candidate winner of two consecutive 12 h shifts, the model will award at most one of them. Also, if a nurse is a candidate winner of a shift, but awarding it would cause another nurse to have insufficient rest time between shifts, the model will not award the shift.

Finally, the auction stage outputs a set of auction winners. These are the candidate winning bids which can be awarded while maintaining overall feasibility, and which maximize the total bid points awarded.

3.2 Schedule completion stage

The schedule completion stage of the model schedules additional shifts for nurses who have not met their working hour requirements by winning shifts in the auction. This stage guarantees that all shifts have adequate coverage and that the minimum nursing hours are satisfied for each nurse. The schedule completion stage also uses an optimization model. This second optimization model includes all the constraints used in the award model. Additional hard constraints require that wins from the auction stage are honored.

The assignment optimization model has a different objective function than was used in the award step. This objective function maximizes the total point value of assigned shifts, using only the bids that were not selected as candidate winners in the auction stage. Thus, it seeks a schedule in which losing bids are satisfied insofar as possible. For example, if a candidate winner cannot be awarded a shift in the auction stage, then another nurse will be assigned to that shift. The model will attempt to assign the highest bidding non-winner, consistent with schedule feasibility.

It is possible to construct a single optimization model that both awards bids and completes the schedule. This can be accomplished by suitably weighting the candidate winning bids in the objective function. However, we believe that a two-stage approach is preferable, for several reasons. First, the nurse manager could review the winning bids, and might choose to reject some of them. Second, a distinct auction stage can provide a basis for multiple auction rounds (this is discussed in Section 6). Third, solving two smaller problems reduces computational requirements, although for our case study described in Section 4 the reduction in solution time was minimal.

4 Case study

The method is now demonstrated using data from an emergency department (ED) at York Hospital in York, PA. The case study included registered nurses (RNs) in the emergency department at York Hospital for the schedule period of March 18–April 14, 2007. The schedule for this time period had already been created using their current self-scheduling method, which they have been using since January of 2007. Self-scheduling demands approximately 8 h/week for the head nurse in charge of scheduling. Seven to 8 weeks prior to the date a schedule is available, nurses are given blank scheduling sheets to fill in the shifts they prefer to work. According to the self-scheduling guidelines, nurses should sign up for all of their required hours. Nurses may also indicate up to 4 days they do not wish to work; any time off above 4 days in a week must be submitted as vacation time. Once the head nurse has all of the self-scheduling request forms, she manually enters them into Automated Nurse Scheduling Office System (ANSOS), a system which reports a 4-week schedule for a specified set of nurses. ANSOS was reported in [29] to have optimization capabilities; however York Hospital does not have the optimization feature in their system. ANSOS is mainly used for reporting the schedule and determining the vacancies on each shift after the self-scheduling requests are submitted. Using the vacancies, the program reports the “Needs List” which is made available for nurses who would like to sign up for overtime. The schedule is then completed and posted 3 to 4 weeks in advance.

The study was performed to see how well the auction procedure would work using constraints at a real hospital. The self-scheduling requests from that schedule period were converted into bids, and the schedule produced using our method was compared to the official schedule. This approach avoided disruption of normal hospital operating procedures or placing extra demands on the nurses. In this section the problem definition is presented, followed by our implementation of the bidding step, the optimization models used for the award and assignment, and the overall algorithm implementation.

4.1 Problem definition

York’s ED has different types of nurses who have various time commitments and experience levels. This particular case study includes registered nurses representing the following types:

- 28 full-time nurses who work 36 h/week,
- six full-time nurses who work 40 h/week,
- six part-time nurses; of these, one works 20 h/week, four work 24 h/week, and one works 28 h/week,
- 28 nurses under PRN contract. PRN means “pro re nata”, a Latin phrase meaning “occasionally” or “according to circumstances” [30]. These nurses are contracted to work either 16 or 24 h/month, with certain requirements for weekend hours,
- five traveler nurses who work 36 h/week during specified time periods.

The demand for staffing in the ED is measured in 4-h time blocks, as certain portions of the day are busier than others. In addition, nurses can work shifts of differing length, including 4-, 8-, and 12-h shifts. Table 1 shows the nurse requirement for each 4-h time block.

Since the demand for nurses is based on a particular time period rather than shifts, it is necessary to determine the number of winners allowed per time period. This requires careful consideration of overlapping shifts. As an example, consider Table 2.

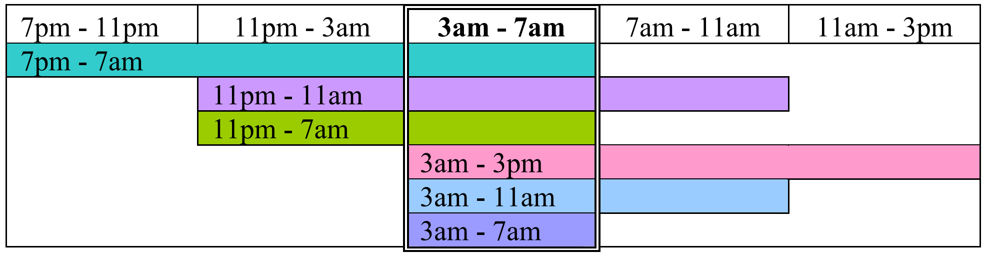
In Table 2, there are six shifts which overlap the 3 A.M.–7 A.M. time period. A bid on any of those shifts would be competing for the same time slot.

Selecting candidate winners is based upon whether the bid is the highest in every time period which it spans. For instance, a nurse who bids on the 3 A.M.–7 A.M. shift and is

Table 1 Nurse requirement for 4-h time blocks

Time period	Nurse requirement
3 A.M.–7 A.M.	10
7 A.M.–11 A.M.	10
11 A.M.–3 P.M.	16
3 P.M.–7 P.M.	18
7 P.M.–11 P.M.	18
11 P.M.–3 A.M.	14

Table 2 Example of shifts spanning multiple demand periods



the top bidder during all six time slots is a candidate winner. If the nurse is not the top bidder for each of the spanning slots but ranks high enough to fall within the number of winners allowed per slot, the bid is considered to be a candidate winner. In this example, the bid would need to be at least in tenth place since ten is the minimum nurse requirement for shifts that span this time slot. Note that nurses only place one bid for a shift; their one bid amount will be used to compare to any other bidder whose shift spans a common time slot.

The number of winners allowed for "off" or rest days must also be specified. After examining York's schedule the number of winners was set to 15 for Saturdays and Sundays and five for weekdays.

4.2 Bidding stage

While permission was not granted to perform active bidding with the RNs, the study was completed by using the self-scheduling requests. Since active bidding could not be done at this time, the bids were inferred from the self-scheduling requests. As an example, consider the partial schedule request shown in Table 3. This particular nurse requested three working shifts and 1 day off. To translate this request into a bid, we assumed that bids carry a weight in proportion to shift length, and that "off" bids are weighted more heavily than "on" bids. Specifically, we used weights of 1 (4-h shifts), 2 (8-h shifts), 3 (12-h shifts), and 6 (days off). These weights were normalized and converted to bid points. The example of Table 3, assuming a total of 100 points for the week, results in the bids shown in Table 4.

York hospital creates schedules every 4 weeks, so the self-scheduling requests covered a 28 day time period. We assumed that each nurse had 400 points for bidding over the period, and we calculated bids as described above. We used this approach in order to translate schedule requests which had already been submitted into bids for the award step. Note that in active bidding, nurses can make their bids however they want and would not have to follow any particular rule regarding allocation of points based on shift length.

4.3 Optimization models

Optimization models are used for the award step of the auction and for the schedule completion stage. The optimization models must be configured to match a particular hospital's work rules, employee contracts, and other constraints or preferences. In this section, we show the models used for the York Hospital ED.

4.3.1 Model notation

n	number of days in schedule
m	number of nurses available for the unit of interest
s	number of 4-h time slots per day
ℓ	number of weeks in the schedule, $\ell=n/7$
i	index for days, $i=1,\ldots,n$
j	index for 4-h time slots, $j=1,\ldots,s$
k	index for nurses, $k=1,\ldots,m$
t	index for weeks, $t=1,\ldots,\ell$
NR_{ij}	staff requirement for slot j of day i, $i=1,\ldots,n$ and $j=1,\ldots,s$
WE_{jt}	$\{i\|$ shift beginning on slot j of day i falls on a weekend on week $t\}$
36 h full-time	$\{k\|$ nurse k works 36 h/week$\}$
40 h full-time	$\{k\|$ nurse k works 40 h/week$\}$
PRN0	$\{k\|$ nurse k is PRN 0 designation$\}$
PRN1	$\{k\|$ nurse k is PRN 1 designation$\}$
PRN3	$\{k\|$ nurse k is PRN 3 designation$\}$
PRN3A	$\{k\|$ nurse k is PRN 3A designation$\}$
PRN4	$\{k\|$ nurse k is PRN 4 designation$\}$
trav	$\{k\|$ nurse k is a traveler nurse$\}$

Table 3 Example requests for a partial schedule

	Sun 3/18	Mon 3/19	Tues 3/20	Wed 3/21	Thurs 3/22	Fri 3/23	Sat 3/24
Nurse 1	11 A.M.–7 P.M.				11 A.M.–11 P.M.	11 A.M.–7 P.M.	Off

Table 4 Result of translating a request to a bid in example problem

	Sun 3/18	Mon 3/19	Tues 3/20	Wed 3/21	Thurs 3/22	Fri 3/23	Sat 3/24
Nurse 1	15				23	15	47

The nurses are divided into these subsets because their contractual requirements are different. Full-time nurses work either 36 or 40 h/week, and must work at least two Mondays or Fridays in a month. PRN0 nurses work 16 h/month. PRN1 nurses work 24 h/month; eight of those hours must be on a weekend. PRN3 and PRN3A nurses work 24 h/month on weekends only; the former work alternate weekends and the latter work three of four weekends. PRN4 nurses work two 12-h shifts every weekend. Traveler nurses work three 12-h shifts per week, on the 3 P.M.–3 A.M. or 7 P.M.–7 A.M. shifts only, and must work at least two Mondays or Fridays in a month.

Additional notation will be introduced where appropriate

4.3.2 Decision variables

At York Hospital, standard shifts consist of either 8 or 12 h. The decision variables were created to reflect the shift start time and number of hours on the shift:

$$X_{ijk} = \begin{cases} 1 \text{ if nurse } k \text{ is scheduled for the } 8-\text{h shift} \\ \text{beginning on slot } j \text{ of day } i \\ 0 \text{ otherwise} \end{cases}$$

$$Y_{ijk} = \begin{cases} 1 \text{ if nurse } k \text{ is scheduled for the} 12-\text{h shift} \\ \text{beginning on slot } j \text{ of day } i\,. \\ 0 \text{ otherwise} \end{cases}$$

In addition to these standard shifts, some part-time and PRN nurses work 4-h shifts. The following variables represent the 4-h shifts:

$$Z_{ijk} = \begin{cases} 1 \text{ if nurse } k \text{ is scheduled for the } 4-\text{h shift} \\ \text{beginning on slot } j \text{ of day } i \quad. \\ 0 \text{ otherwise} \end{cases}$$

Finally, a variable was created to represent rest days:

$$R_{ik} = \begin{cases} 1 \text{ if nurse } k \text{ is scheduled a day off on day } i \\ 0 \text{ otherwise} \end{cases}.$$

The hospital constraints at York Hospital are derived from hospital needs as well as considerations for nurses' health and well-being.

4.3.3 Constraints and objectives

1. *Satisfy daily staff requirements*. The staff requirements at York are determined over 4-h time periods as stated in the previous section. Nurses may work 4-, 8-, or 12-h shifts, thus some shifts overlap several time blocks:

$$\sum_{k=1}^{m}\sum_{b=j-1}^{j} X_{ibk} + \sum_{k=1}^{m}\sum_{b=j-2}^{j} Y_{ibk} + \sum_{k\notin \text{PRN4,trav}} Z_{ijk} \geq NR_{ij}, \quad \forall i=1,\ldots,n, \forall j=1,\ldots,s \tag{4.1}$$

Note that the index j wraps around to the previous day for $j \leq 2$. When $i=1$ and $b=-1$ or 0, it represents the last slots of the previous schedule. Thus, $Y_{i,0,k}$ corresponds to $Y_{28,6,k}$, or the sixth slot on the last day (day 28) of the previous schedule. Likewise $Y_{i,(-1)k}$ corresponds to $Y_{28,5,k}$, or the fifth slot on the last day (day 28) of the previous schedule. The values of $Y_{28,5,k}$ and $Y_{28,6,k}$ are known from the previous schedule and thus can be used as input for the current schedule as hard constraints. This idea will apply to all constraints which overlap the previous schedule henceforth.

2. *Minimum rest time between shifts*. Nurses must be given the proper amount of rest between working shifts. The minimum time between shifts for nurses at York Hospital is 8 h. Constraints 4.2a, 4.2b, 4.3a, 4.3b, 4.4a, 4.4b, 4.5a, 4.5b, 4.6a, and 4.6b were developed from the 4-h time blocks in Table 1 to ensure that each nurse has 8 h of rest between shifts:

$$Y_{(i-1)(j+4)k} + X_{(i-1)(j+5)k} + Y_{(i-1)(j+5)k} + \sum_{b=j}^{j+2}(X_{ibk} + Y_{ibk} + Z_{ibk}) \leq 1, \quad \forall i=1,\ldots,n, \forall k \notin \text{PRN4, trav}, j=1 \tag{4.2a}$$

$$Y_{(i-1)(j+4)k} + X_{(i-1)(j+5)k} + Y_{(i-1)(j+5)} + \sum_{b=j}^{j+2}(X_{ibk} + Y_{ibk}) \leq 1, \quad \forall i=1,\ldots,n, \forall k \in \text{PRN4, trav}, j=1 \tag{4.2b}$$

$$Y_{(i-1)(j+4)k} + \sum_{b=j-1}^{j+2}(X_{ibk} + Y_{ibk}) + \sum_{b=j}^{j+2} Z_{ibk} \leq 1, \quad \forall i=1,\ldots,n, \forall k \notin \text{PRN4, trav}, j=2 \tag{4.3a}$$

$$Y_{(i-1)(j+4)k} + \sum_{b=j-1}^{j+2} (X_{ibk} + Y_{ibk}) \leq 1, \quad \forall i = 1, ..., n, \forall k \in \text{PRN4, trav}, j = 2 \tag{4.3b}$$

$$\sum_{b=j-1}^{j+2} X_{ibk} + \sum_{b=j-2}^{j+2} Y_{ibk} + \sum_{b=j}^{j+2} Z_{ibk} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \text{PRN4, trav}, j = 3, 4 \tag{4.4a}$$

$$\sum_{b=j-1}^{j+2} X_{ibk} + \sum_{b=j-2}^{j+2} Y_{ibk} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \text{PRN4, trav}, j = 3, 4 \tag{4.4b}$$

$$\sum_{b=j-1}^{j+1} X_{ibk} + \sum_{b=j-2}^{j+1} Y_{ibk} + \sum_{b=j}^{j+1} Z_{ibk} + X_{(i+1)(j-4)k} + Y_{(i+1)(j-4)k} + Z_{(i+1)(j-4)k} \leq 1, \quad \forall i = 1, ..., n-1, \forall k \notin \text{PRN4, trav}, j = 5 \tag{4.5a}$$

$$\sum_{b=j-1}^{j+1} X_{ibk} + \sum_{b=j-2}^{j+1} Y_{ibk} + X_{(i+1)(j-4)k} + Y_{(i+1)(j-4)k} \leq 1, \quad \forall i = 1, ..., n-1, \forall k \in \text{PRN4, trav}, j = 5 \tag{4.5b}$$

$$\sum_{b=j-1}^{j} X_{ibk} + \sum_{b=j-2}^{j} Y_{ibk} + Z_{ijk} + \sum_{b=j-5}^{j-4} X_{(i+1)bk} + \sum_{b=j-5}^{j-4} Y_{(i+1)bk} + \sum_{b=j-5}^{j-4} Z_{(i+1)bk} \leq 1, \quad \forall = 1, ..., n-1, \forall k \notin \text{PRN4, trav}, j = 6 \tag{4.6a}$$

$$\sum_{b=j-1}^{j} X_{ibk} + \sum_{b=j-2}^{j} Y_{ibk} + \sum_{b=j-5}^{j-4} X_{(i+1)bk} + \sum_{b=j-5}^{j-4} Y_{(i+1)bk} \leq 1, \quad \forall = 1, ..., n-1, \forall k \in \text{PRN4, trav}, j = 6. \tag{4.6b}$$

Constraint 4.7 allows no more than one 4-h shift per nurse on a single day; this is to ensure that nurses, for example, do not get scheduled for a 7 A.M.–11 A.M. shift and then a 7 P.M.–11 P.M. shift in the same day. While this combination is technically valid in the sense of giving enough rest time between shifts, it is quite impractical to come in and work two 4-h shifts which are spread out. Two sequential 4-h shifts are not allowed because this would be assigned as one 8-h shift. Since nurses who are of PRN4 designation or traveler nurses work 12-h shifts, they will not be eligible to work 4-h shifts:

$$\sum_{j=1}^{6} Z_{ijk} \leq 1, \forall i = 1, ..., n, \forall k \notin \text{PRN4, trav}. \tag{4.7}$$

In addition, constraints were needed to accommodate rest days; these constraints are different for PRN4 and traveler nurse because they only work 12-h shifts:

$$\sum_{j=1}^{3} \left(X_{ijk} + Y_{ijk} + Z_{ijk}\right) + R_{ik} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \text{PRN4, trav}, \tag{4.8a}$$

$$\sum_{j=1}^{3} \left(X_{ijk} + Y_{ijk}\right) + R_{ik} \leq 1, \forall i = 1, ..., n, \forall k \in \text{PRN4, trav}, \tag{4.8b}$$

$$\sum_{j=4}^{6} \left(X_{ijk} + Y_{ijk} + Z_{ijk}\right) + R_{ik} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \text{PRN4, trav}, \tag{4.9a}$$

$$\sum_{j=4}^{6} \left(X_{ijk} + Y_{ijk}\right) + R_{ik} \leq 1, \forall i = 1, ..., n, \forall k \in \text{PRN4, trav}. \tag{4.9b}$$

A consideration for rest days is that nurses do not prefer to be scheduled beyond 11 P.M. the night before a rest day or before 7 A.M. the morning after a rest day. This is not a strict policy, and nurses may work during the aforementioned times if they prefer. Thus, the following constraints apply unless otherwise requested in the bid package:

$$\sum_{j=5}^{6} X_{(i-1)jk} + \sum_{j=4}^{6} Y_{(i-1)jk} + Z_{(i-1)6k} + R_{ik} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \text{PRN4, trav}, \tag{4.10a}$$

$$\sum_{j=5}^{6} X_{(i-1)jk} + \sum_{j=4}^{6} Y_{(i-1)jk} + R_{ik} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \text{PRN4, trav}, \tag{4.10b}$$

$$R_{ik} + X_{(i+1)1k} + Y_{(i+1)1k} + Z_{(i+1)1k} \leq 1, \quad \forall i = 1, ..., n-1, \forall k \notin \text{PRN4, trav}, \tag{4.11a}$$

$$R_{ik} + X_{(i+1)1k} + Y_{(i+1)1k} \leq 1, \quad \forall i = 1, ..., n-1, \forall k \in \mathrm{PRN4, trav.} \tag{4.11b}$$

Most nurses at York Hospital do not prefer to "double-back", meaning they do not prefer to be scheduled at 7 A.M. if they worked until 11 P.M. on the previous day. This is not a strict requirement but will be enforced unless otherwise requested:

$$X_{(i-1)4k} + Y_{(i-1)3k} + Z_{(i-1)5k} + X_{i2k} + Y_{i2k} + Z_{i2k} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \mathrm{PRN4, trav,} \tag{4.12a}$$

$$X_{(i-1)4k} + Y_{(i-1)3k} + X_{i2k} + Y_{i2k} \leq 1, \quad \forall i = 1, ..., n, \forall k \in \mathrm{PRN4, trav.} \tag{4.12b}$$

Similarly, nurses will not be scheduled at 11 A.M. if they worked until 3 A.M. on the previous day unless otherwise requested:

$$X_{(i-1)5k} + Y_{(i-1)4k} + Z_{(i-1)6k} + X_{i3k} + Y_{i3k} + Z_{i3k} \leq 1, \quad \forall i = 1, ..., n, \forall k \notin \mathrm{PRN4, trav,} \tag{4.13a}$$

$$X_{(i-1)5k} + Y_{(i-1)4k} + X_{i3k} + Y_{i3k} \leq 1, \quad \forall i = 1, ..., n, \forall k \in \mathrm{PRN4, trav.} \tag{4.13b}$$

3. *Working days per schedule*. The number of work days for each nurse is dependent on whether the nurse is full-time, part-time, or PRN. Full-time nurses are under contract to work either 36 or 40 h/week. Nurses may work shifts in any combination that adds up to their required hours. For example those who work 36 h/week may work three 12-h shifts or three 8-h shifts and one 12-h shift per week:

$$\sum_{i=7t-6}^{7t} \sum_{j=1}^{s} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 36, \forall t = 1, ..., \ell, \quad \forall k \in 36\,\mathrm{h\ full-time,} \tag{4.14}$$

$$\sum_{i=7t-6}^{7t} \sum_{j=1}^{s} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 40, \forall t = 1, ..., \ell, \quad \forall k \in 40\,\mathrm{h\ full-time.} \tag{4.15}$$

PRN nurses work shifts according to their PRN designation. Note that, according to payroll, the "weekend" begins Friday at 3 P.M. and ends on Monday at 7 A.M.

PRN 0: 16 h/month

$$\sum_{i=1}^{n} \sum_{j=1}^{s} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 16, \forall k \in \mathrm{PRN0} \tag{4.16}$$

PRN 1: 24 h/month of which 8 h must be on a weekend

$$\sum_{i=1}^{n} \sum_{j=1}^{s} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 8, \forall k \in \mathrm{PRN1} \tag{4.17}$$

$$\sum_{t=1}^{\ell} \sum_{j=1}^{s} \sum_{i \in \mathrm{WE}_{jt}} \left(8X_{ijk} + 4Z_{ijk}\right) = 8, \forall k \in \mathrm{PRN1} \tag{4.18}$$

PRN 3: 24 h every other weekend

$$\sum_{j=1}^{s} \sum_{i \in \mathrm{WE}_{jt}} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 24 \times p_{1k}, \quad \forall k \in \mathrm{PRN3}, t = 1 \tag{4.19}$$

$$\sum_{j=1}^{s} \sum_{i \in \mathrm{WE}_{jt}} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 24 \times p_{2k}, \quad \forall k \in \mathrm{PRN3}, t = 2 \tag{4.20}$$

$$\sum_{j=1}^{s} \sum_{i \in \mathrm{WE}_{jt}} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 24 \times p_{3k}, \quad \forall k \in \mathrm{PRN3}, t = 3 \tag{4.21}$$

$$\sum_{j=1}^{s} \sum_{i \in \mathrm{WE}_{jt}} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) = 24 \times p_{4k}, \quad \forall k \in \mathrm{PRN3}, t = 4 \tag{4.22}$$

$$p_{1k} + p_{2k} = 1, \forall k \in \mathrm{PRN3} \tag{4.23}$$

$$p_{2k} + p_{3k} = 1, \forall k \in \mathrm{PRN3} \tag{4.24}$$

$$\begin{aligned} &p_{3k} + p_{4k} = 1, \forall k \in \mathrm{PRN3} \\ &p_{1k}, p_{2k}, p_{3k}, p_{4k} \text{ binary} \end{aligned} \tag{4.25}$$

PRN 3A: 24 h three out of four weekends

$$\sum_{j=1}^{s}\sum_{i\in \mathrm{WE}_{jt}}\left(8X_{ijk}+12Y_{ijk}+4Z_{ijk}\right)=24\times p_{5k},\quad \forall k\in \mathrm{PRN3A}, t=1 \tag{4.26}$$

$$\sum_{j=1}^{s}\sum_{i\in \mathrm{WE}_{jt}}\left(8X_{ijk}+12Y_{ijk}+4Z_{ijk}\right)=24\times p_{6k},\quad \forall k\in \mathrm{PRN3A}, t=2 \tag{4.27}$$

$$\sum_{j=1}^{s}\sum_{i\in \mathrm{WE}_{jt}}\left(8X_{ijk}+12Y_{ijk}+4Z_{ijk}\right)=24\times p_{7k},\quad \forall k\in \mathrm{PRN3A}, t=3 \tag{4.28}$$

$$\sum_{j=1}^{s}\sum_{i\in \mathrm{WE}_{jt}}\left(8X_{ijk}+12Y_{ijk}+4Z_{ijk}\right)=24\times p_{8k},\quad \forall k\in \mathrm{PRN3A}, t=4 \tag{4.29}$$

$$\begin{aligned}&p_{5k}+p_{6k}+p_{7k}+p_{8k}=3, \forall k\in \mathrm{PRN3A}\\&p_{5k},p_{6k},p_{7k},p_{8k}\ \text{binary}\end{aligned} \tag{4.30}$$

PRN 4: two 12-h shifts every weekend

$$\sum_{j=1}^{s}\sum_{i\in \mathrm{WE}_{jt}}12Y_{ijk}=24, \forall k\in \mathrm{PRN4}, \forall t=1,\ldots,\ell \tag{4.31}$$

Also, since it is known that PRN 4 nurses do not normally work 8-h shifts or weekday shifts, those variables can be set to zero. PRN 4 nurses also do not work 4-h shifts, but this was accounted for in previous constraints:

$$X_{ijk}=0, \forall i=1,\ldots,n, \forall j=1,\ldots,s, \forall k\in \mathrm{PRN4} \tag{4.32a}$$

$$Y_{ijk}=0, \forall i\notin WE_{jt}, \forall j=1,\ldots,s, \forall k\in \mathrm{PRN4}, \forall t=1,\ldots,\ell \tag{4.32b}$$

Traveler nurses work 36 h/week and are designated to the following shifts: 3 P.M.–3 A.M. or 7 P.M.–7am (j=4 or 5):

$$\sum_{i=7t-6}^{7t}12Y_{i,4,k}+\sum_{i=7t-6}^{7t}12Y_{i,5,k}=36, \forall k\in \text{trav}, \forall t=1,\ldots,\ell \tag{4.33}$$

Similar to the PRN 4 constraint set, the number of variables can be reduced by eliminating shifts that traveler nurses cannot work, including all 8-h shifts and any 12-h shifts which do not fall in their assigned time slot:

$$X_{ijk}=0, \forall i=1,\ldots n, \forall j=1,\ldots,s, \forall k\in \text{trav} \tag{4.34a}$$

$$Y_{ijk}=0, \forall i=1,\ldots,n, \forall j\neq 4,5, \forall k\in \text{trav} \tag{4.34b}$$

Part-time nurses work either 20, 24, or 28 h/week:

$$\sum_{i=7t-6}^{7t}\sum_{j=1}^{s}\left(8X_{ijk}+12Y_{ijk}+4Z_{ijk}\right)=20, \forall k\in \mathrm{PT20},\quad \forall t=1,\ldots,\ell \tag{4.35}$$

$$\sum_{i=7t-6}^{7t}\sum_{j=1}^{s}\left(8X_{ijk}+12Y_{ijk}+4Z_{ijk}\right)=24,\quad \forall k\in \mathrm{PT24}, \forall t=1,\ldots,\ell \tag{4.36}$$

$$\sum_{i=7t-6}^{7t}\sum_{j=1}^{s}\left(8X_{ijk}+12Y_{ijk}+4Z_{ijk}\right)=28, \forall k\in \mathrm{PT28},\quad \forall t=1,\ldots,\ell \tag{4.37}$$

4. *Monday/Friday constraint*. Every nurse except PRN nurses must work at least two Mondays and/or Fridays in a month. Let MF represent the set days in the month that are Mondays or Fridays:

$$\sum_{i\in \mathrm{MF}}\sum_{j=1}^{s}\left(X_{ijk}+Y_{ijk}+Z_{ijk}\right)\geq 2,\quad \forall k\notin \mathrm{PRN0}, \mathrm{PRN1}, \mathrm{PRN3}, \mathrm{PRN3A}, \mathrm{PRN4} \tag{4.38}$$

5. *Treatment of nurse shortage*. Since it is known that the York Hospital Emergency Department is understaffed, there will not be enough nurses to meet coverage requirements. To combat this issue, a dummy variable D_{ij} is introduced for each time slot on each day. Thus, when the model completes, D_{ij} represents the number of nurses still needed on a particular 4-h time slot. This will be used to determine the blocks of time which are suitable for any fill-in agency nurses or overtime sign-ups. With a nurse shortage, Eq. 4.1 is rewritten as Eq. 4.39:

$$\sum_{k=1}^{m}\sum_{b=j-1}^{j}X_{ibk}+\sum_{k=1}^{m}\sum_{b=j-2}^{j}Y_{ibk}+\sum_{k=1}^{m}Z_{ijk}+D_{ij}\geq NR_{ij},\quad \forall i=1,\ldots,n, \forall j=1,\ldots,s. \tag{4.39}$$

Another constraint is added to prevent D_{ij} from becoming too large. The staff requirement for each 4-h time slot is known. For example, it is ideal for the 3 P.M.–7 P.M. slot to have 18 nurses working, but this goal is rarely achieved without offering overtime. To restrict the dummy variable, the model requires that at least half of the nurse requirement is met by nurses under regular hours:

$$D_{ij} \leq \frac{\mathrm{NR}_{ij}}{2}, \forall i = 1, ..., n, \forall j = 1, ..., s \tag{4.40}$$

D_{ij} integer $\forall i = 1, ..., n, \forall j = 1, ..., s.$

6. *Other considerations*. Nurses often have mandatory education days or "project" days in which they are required to be at the hospital but are unavailable to care for patients. In most cases, the nurses will be on the schedule to work a total of 8 h. These meetings are counted in their weekly hours but the nurses are not counted toward the staff requirement for primary care. To model this, the rest variable R_{ik} is used in the constraints on working hours. Let PM_k represent the set of project or education days for nurse k. For example, constraint 4.14 would change to the following if a project or education day was specified:

$$\sum_{i=7t-6}^{7t} \sum_{j=1}^{s} \left(8X_{ijk} + 12Y_{ijk} + 4Z_{ijk}\right) + \sum_{i \in \mathrm{PM}_k} 8R_{ik} = 36, \forall t = 1, ..., \ell, \quad \forall k \in 36\,\mathrm{h\,full-time}. \tag{4.41}$$

Thus, the 8 h would be reflected in the nurse's hours but not falsely counted towards the coverage requirement. Since project/education days are mandatory, they will be set to one in the formulation:

$$R_{ik} = 1, \forall i \in \mathrm{PM}_k. \tag{4.42}$$

Paid time off (PTO) may also be requested while bidding for the schedule. PTO includes vacation days or time that has accrued according to hours worked. Nurses who would like to use PTO will not have to use points to bid for it—it will automatically be considered a day off. (Requests for days off which are not based on PTO must be done as bids.) Similar to project and education days previously described, PTO is counted towards the nurses' weekly time commitments. Let PTO_k represent the case where nurse k would like to use paid time off. The following constraint would be added to the formulation for a PTO request:

$$R_{ik} = 1, \forall i \in \mathrm{PTO}_k. \tag{4.43}$$

7. *Objective function for the award model*. Additional notation is needed to construct the objective function. In the following definitions, "win" refers to a candidate winner:

WX_{jk} $i|$ nurse k wins an 8-h shift starting on slot j, i=1,…,n, j=1,…,s, k=1,…,m}
WY_{jk} $i|$ nurse k wins a 12-h shift starting on slot j, i=1,…,n, j=1,…,s, k=1,…,m}
WZ_{jk} $i|$ nurse k wins a 4-h shift starting on slot j of day i, i=1,…,n, j=1,…,s, k=1,…,m}
WR_k $i|$ nurse k wins a day off on day i, i=1,…,n}
LX_{jk} $i|$ nurse k loses an 8-h shift starting on slot j of day i, i=1,…,n, j=1,…,s, k=1,…,m}
LY_{jk} $i|$ nurse k loses a 12-h shift starting on slot j of day i, i=1,…,n, j=1, …,s, k=1,…,m}
LZ_{jk} $i|$ nurse k loses a 4-h shift starting on slot j of day i, i=1,…,n, j=1,…,s, k=1,…,m}
LR_k $i|$ nurse k loses a day off on day i, i=1,…,n}
PX_{ijk} points bid by nurse k on the 8-h shift starting on slot j of day i, i=1,…,n, j=1,…,s, k=1,…,m
PY_{ijk} points bid by nurse k on the 12-h shift starting on slot j of day i, i=1,…,n, j=1,…,s, k=1,…,m
PZ_{ijk} points bid by nurse k on the 4-h shift starting on slot j of day i, i=1,…,n, j=1,…,s, k=1,…,m
PR_{ik} points bid by nurse k on the day off of day i, i=1,…,n, k=1,…,m

Note that the set of winners WX_{jk}, WY_{jk}, WZ_{jk}, and WR_k represent the candidate winners in the auction, in other words, those who successfully "placed" for a given time slot during the auction step.

The objective function in the award step focuses on maximizing the point value of bids awarded to the candidate winners:

$$\begin{aligned} \text{Maximize } Z = & \sum_k \sum_j \sum_{i \in \mathrm{WX}_{jk}} PX_{ijk} \cdot X_{ijk} \\ & + \sum_k \sum_j \sum_{i \in WY_{jk}} PY_{ijk} \cdot Y_{ijk} \\ & + \sum_k \sum_j \sum_{i \in WZ_{jk}} PZ_{ijk} \cdot Z_{ijk} \\ & + \sum_k \sum_{i \in WR_{jk}} PR_{ik} \cdot R_{ik}. \end{aligned} \tag{4.44}$$

8. *Objective function for the assignment model*. The schedule completion stage assigns shifts to nurses to meet hourly work requirements remaining after the auction stage. Hard constraints are used to represent the bids won in the auction. The objective function for the schedule completion stage seeks to award bids which were not selected as candidate winners but can create a

feasible assignment. The objective function in this step is the following:

$$\begin{aligned}\text{Maximize } Z = &\sum_k \sum_j \sum_{i \in LX_{jk}} PX_{ijk} \cdot X_{ijk} \\ &+ \sum_k \sum_j \sum_{i \in LY_{jk}} PY_{ijk} \cdot Y_{ijk} \\ &+ \sum_k \sum_j \sum_{i \in LZ_{jk}} PZ_{ijk} \cdot Z_{ijk} \\ &+ \sum_k \sum_{i \in LR_{jk}} PR_{ik} \cdot R_{ik} .\end{aligned} \tag{4.45}$$

4.4 Algorithm for nurse scheduling

The bidding process and optimization model were described in the previous sections; the general solution process is outlined in Algorithm 1. First, variables such as project days, which are determined outside of the bidding process, are fixed. Then, the candidate winning bids are found and the optimization is applied to award those that are feasible. Remaining shifts are then assigned using optimization.

Algorithm 1

1. *Pre-processing*

   Determine which nurses have PTO, project, and/or education days. Add Eqs. 4.42 and 4.43 to the formulation for the appropriate nurses. Adjust constraints 4.16–4.37 to account for PTO, project, and/or education days as shown in the example in Eq. 4.41.
2. *Auction stage: bidding, sorting, and winner determination*

   After bidding has completed, sort the bids in descending order and determine those bids which are high enough to place as candidate winners. Determine which winning bids are feasible by solving the formulation in Section 4.3.3 using the objective function 4.44. The output is a set of winning bids.
3. *Schedule completion stage*

   For the winning bids, add a constraint for each to guarantee the award:

$$X_{ijk} = 1, \forall j, k, \ i \in WX_{jk}, \tag{4.46}$$

$$Y_{ijk} = 1, \forall j, k, \ i \in WY_{jk}, \tag{4.47}$$

$$Z_{ijk} = 1, \forall j, k, \ i \in WZ_{jk}, \tag{4.48}$$

$$R_{ik} = 1, \forall k, \forall \, i \in WR_k. \tag{4.49}$$

Since the winning bids are guaranteed using the constraints above, we now consider the losing bids by using the objective function in Eq. 4.45. Solve the full formulation with the additional constraints in Eqs. 4.46, 4.47, 4.48, and 4.49.

4.5 Case study results

The model contains 29,344 variables and 32,892 constraints, and is generated automatically. We developed a program to read the bids, select the candidate winners, generate the formulation, and call an optimization solver. Determining the candidate winners and generating the formulation took 2.073 s. The award stage took 2 min and 55 s, and the schedule completion stage took 5.74 s. LINGO was used for optimization, and the software was run on an Intel Core 2 Duo processor T7200 (2 GHz, 1 GB RAM). Altogether, the methodology required slightly more than 3 min of CPU time to generate a schedule, a significant time savings as compared to the manual method currently in use at York Hospital.

As evident in Table 5, the two methods are comparable in terms of percentage of requests fulfilled, but the auction-optimization model performed slightly better for "on" bids. The manual method awarded more "off" requests; however, after further investigation, it was found that the only losing "off" bids in the auction-optimization model were attributed to a traveler nurse who was ending a contract. This particular traveler nurse had written an "X" on his request form, which was meant to represent an end in contract but was mistakenly read as an "off" bid during the bid translation. Thus, four of the "off" bids which were not granted to the traveler nurse were actually days he was not supposed to be included on the schedule. When the constraints in the model were altered to reflect the actual situation, the unfulfilled "off" bids were avoided. Thus, the methods are equal in terms of awarding "off" shifts for this schedule. We chose to present this issue rather than correct it in the table to bring attention to such issues which may arise in practice. For implementation to be successful, nurse managers must be able to input any changes to the constraints on the front-end before the model is run each scheduling period.

This study has demonstrated that the auction-optimization approach can capture both realistic hospital constraints and

Table 5 Comparison of overall success

Request Type	Percent requests fulfilled self-scheduling	Percent requests fulfilled auction-optimization model
"On" requests	90.48	98.27
"Off" requests	100	95.51

individual preferences, and can use them to generate a good schedule. Our schedule is comparable to one generated by an experienced nurse manager. It took approximately 3 min to produce the schedule once the bids were input, whereas the current self-scheduling process demands approximately 8 h/week of the head nurse's time per schedule.

An advantage of the auction-optimization approach is its similarity to self-scheduling: the bidding stage is very similar to the request submission process already in place. The only difference is that nurses will submit point values with their shift requests to reflect their strength of preference. Optionally, nurses could be allowed to bid on their strongest preferences rather than specify all of their working hours as with self-scheduling. If they bid on fewer hours than required, the assignment model would add shifts up to their required hours.

It should be noted that the emergency department at York Hospital is understaffed. Deficits are filled by allowing nurses to sign up for overtime hours or by allowing agency nurses to pick up shifts. If the hospital had adequate staffing, there could be more competition for shifts or rest days, and it is possible that the win percentages would be lower.

5 Factors affecting performance

The auction-optimization approach was successful in creating a schedule that fulfilled most of the York Hospital ED nurses' requests. As previously noted, the department was understaffed at that time. Options for scheduling were somewhat constrained due to the different designations of nurses, and considerable flexibility was allowed in specifying the length of a work shift. The model also included some rather complex rules for rest periods. In this section, we further examine performance of the auction-optimization approach using simulated bidding and a much simpler scheduling environment.

The simplified problem consists of 7 days with two 12-h shifts per day. Coverage requirements are a minimum of two and a maximum of six nurses per shift. Individual nurses can work three or four shifts per week. A work shift must be followed by a rest shift, and no nurse can work more than three consecutive days. Bidding was performed via simulation, and each simulated nurse was allotted 100 points. Experiments were conducted to understand the effects of staff availability, bidding preferences, and number of bids for rest days.

5.1 Staff availability

If individual preferences are not considered, a feasible schedule that meets the minimum coverage requirement can be constructed with six nurses. We simulate the bidding process for seven to 14 nurses, with 100 replications at each staff level. In this case, we assume that nurses will bid for work shifts only (we investigate rest shifts in Section 5.3). We assume randomly distributed preferences: each simulated nurse chose between one and four shifts at random, and then distributed their bid points randomly among the selected shifts. After the simulated bidding was completed, the award step and schedule completion stage were executed.

Figure 1 shows the percentage of bids satisfied in the experiment. The lowest percentage was for seven nurses, at 81.4% either awarded in the auction stage or assigned later. Note that some bids were voided in this case also. As the number of nurses increased, the percentage of awards decreased, but the schedule completion stage was able to satisfy most of the requests, resulting in an overall increase in satisfaction of preferences, reaching 99.9% with 14 nurses.

5.2 Weighted preferences

Purely random bidding behavior is unlikely, so we investigate a situation where preferences are higher for certain shifts. Simulated nurses select work shifts using the following probabilities: week day first shift=4/9, week day second shift=2/9, weekend first shift=2/9, and weekend second shift=1/9. As in the previous experiment, we conduct 100 replications at each staffing level.

Figure 2 shows the same general trend of awards and assignments, but with seven nurses only 59.7% of the preferences were satisfied, rising to 92.7% with 14 nurses.

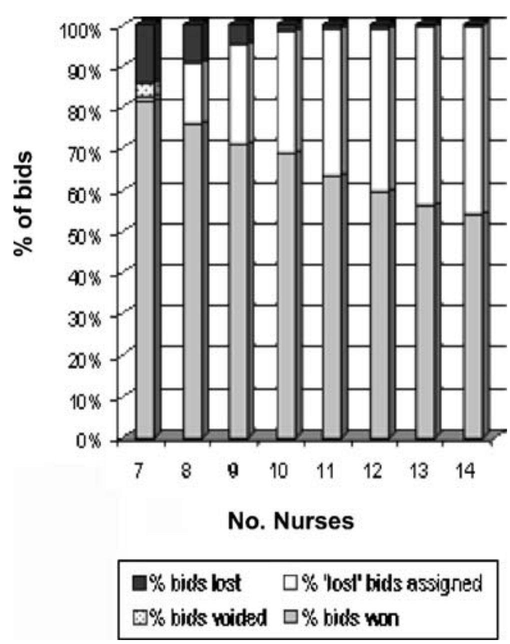


Fig. 1 Percentage of bids as a function of the number of nurses for preferences following a discrete uniform distribution

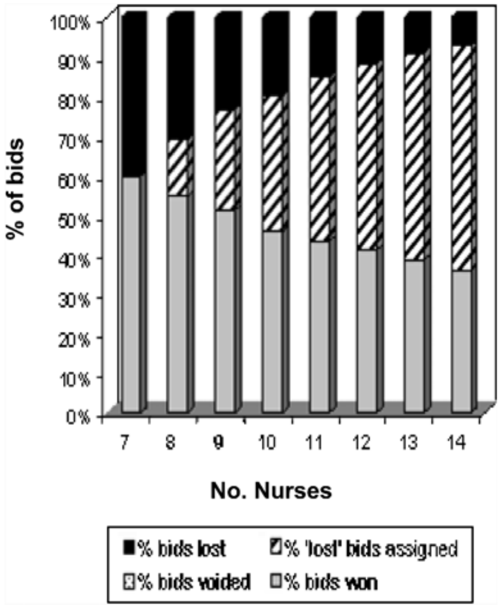


Fig. 2 Percentage of bids as a function of the number of nurses for preferences favoring weekdays

With more people favoring certain shifts, the ability to satisfy preferences will necessarily be lower. In the case of seven nurses, ten of the 100 replications included one nurse for whom no preferred shifts were assigned in the final schedule. Those simulated nurses had bid for either three or four shifts (and thus allocated fewer points to any particular choice). This suggests that in a situation with tight scheduling constraints, it might be beneficial to bid on a few highly preferred choices rather than a complete schedule. In practice, an administrator might choose to restrict the minimum or maximum number of bids by any one individual if this is a concern.

5.3 Bidding for rest shifts

In many situations, staff may have stronger preferences for specific time off rather than for specific working shifts. We investigate this issue by allowing the simulated nurses to bid for one to ten off shifts (ten was chosen because the schedule has 14 shifts total, and an individual can be scheduled for four work shifts). For each scenario, the number of "off" bids was fixed; simulated nurses randomly chose "off" shifts and bid points on each. After bidding for "off" shifts, they used their remaining points to bid for a random number of working or "on" shifts, ranging from zero to four. Each selected shift was randomly assigned a preference value and the bid points were allocated to shifts in proportion to the strength of preference. One hundred replications were performed at each design point.

We report results for seven nurses, since this case is most tightly constrained. Over the entire experiment, 92.6% of bids were either awarded or assigned, but the loss percentage varies depending on the nature of the bids. Figure 3 shows the likelihood of losing an "off" bid as a function of the number of bids placed. As the number of "off" bids increases, the average percentage of lost bids increases. As nurses try to get more shifts off, they will lose bids at a higher rate because they will have fewer points per shift to bid. Again, this suggests that in a tight scheduling scenario, a strategy of making fewer high value bids might be beneficial.

However, the curves level off when nurses bid for three or four "on" shifts. In these cases, the simulated nurses have placed bids on 11 or more of the 14 shifts. We surmise that when nurses bid on a very large number of shifts it may be more costly to the objective function to deny a bid; the nurse who places a large number of bids would reduce the objective function in two ways: for not being assigned to a shift with an "on" bid and for assignment to a shift with an "off" bid.

In summary, the auction-optimization model appears to perform quite well in this experiment, in that it is able to award or assign a large percentage of the shifts that were bid. Overall "win" percentage decreases if there is no excess capacity and also if most individuals have similar preferences, but still exceeded 59% in the worst case we tested. In general, it appears that it is not advantageous to bid on a large number of shifts, although the case of placing a bid on all or almost all shifts is not quite as clear. Note that the experiment assumed that each nurse placed bids independently of other nurses. In actual practice, nurses might choose to resolve some conflicts among themselves before submitting a bid package.

6 Practical challenges and future research

The case study used self-scheduling requests to test the behavior of the auction-optimization model in a real

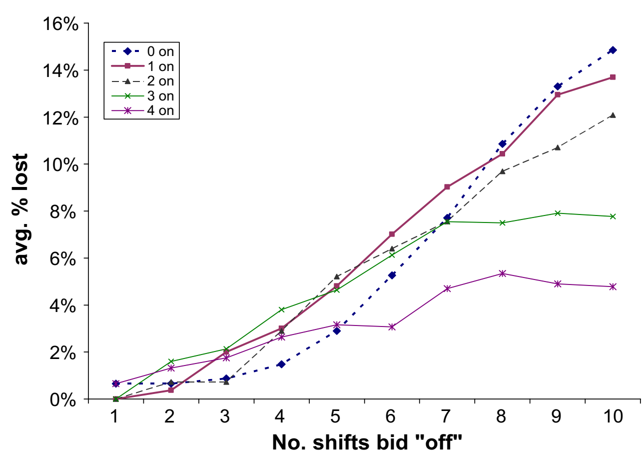


Fig. 3 Average percent bids lost vs. no. of "off" shifts for seven nurses

hospital setting. To avoid disrupting the normal operations at York Hospital, an experiment with active bidders was not performed. Further study is required with actual bidding, perhaps using a web-based application for bid submission.

The objective functions for the optimization models focused on maximizing points, and in general, the highest bidders are awarded or assigned their requests. While this objective is fair in that the highest bidders will win, there are situations where it may not be ideal. Suppose a nurse bids on two preferred shifts, one of which is more popular with the other nurses. In order to win, the nurse would have to bid more points on the popular shift, even if he/she ranked the less popular shift as personally more desirable. If conflicts were such that the nurse could be awarded either one of the two shifts, but not both, he/she would be awarded the higher bid value (and less preferred) shift.

Sönmez and Ünver [31] discussed this issue in the context of bidding for course schedules in business schools. In course bidding (as in our approach), bids are used to infer preferences as well as select winners. The authors show that the two roles of bids can conflict, because the strength of bid must reflect the competition for a particular course and thus can differ from the individual's preference for that course. The authors discuss making a distinction between the two roles of bids by asking students to state preference for courses in addition to bid amounts.

In our nurse scheduling methodology, selection of winners is not based solely on the value of the bid, so it is unclear how often conflicts between preferences and bid amounts will occur. In addition, shift popularity is likely to change across days and weeks and so is a less reliable indicator of competition. Even so, individual nurses might bid based on their beliefs about shift popularity. Further study could evaluate a system that allows nurses to rank shifts they bid on in order of preference so that preferences do not have to be inferred by bid amount.

In this paper, we assumed that each nurse had the same number of points to use for bidding. As an alternative, certain nurses could be assigned more points for reasons such as seniority, exceptional job performance, etc. Another alternate treatment is to allow unsuccessful bidders in one particular schedule period to roll over the unused points to the next schedule period; this might balance shifts awarded over the long run.

The optimization model used in the case study reflected a situation in which all nurses have similar competencies. If this were not the case, additional constraints could be added to, for example, require nurses with special competencies to work together. Bidding could be modified to allow a nurse to express a preference for working with another nurse, rather than on a particular shift.

Finally, we employed a single-round sealed-bid auction for the case study and subsequent experiments, because of its close parallel to self-scheduling. This could easily be extended to multiple rounds by repeating the auction stage. An interesting extension would involve determining which shifts each nurse is eligible to bid on, given previous auction winners.

7 Conclusions

Unlike other scheduling methods in the literature, the auction-optimization approach directly accommodates the preferences of individual nurses. Having more influence on the scheduling process has been shown to promote feelings of autonomy and lead to increased job satisfaction. The method improves upon self-scheduling by allowing nurses to express their strength of preference through the amounts that they bid.

The results from applying the auction-optimization model to the emergency department at York Hospital are very encouraging. Most nurse requests were fulfilled, and the schedule was generated in a reasonable amount of computer time. It should be stressed that the shortage of nurses makes it easier to fulfill bid requests. (Our simulation experiments show that win percentages are high, even with adequate staff levels.) We note that in the current health care environment, nurse shortages are quite common, and therefore our case study results represent today's reality.

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