UNIVERSITY OF TWENTE.

Operations Research and Health Care

Operating Room Planning and Scheduling

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Agenda

- Introduction: operating rooms
- Elective surgery scheduling algorithms
- Master Surgical Scheduling (MSS)
- Elective surgery sequencing to deal with emergency patients

INTRODUCTION

Operating Room (OR) management

Introduction: Operating rooms

- Significant source of hospital’s income
- Majority of hospital admissions undergo surgery
- Cost intensive (capital and labor)
- Determines “the pace” of the hospital
  - *“If the OR sneezes, the hospital has a cold”*
- Are a dangerous place
  - >10% of the patients experience complications or an incident
- Increased less invasive surgery (endoscopic, robotic)
  - more “day care” (outpatient) treatments
- Have a lot of variability
  - Diversity surgical procedures, complications, every patient is different, emergencies
- Capacity is determined by availability of trained staff

Introduction: Shortage of OR staff (cf. *F. Boer, LUMC)*

- Cyclical shortage (4-5 years) of personnel
- Causes of shortage
  - Oscillation in training capacity due to shortsighted planning
  - Drop-out in training school
  - Increase of part-time percentage during occupational life
  - Aging
- Fortifying effect: occurrence of employment agencies
- Effects:
  - Closure of operating rooms
  - Increase working pressure
  - Increase of labor costs: employment agencies, salary raises, additional income elements

Introduction:
Operating room staff

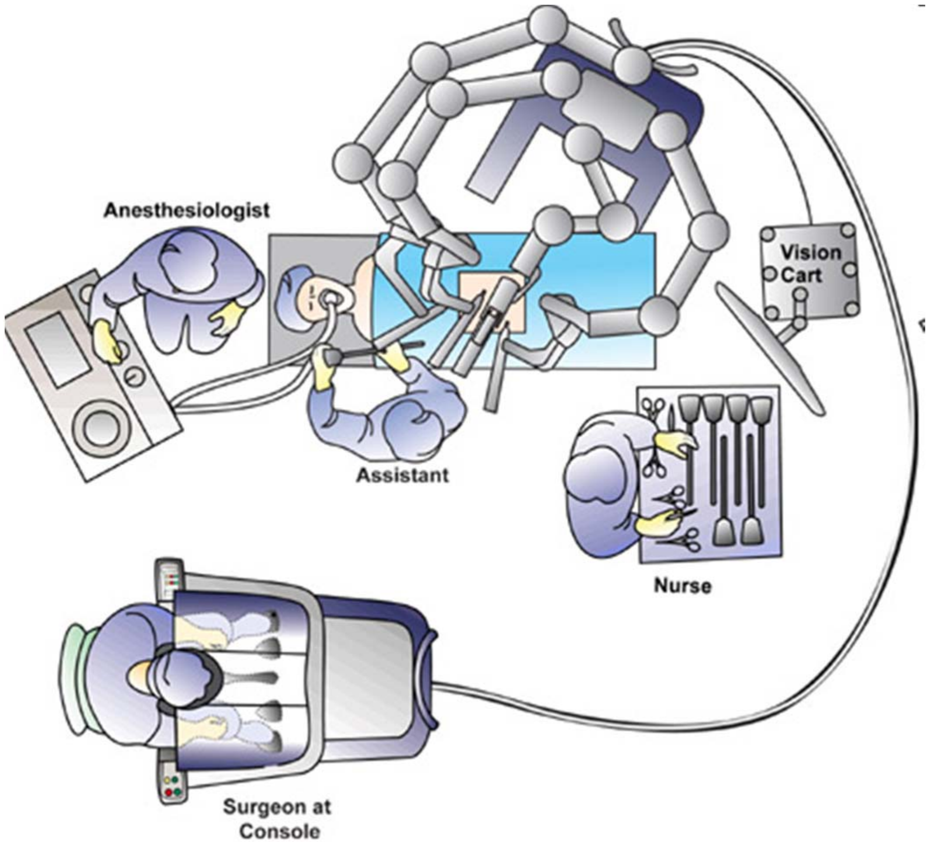
- Surgeons
- Anesthesiologists (responsible for patient)
- Surgery assistants
- Anesthesia assistants
- Day coordinator
- Logistical support (material, prostheses, blood, instruments)
- Staff in training / interns

Surgery is a complex process where many resources act together

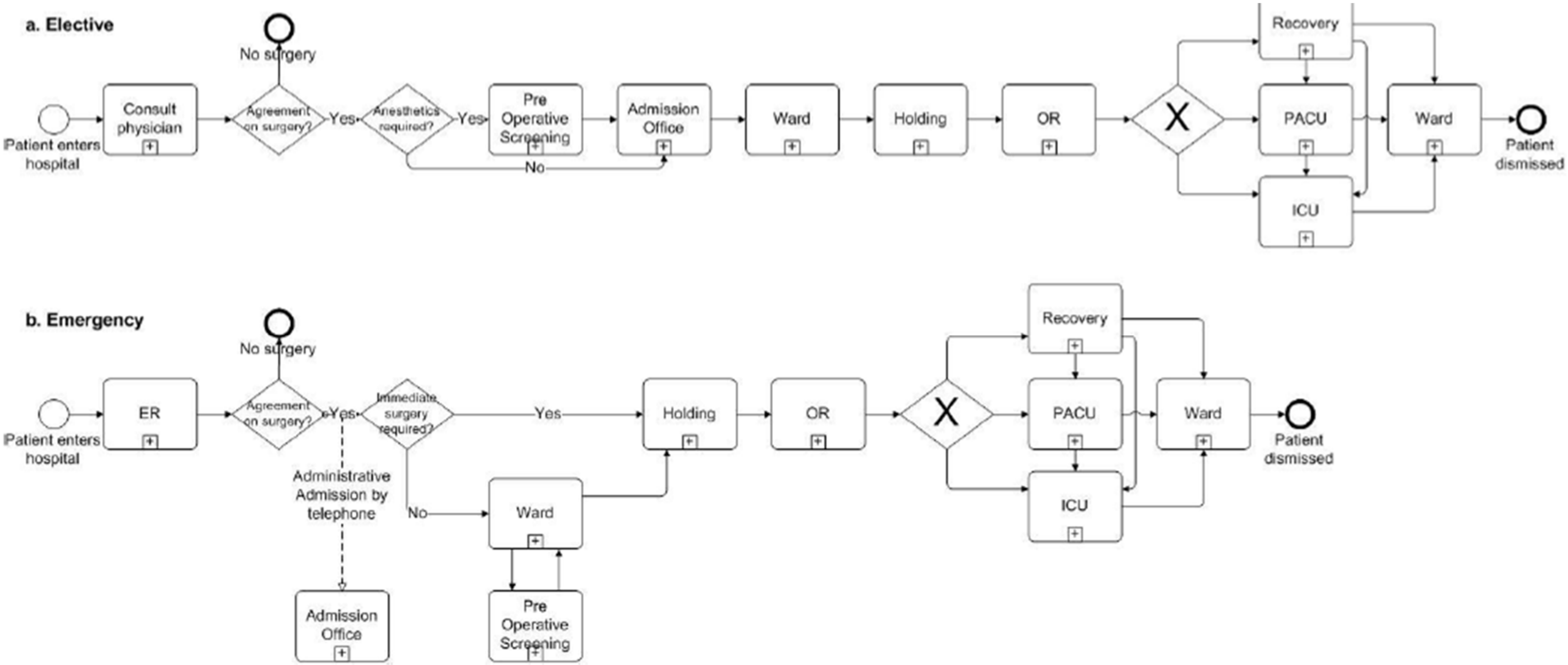
Introduction:
Operating room layout

Introduction:
Operating room layout

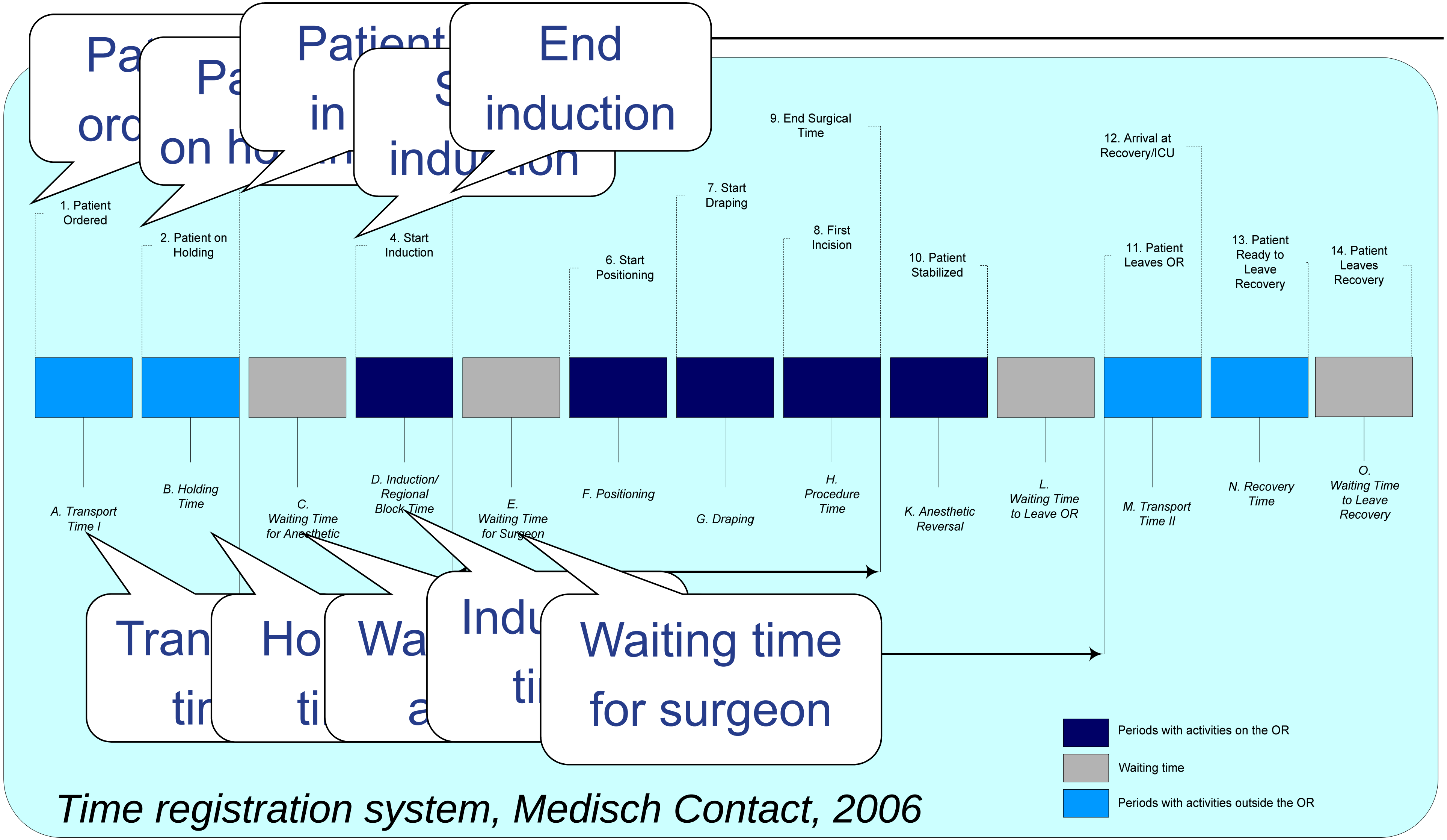
Robotic (endoscopic) surgery



Introduction:
Process from admission to discharge



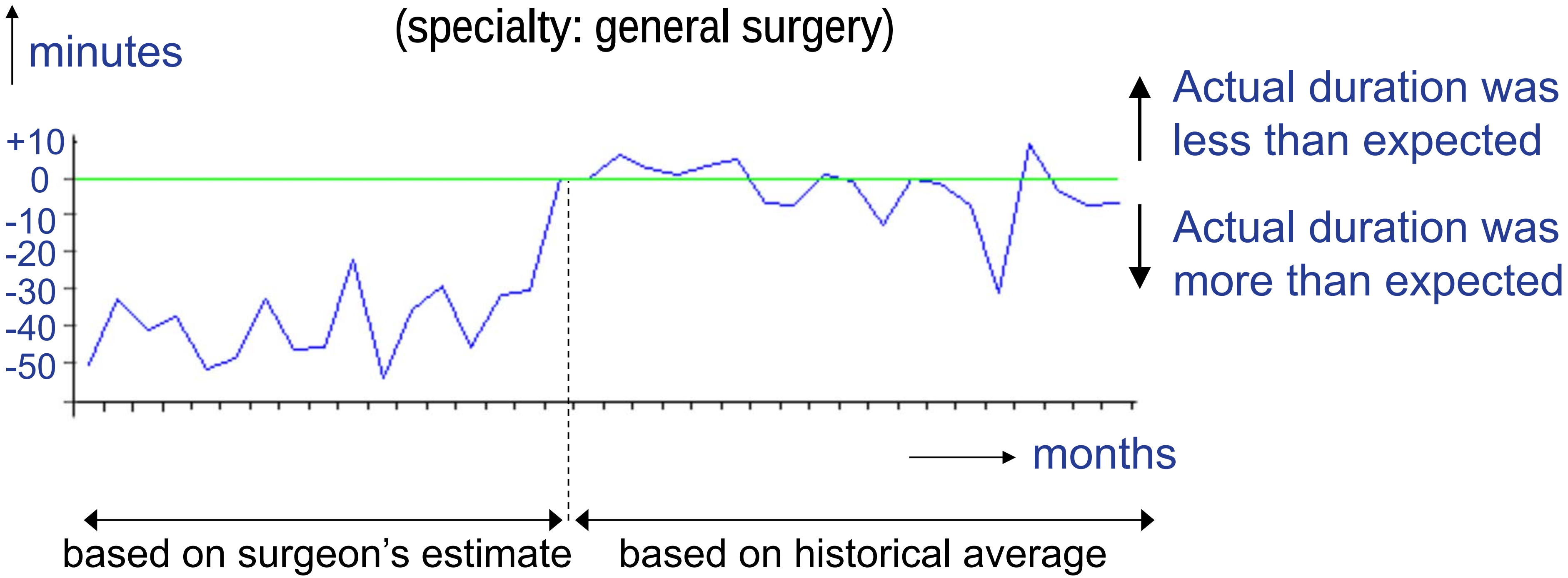
Introduction:
Process within an OR session



Time registration system, Medisch Contact, 2006

Introduction:
Planning based on surgeon’s estimate

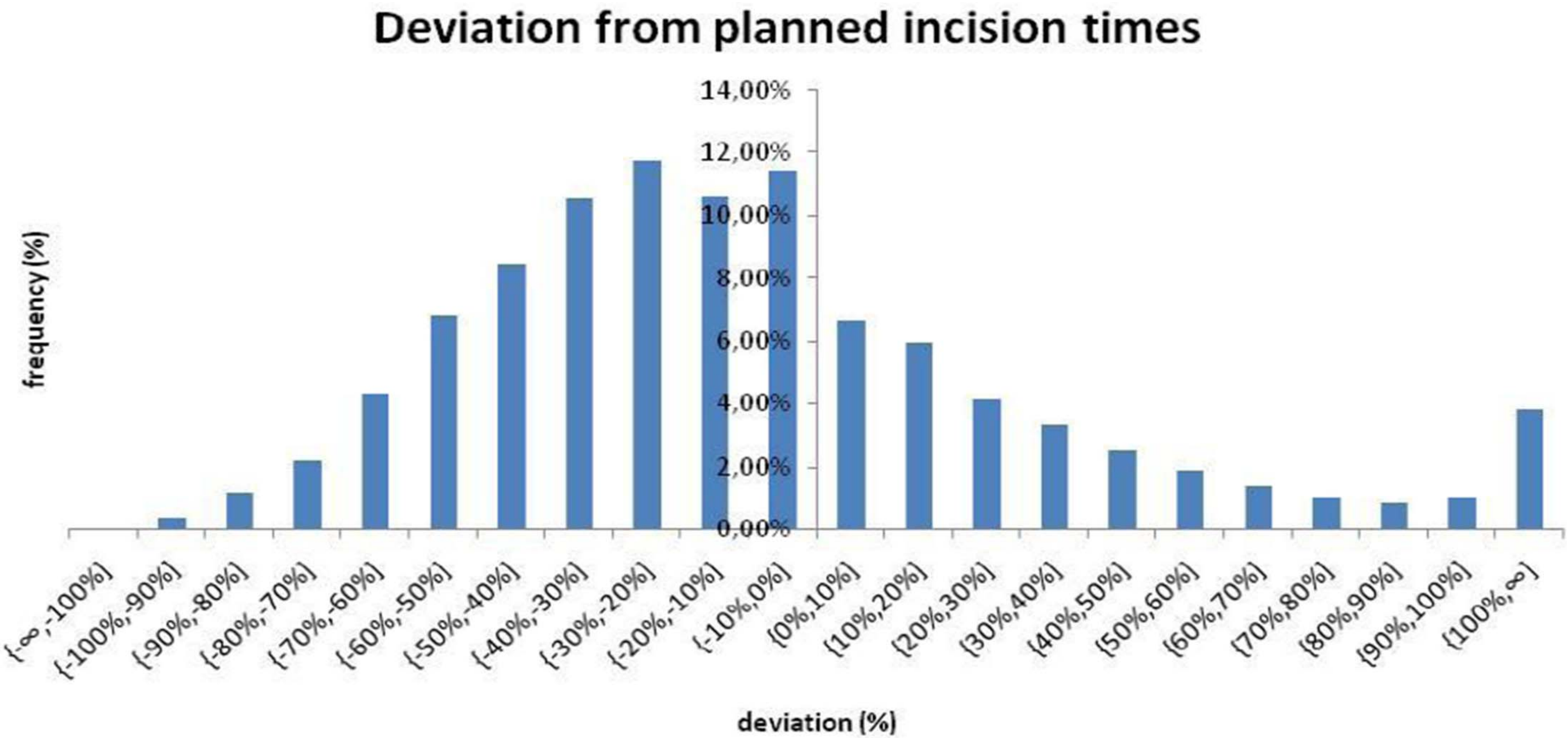
Average deviation from the expected surgery duration
(specialty: general surgery)



Introduction:
Process within an OR session (cf. *F. Boer, LUMC)*

Process step	Share of total session time (%)	Surgeon's activities	Anesthesiologist's activities	Assistant's activities
Induction	10	0	++	++
Positioning	10	+	+	+/++
Procedure time	60	++	+	+/++
Anesthetic revival	10	0	++	++
Changeover	10	0	++	++

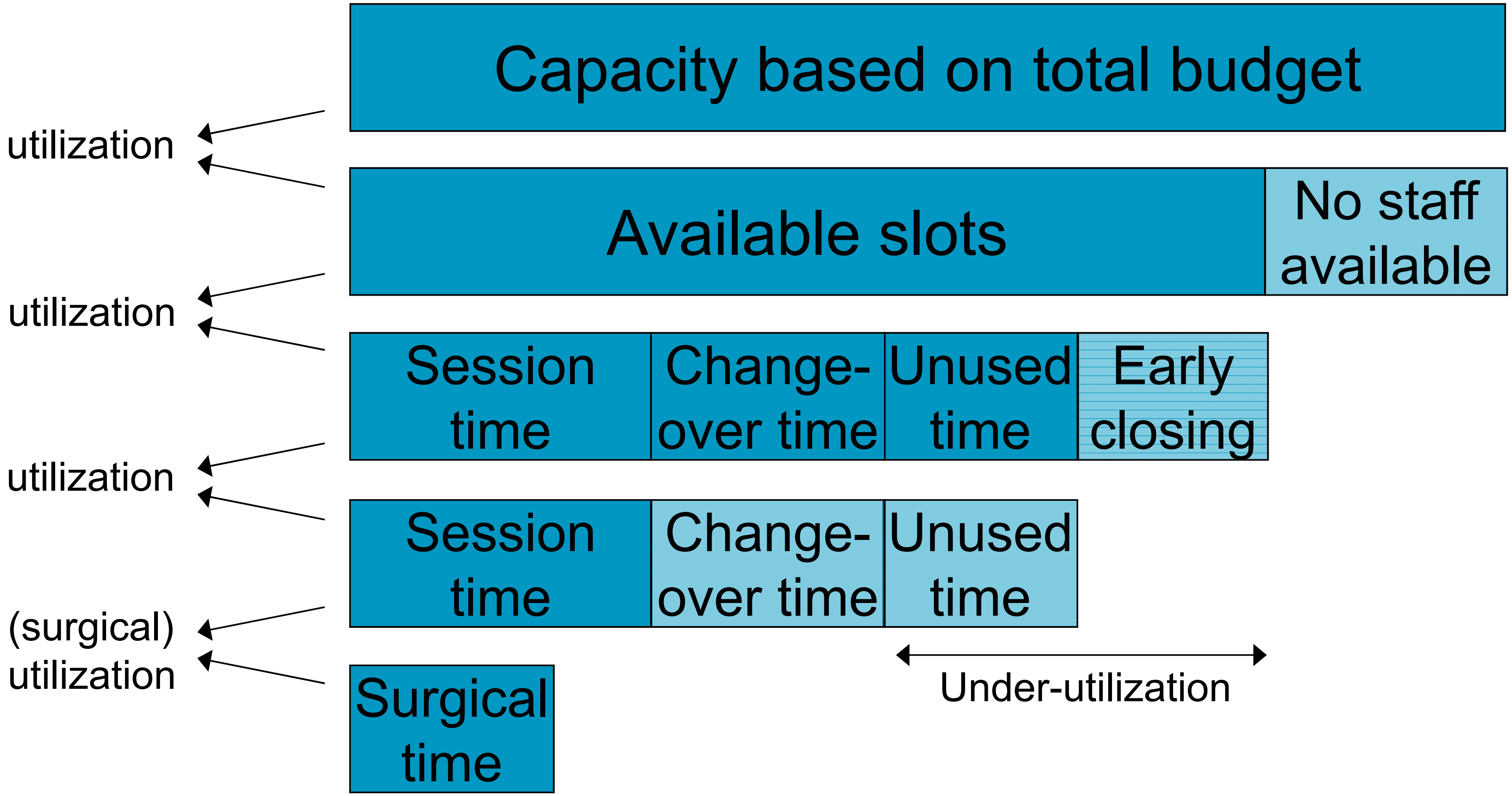
Introduction:
Stochastic surgery durations



Introduction:
Performance of an operating room

- Productivity, e.g.
  - Utilization →
  - Ratio: procedure time / capacity
- Changeover time
- Throughput time
- % Cancellations, related to
  - patient
  - anesthesia preparation
  - organization
- Waiting time of emergency patients
- Overtime
- Effectiveness (eg. revisits of patient, complications)

Introduction:
OR utilization



Introduction: Performance of an operating room

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Hierarchical positioning framework for hospital planning & control

	Medical planning	***Resource capacity planning***	***Material planning***	***Financial planning***
Society				
Strategic	Research planning, introduction of new treatment methods	Case mix planning, layout planning, capacity dimensioning	Supply chain and warehouse design	Agreements with insurance companies, capital investments
Tactical	Care pathway planning	Allocation of time and resources to specialties, rostering	Supplier selection, tendering, forming purchasing consortia	Budget and cost allocation
Operational offline	Diagnosis and planning of an individual treatment	Elective patient scheduling workforce planning	Purchasing, determining order sizes	DRG billing, cash flow analysis
Operational online	Triage, diagnosing complications	Monitoring, emergency rescheduling	Rush ordering, inventory replenishing	Expenditure monitoring, handling billing complications

← hierarchical decomposition →

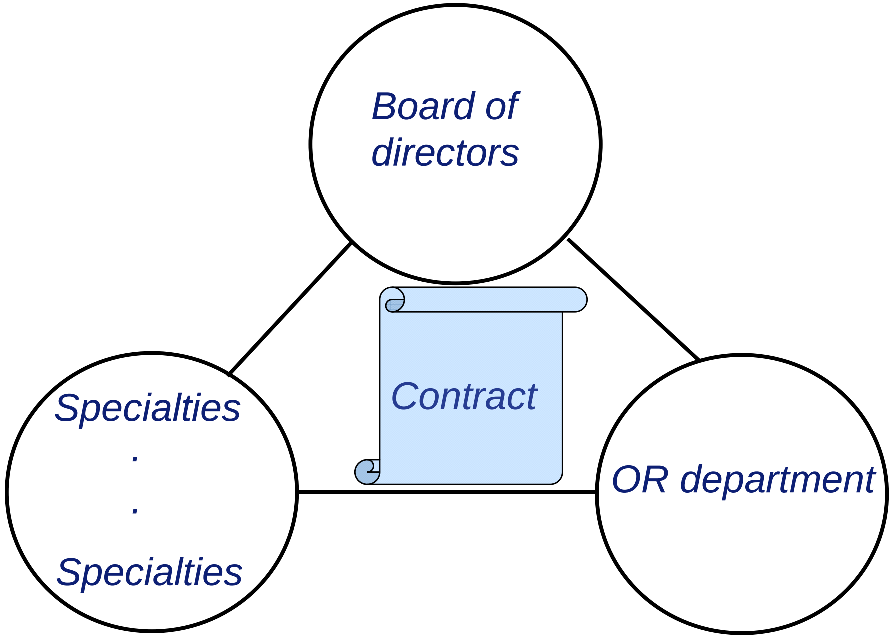
← managerial areas →

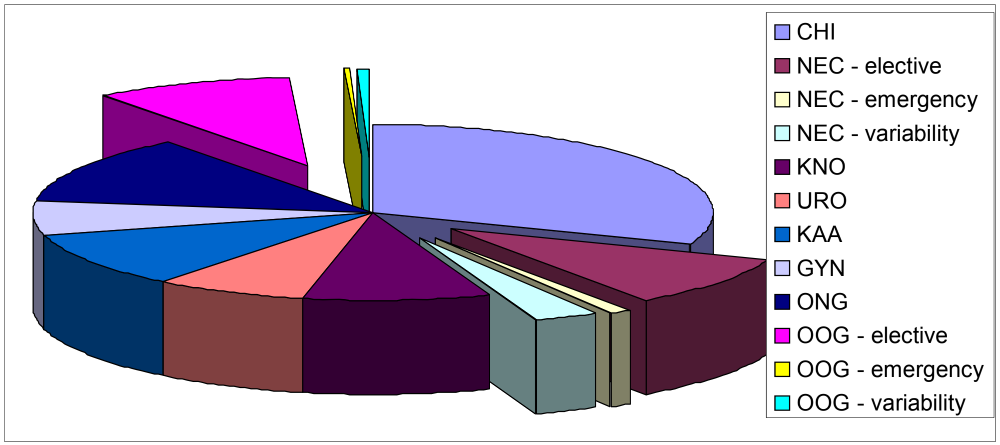
Introduction:
OR planning & scheduling

- Strategic level (year, quarter)
  - Allocation of OR capacity to surgical specialties
- Tactical level (month)
  - Weekly allocation of “OR-days” to specialties
  - Master Surgical Scheduling
- Operational (offline) level (weeks)
  - Elective & semi-urgent surgery scheduling
- Operational (online) level (days)
  - Monitoring and control
  - Emergency surgery scheduling

Introduction:
Strategic OR planning

- Capacity dimensioning
  - Operating rooms, equipment
  - Staff
- Division of the "capacity pie"
  - Contract: board – OR management – specialties



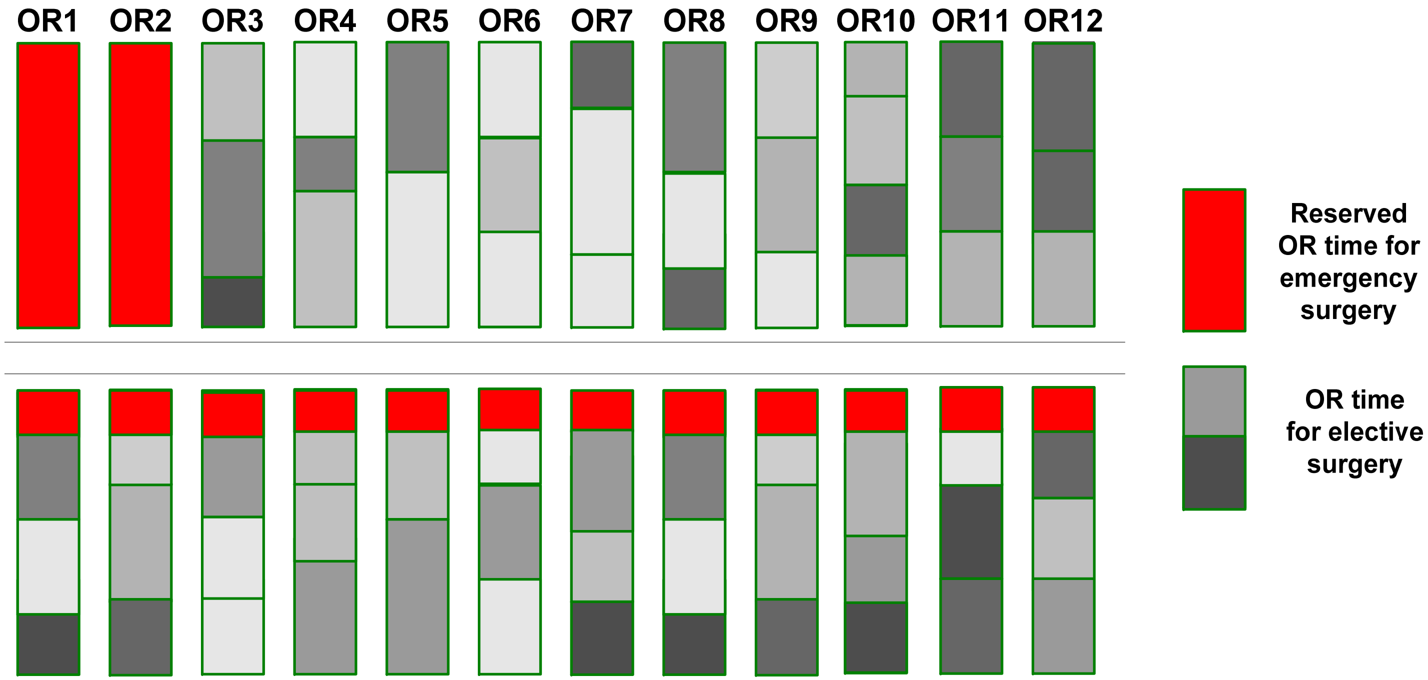


Introduction:
Strategic OR planning

- Emergency operating rooms or not?

Concept: "emergency ORs"

Concept: "No emergency ORs"



Introduction: Tactical OR planning

- Open block planning (common in US)
  - First come first serve operation
  - Different specialties operate successively in OR
  - Long changeover time, unbalanced workload, overtime
  - Emergency operating room
- Closed block planning (common in Netherlands)
  - Each specialty / surgeon gets blocks of time (ORday, morning session, afternoon session)
  - Each specialty / surgeon schedules its patients in these blocks, at least 1 week in advance
  - More efficient, less waiting time for patients
  - Remaining time cannot be redistributed
- Semi-open block planning: combination

Introduction: Offline operational scheduling

We assume closed block planning approach

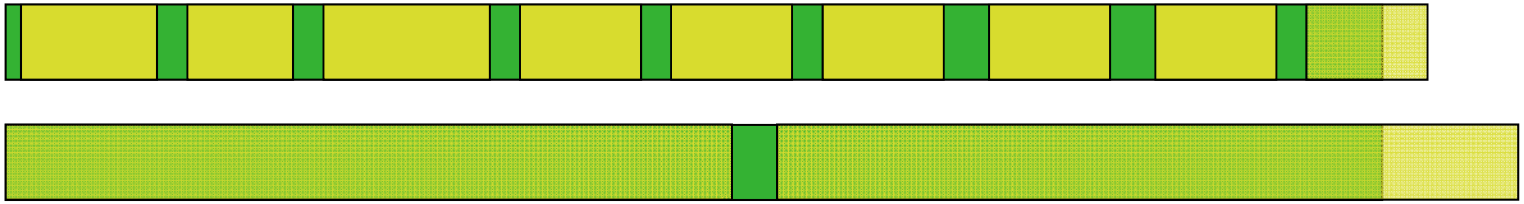
- Elective patients: scheduled into specialty's blocks at least a week in advance
- Semi-urgent patients: scheduled days before
- Emergency patients: scheduled upon arrival

Introduction: Offline operational scheduling

- Overtime is:
  - Costly (collective labor agreements)
  - Propagated in the hospital
  - To be avoided in elective scheduling
- Whether overtime costs cover the marginal costs is usually unknown

Introduction:
Offline operational scheduling

- Specialties with short procedures are able to combine high occupancy with less overtime



- The ability to achieve a high occupancy also depends on the case mix (short-short, short-long, long-long)
- Waiting lists allow for better solutions, but are increasingly unethical

Introduction: Constraints offline operational scheduling

- Several surgeons (of different specialties)
- Non-identical ORs
- Availability of:
  - (Movable) equipment
  - Instrument trays
  - Prostheses (ordering lead-time)
- Impact of staff training
- Preferences of staff
- Et cetera…

- An inventory of the ‘hard’ and ‘soft’ constraints in a small regional hospital yielded 138 constraints

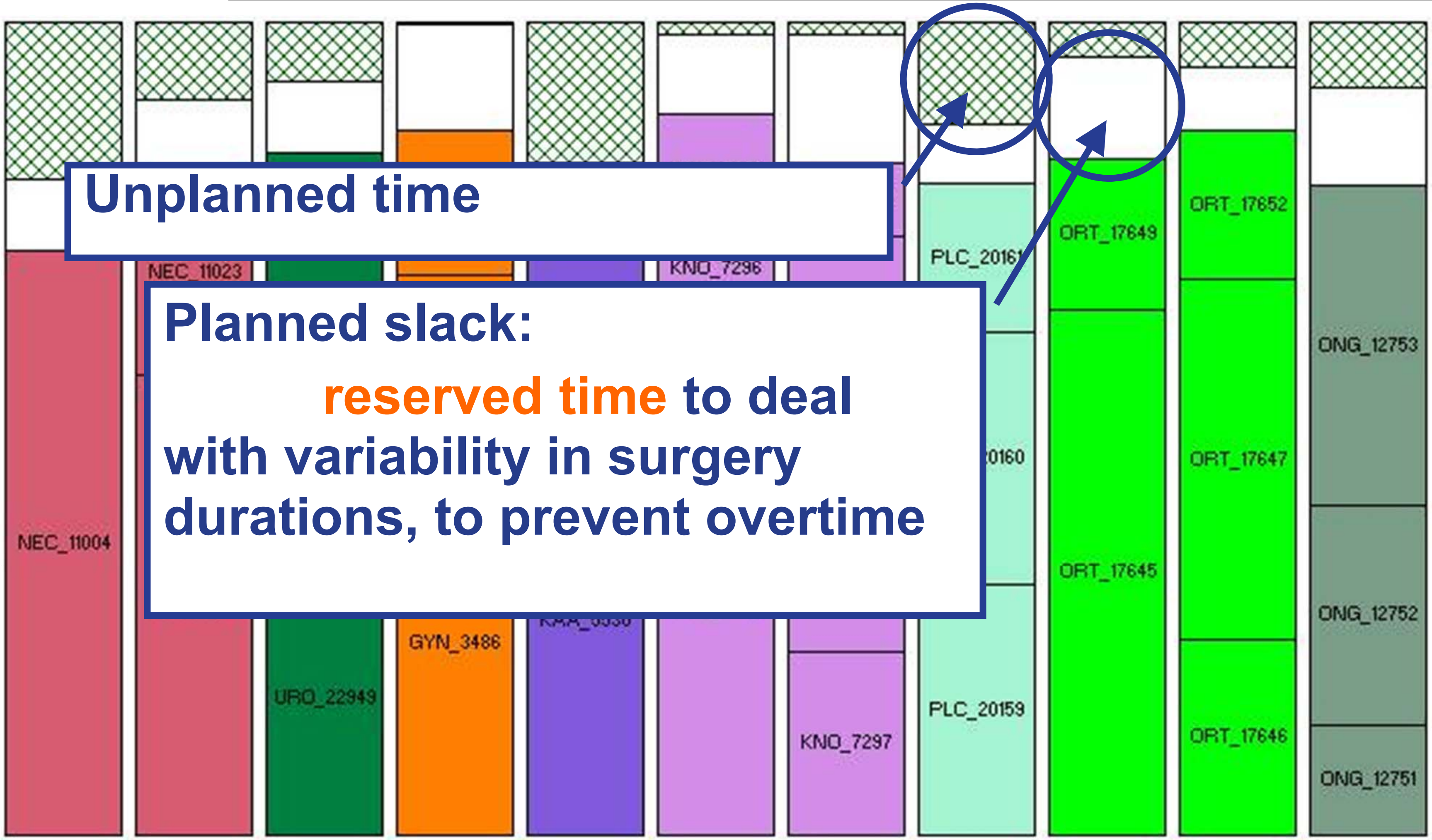
Introduction:
Online operational scheduling

- Sequencing elective surgeries
  - Children are operated at start of the program
  - Solving problems with movable equipment
  - “Dirty” surgeries (bacteria or air pollution involved) at end of the program
  - Instrument tray availability
    - Re-use of instrument tray after sterilization
  - Several surgeons (of different specialties)

- Monitoring and control (re-scheduling of surgeries)

“Straightforward” ELECTIVE SURGERY SCHEDULING ALGORITHMS

Example elective schedule(11 ORs)



Unplanned time

Planned slack:

reserved time to deal with variability in surgery durations, to prevent overtime

Elective surgery scheduling: "easy version"

- Closed block planning:
  - Problem decomposes into subproblem per specialty
- Horizon: typically one week
- Stochastic binpacking problem
- Parallel identical machine scheduling
- Outcome:
  - List of elective surgeries per block
- Objective:
  - Target utilization
- Common approach:
  - (Probabilistic) constructive heuristic, then local search

Elective surgery scheduling: “easy version”

Constructive “list scheduling” heuristic requires:

- **Job priority rule**
  - Expected duration, variance, random
- **Job selection rule**
  - Ascending, descending, random
- **Machine priority rule**
  - First Fit, Best Fit, Random Fit
- **Machine selection rule**
  - Ascending, descending, random

Randomized "list scheduling" regret-based random sampling

- For randomized job selection and/or machine selection
- Probability (>0) related to priority
- Job priority: q_j
- Worst of all job priorities: $W = \min q_j$
- Regret factor of job j: $r_j = |W - q_j|$
  - "regret if job j is not selected"
- Probability to select job j:
  - Where α = bias factor

$$P_j = \frac{(1 + r_j)^\alpha}{\sum_i (1 + r_i)^\alpha}$$

Normalization constant

Randomized "list scheduling" regret-based random sampling

- Example: longest duration first

$$P_j = \frac{(1+r_j)^\alpha}{\sum_i (1+r_i)^\alpha}$$

Job j	Duration q_j	Regret factor r_j	$(1+r_j)^\alpha$	P_j (α=0)	P_j (α=2)	P_j ($\alpha=\infty$)
1	5	2	3^α	0.333	0.643	1
2	3	0	1^α	0.333	0.071	0
3	4	1	2^α	0.333	0.286	0
	W = 3	$r_j = \|W-q_j\|$				

"Random Sampling" (α=0)

"Deterministic Sampling" ($\alpha=\infty$)

Randomized "list scheduling" regret-based random sampling

Suppose job 2 was selected in the previous iteration

➔ recalculate the probabilities for the remaining jobs

$$P_j = \frac{(1+r_j)^\alpha}{\sum_i (1+r_i)^\alpha}$$

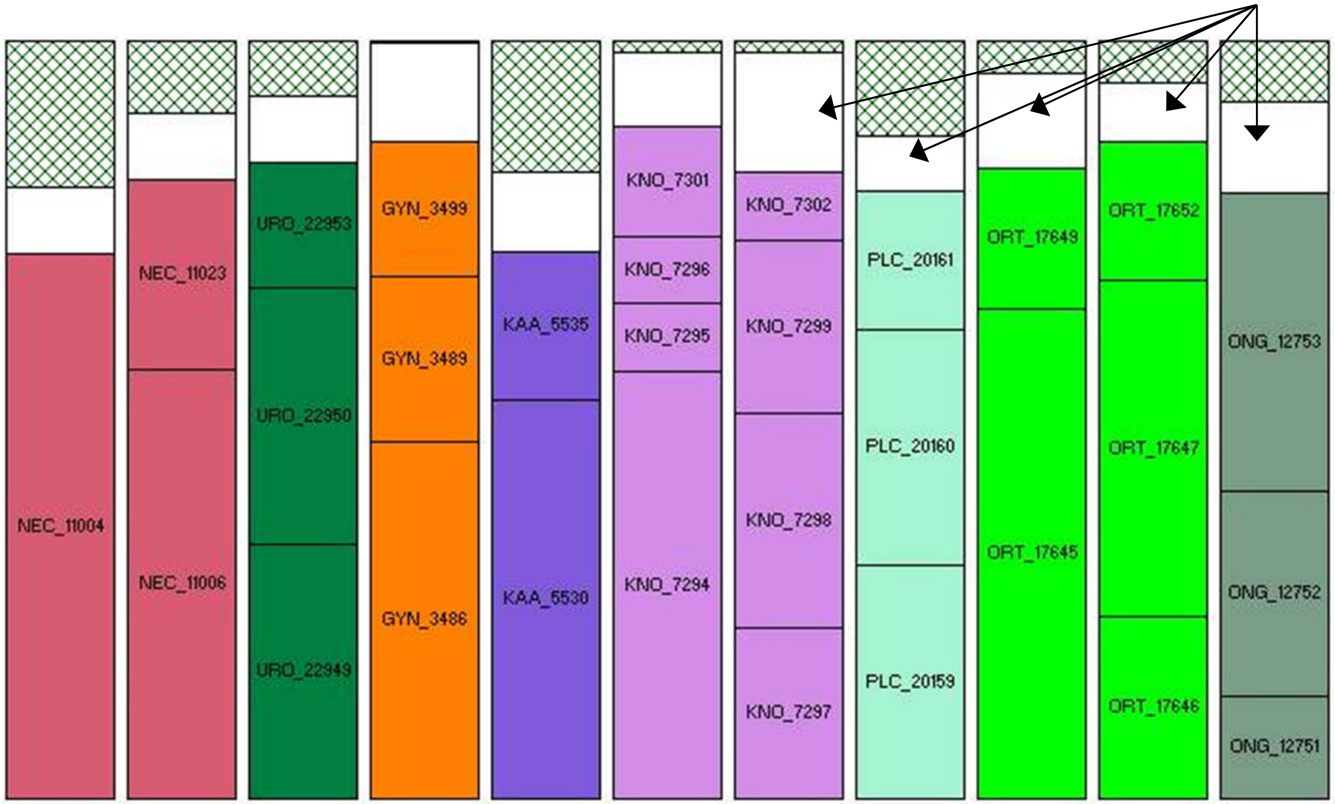
Job j	**Duration q_j**	**Regret factor r_j**	**$(1+r_j)^\alpha$**	**P_j (α=0)**	**P_j (α=2)**	**P_j (α=∞)**
1	5	1	2^α	0.5	0.8	1
~~**2**~~	~~3~~					
3	4	0	1^α	0.5	0.2	0
	W = 4	$r_j = \lvert W-q_j \rvert$				

Randomized "list scheduling" regret-based random sampling

- Observations:
  - Bias factor α allows "steering to priority rule"
  - The higher the bias factor α, the more deterministic the method
  - Every job has a probability of selection
- Method was proposed as "Adaptive Search" by Kolisch and Drexl (1996) for the Resource Constrained Project Scheduling Problem (RCPSP)
  - *Kolisch, R. and A. Drexl, 'Adaptive Search for solving hard project scheduling problems', in: Naval Research Logistics, no. 43, pp. 23-40, 1996.*

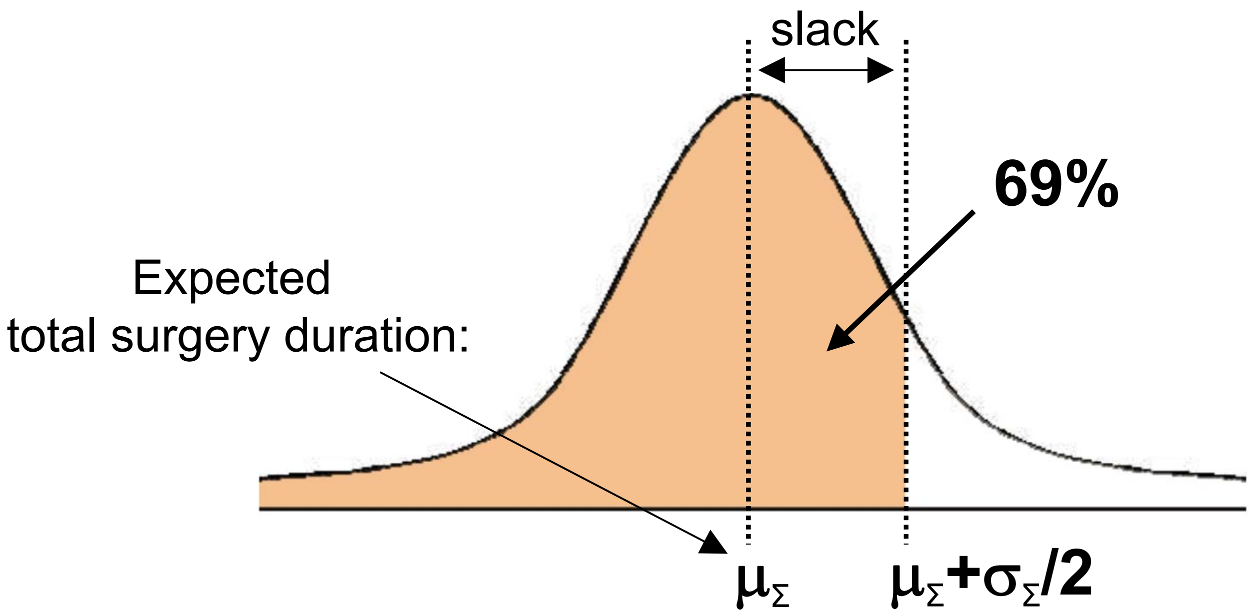
Local search optimization of the OR-schedule using the portfolio effect

- Swapping jobs between OR-days
- Accept swap
  - based on portfolio effect (impact on <u>total planned slack</u>)



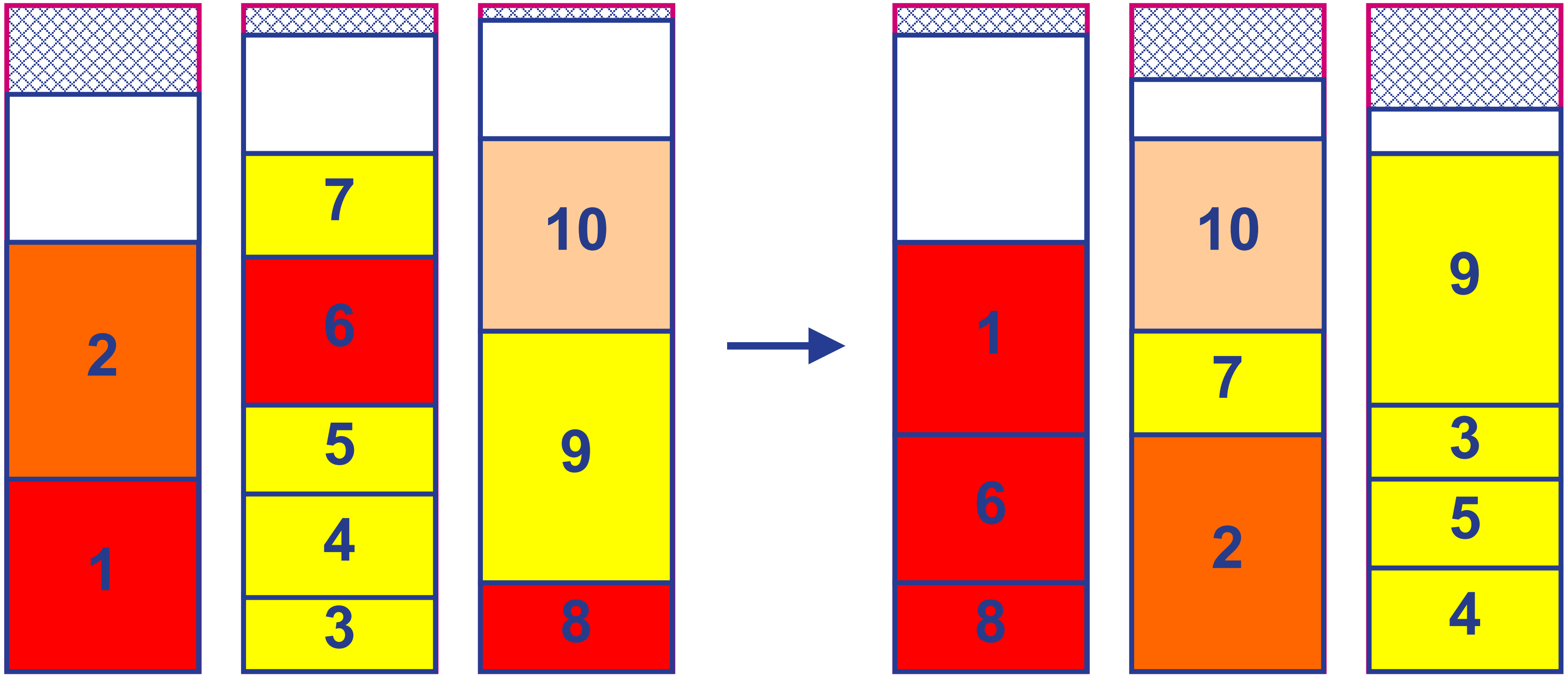
Local search optimization of the OR-schedule using the portfolio effect

- Erasmus MC assumes a normal distributed total surgery duration
- Planned slack is $\sigma_\Sigma/2$, where σ_Σ is the total surgery duration standard deviation



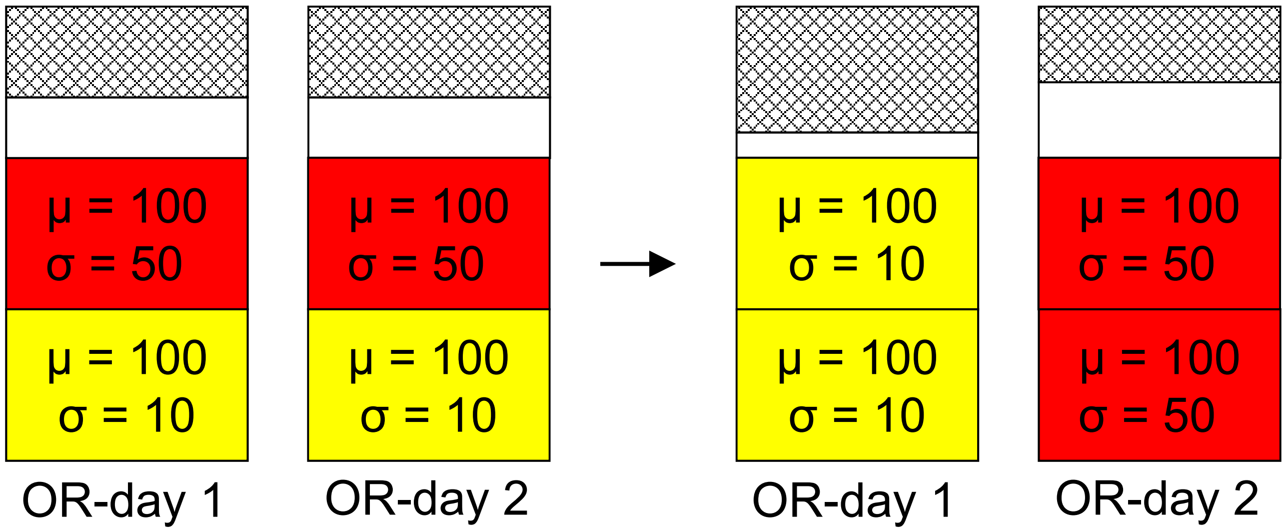
Example

The more 'red' the surgery, the higher its duration variability



These swaps reduce the total slack

Example (2)



OR-day 1: $\sigma^1_\Sigma = \sqrt{50^2 + 10^2}$

OR-day 2: $\sigma^2_\Sigma = \sqrt{50^2 + 10^2}$

$$\sigma_\Sigma = \sigma^1_\Sigma + \sigma^2_\Sigma = \underline{102.0}$$

$$\sigma^1_\Sigma = \sqrt{50^2 + 50^2}$$

$$\sigma^2_\Sigma = \sqrt{10^2 + 10^2}$$

$$\sigma_\Sigma = \sigma^1_\Sigma + \sigma^2_\Sigma = \underline{84.9}$$

Restrictions local search

OR-capacity constraint: $\sum_{i \in N_{skt}} \mu_i + \delta_{skt} \leq c_{kt} + O_{skt} \quad (\forall s, t, k \in K_{st})$

6 degrees of freedom for surgery-OR-assignments:

A surgery from day t (<u>base solution</u>), specialty *s*, unit u, must be planned on:

1. day *t*, within the OR-days assigned to specialty *s*.
2. day *t*, within the OR-days assigned to the unit *u* the surgery belongs to
3. day *t*, within any OR-day.
4. any OR-day assigned to specialty *s* within the week.
5. any OR-day assigned to unit *u* within the week.
6. any OR-day within the week.

Local search method: Simulated Annealing

- with probability P, a 1-exchange is evaluated
- with probability 1-P, a 2-exchange is evaluated
- **improvement**: accept
- **no improvement**: accept with probability: $e^{\frac{-Y}{\tau}}$, where:

  Y = deterioration of objective criterion

  τ: temperature
- proportional cooling scheme, after every "k" swaps:
- Stop if $\tau < \varepsilon$

$$\tau^{NEW} = \tau^{OLD} \cdot \theta \; (0 \le \theta < 1)$$

Local search method: Simulated Annealing Parameter initialization (1/2)

- **Initial temperature:**
  Set so that, in the beginning, almost all swaps are accepted
  - Perform the following experiment:

    STEP 1: Set initial temperature τ to 10

    STEP 2: Count the number of accepted swaps out of 1000 tries

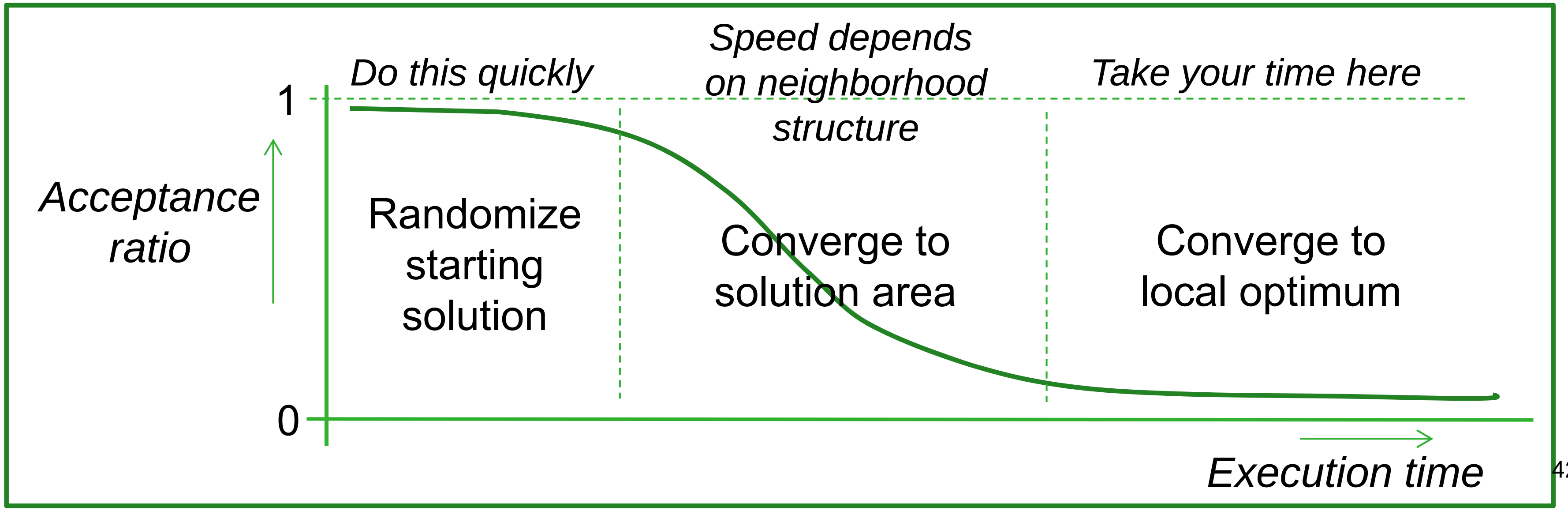
    STEP 3:

    IF acceptance ratio > 0.95 THEN
    use τ as starting temperature

    ELSE
    $\tau := \tau \times 2$, RETURN to STEP 2

Local search method: Simulated Annealing Parameter initialization (2/2)

- **Length of Markov chain (k):**
  Similar in size as the size of the neighborhood structure
  (if there are 10 neighbor solutions, doing 1000 swaps makes no sense)
- **Temperature decrease factor (θ)**
  Temperature lower bound
  These determine the speed of convergence → trial and error



Local search method: Simulated Annealing Remarks

- **What is a good neighborhood structure?**
  - trade-off: *computation speed* and *convergence*

- **Variants are possible**
  - Increasing Markov chain length
  - Stop when working solution at the end of the Markov chain has not changed for *n* Markov chains
  - Multi-start with short runs

- **Convergence property**
  in the limit, SA converges to global optimum

Towards more advanced ELECTIVE SURGERY SCHEDULING ALGORITHMS

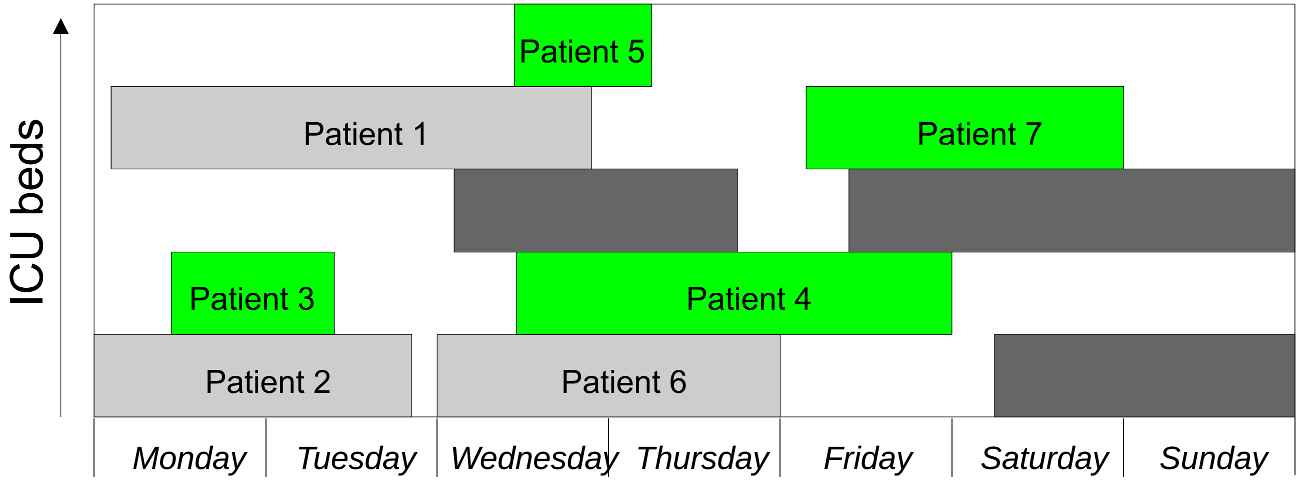
Towards more advanced approaches for elective surgery scheduling

Considerations:

- Strong interaction with subsequent departments
  - Surgery schedule determines workload in wards and ICU →
  - Surgeries are cancelled if ward or ICU is full
  - Less frequent ward discharges in weekends ➔ balancing the surgery schedule reduces bed usage
- Mathematical optimization hard to implement:
  - Interferes with surgeon’s autonomy
  - Leads to “nervous” schedules
- The surgery program is very repetitive

ICU bed requirements after surgery

Expected ICU utilization of elective patients without coordination



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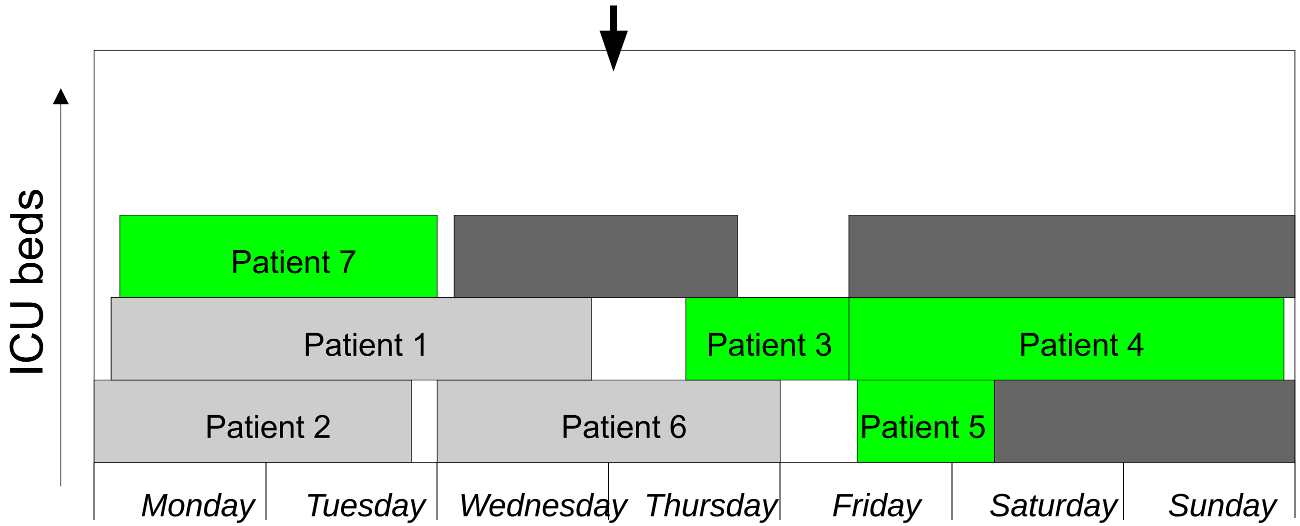
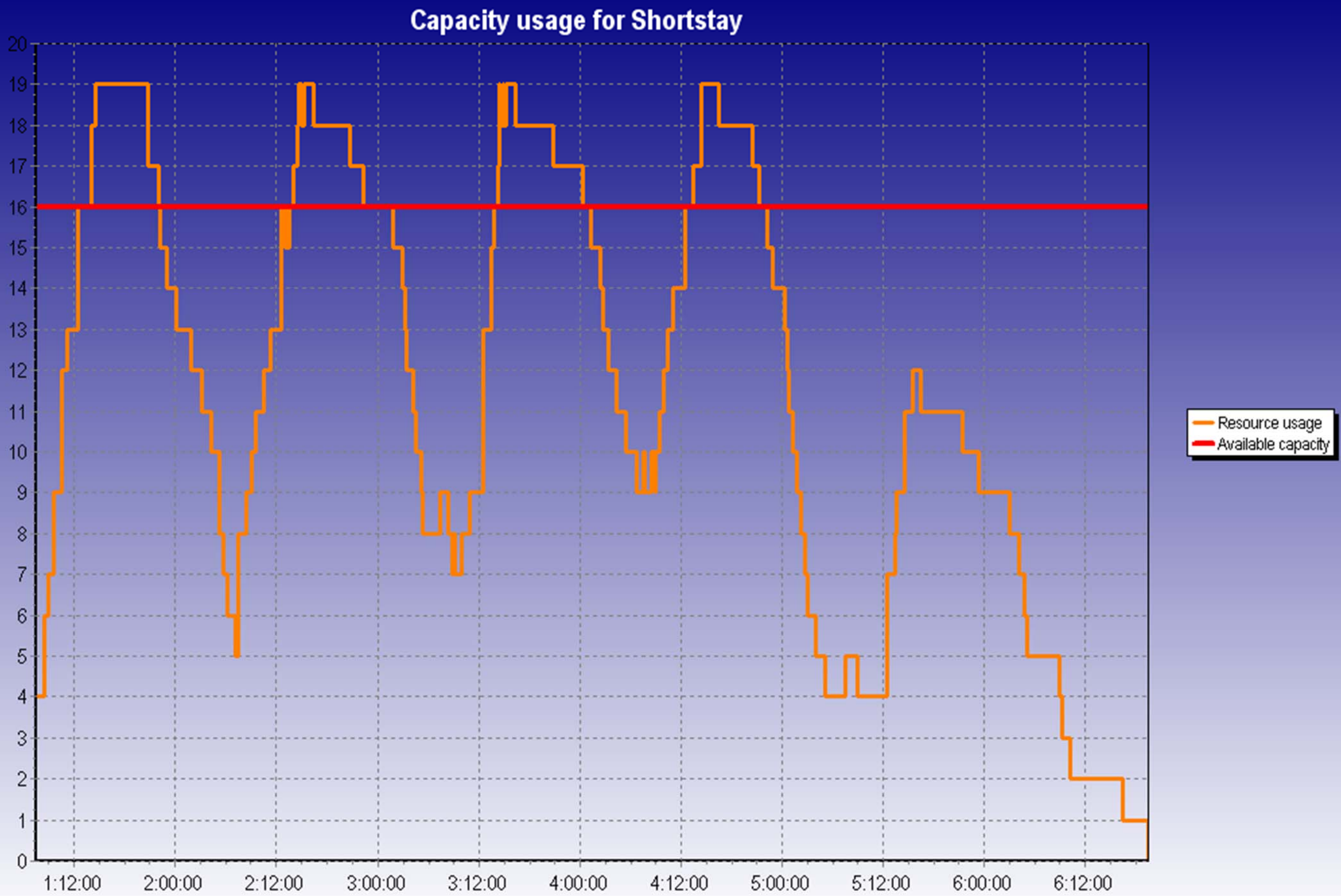


Capacity usage for shortstay ward



Towards more advanced approaches for elective surgery scheduling

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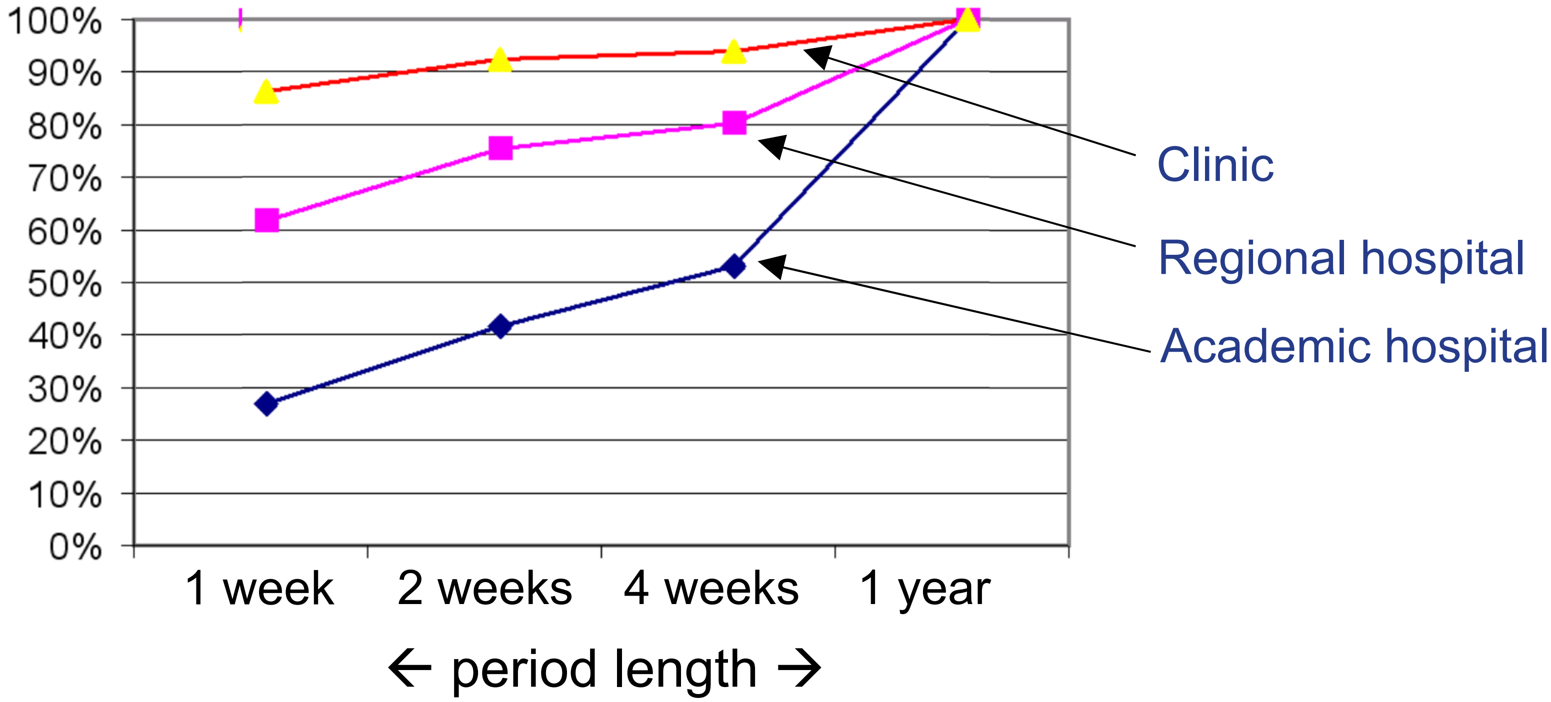
Repetitiveness of surgical case mix

Percentage of surgeries that occur at least once during a period



Master surgical scheduling

a cyclic, integral planning of ORs and ICU department

OR Spectrum 30(2), 2007 (co-work Van Oostrum *et al.*)

Master surgical scheduling: idea

Idea: design a cyclic schedule of surgery types that:

- covers all frequent elective surgery types
- levels the workload of the specialties
- levels the workload of subsequent departments (ICU, wards)
- is robust against uncertainty
- improves OR-utilization
- maintains autonomy of clinicians

Assign patients to the "slots" in the schedule

MSS: problem description

Goal:

- Maximize the OR-utilization
- Level capacity usage of subsequent resources (ICU)

Constraints:

- OR-capacity constraints (probabilistic)
- All surgery types must be planned i.c.w. their frequency

To determine:

- Length of the planning cycle
- A list of surgery types for every OR-day ("OR-day schedule")

Mathematical program

V_{ijt} : #surg of type i assigned to OR (j,t)

W_{jt}: OR (j,t) is open

levels the hospital bed usage

maximizes the OR utilization

$$\min \theta_1 * \sum_{t=1}^{T} \sum_{j=1}^{J} o_{jt} * W_{jt} + \theta_2 * \sum_{b=1}^{B} \left[\frac{c_b}{\left[\sum_{i=1}^{I} q_{ib} * l_{ib} * s_i \right] / T} \right] * \max_{\tau \in T} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=1}^{T} \psi_{t\tau ib} * q_{ib} * V_{ijt}$$

ward importance weight

peak bed usage in ward

Subject to:

$$V_{ijt} \leq s_i * W_{jt}, \quad (i = 1, ..., I, \quad j = 1, ..., J, \quad \tau = 1, ..., T)$$

$$\sum_{t=1}^{T} \sum_{j=1}^{J} V_{ijt} = s_i, \quad (i = 1, ..., I)$$

← All surgeries assigned

$$\Pr[f_{jt}(V) \leq o_{jt}] \geq \alpha, \quad (j = 1, ..., J, \quad t = 1, ..., T)$$

$$\Pr[h_{kt}(V) \leq f_{kt}, \quad (k = 1, ..., K)] \geq \delta, \quad (t = 1, ..., T)$$

Probabilistic constraints for wards, ORs

$$V_{ijht} \in \{0, 1\}$$

$$W_{jt} \in \{0, 1\}$$

Master surgical scheduling: decomposition approach

PHASE 1:
Generation of
"OR-day schedules"

Goal: capacity utilization

→ ILP, solved by column generation and then rounding

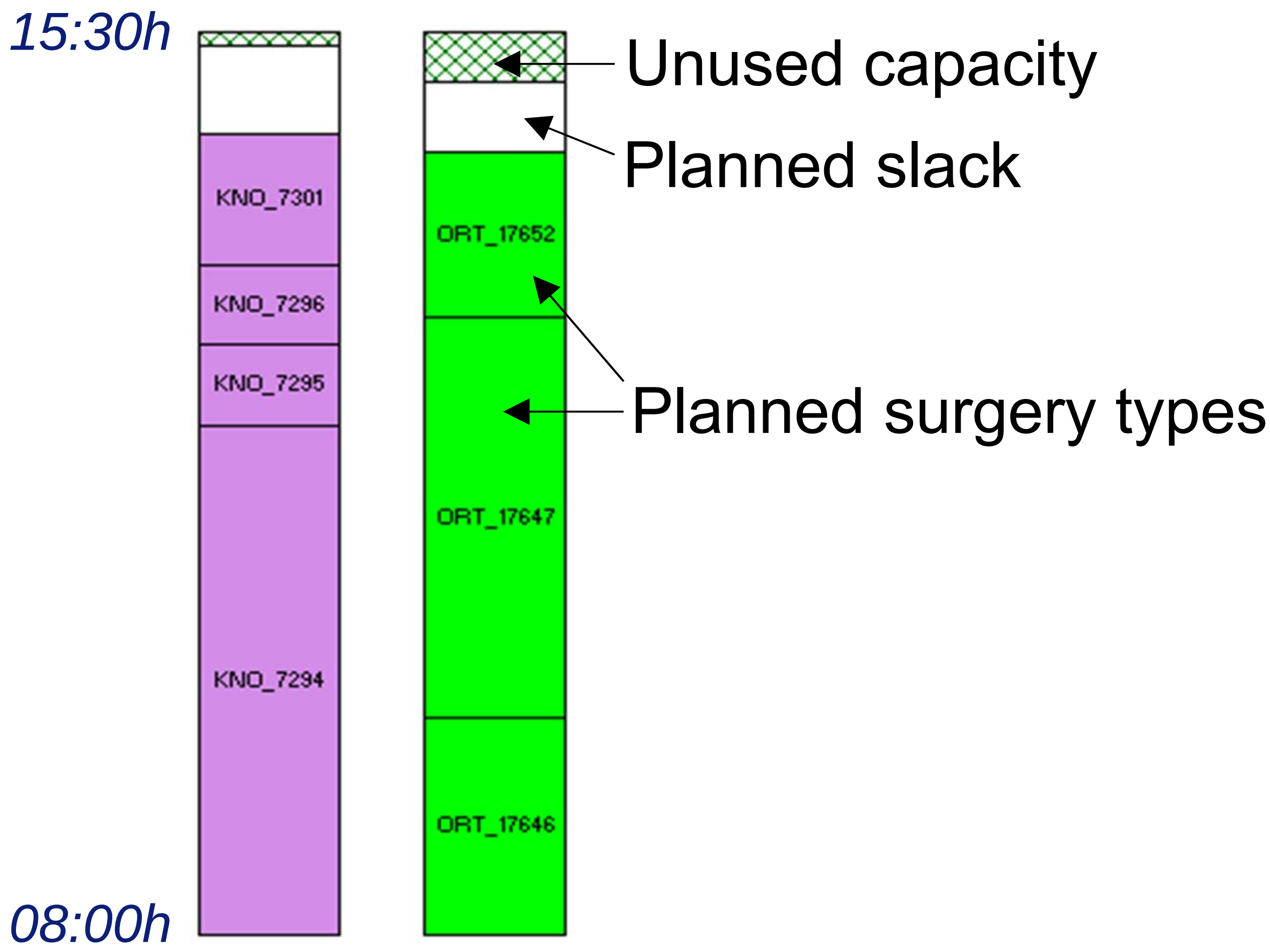
Constraints:
- All surgeries must be planned
- OR-capacity (probabilistic)

PHASE 2:
Assignment of
"OR-day schedules"

Goal: bed usage leveling

→ ILP, solved using CPLEX in AIMMS modeling language

OR-day schedule (ORDS) example



u: ORDS type
r: ORDS capacity size

Phase 1: Minimization of OR capacity

$$\min \sum_{r \in R} \sum_{u \in U_r} d_r \cdot X_u$$

$$\sum_{r \in R} \sum_{u \in U_r} a_{iu} \cdot X_u \geq s_i \quad (\forall i)$$

$$\sum_{u \in U_r} X_u \leq m_r \quad (\forall r \in R)$$

$$X_u \in \mathbf{N} \quad (\forall u \in U)$$

- X_u: Number of ORDSs of type *u*
- d_r: Capacity in OR-day of type *r*
- U_r: Set of ORDSs *u* that fit in an OR-day of type *r*
- s_i: Demand for surgeries of type *i*
- a_{iu}: Number of surg. of type *i* in ORDS *u*
- m_r: Number of OR-days of type *r*

- Issue: exponentially large set U_r
- Solution: column generation approach

Wiki: Cutting stock problem

Column generation principle (minimization problem)

- Used for LPs with a huge number of variables
- Steps:

aka "restricted master problem"

1. Formulate a feasible restricted LP (selection of the variables)
2. Solve restricted LP to obtain shadow prices (dual variables)

aka "pricing problem" or "subproblem"

3. Determine whether there is a variable not contained in restricted LP, with negative reduced costs
   - YES: Add this variable and the corresponding column to the restricted LP, and GO TO 2.
   - NO: STOP

- Upon termination, the optimal LP solution is equal to the optimal restricted LP solution
- Integer solution: combine with branch-and-bound or heuristic

Column generation applied to phase 1: minimization of OR capacity

- Steps:
  - Formulate a feasible restricted LP
    - Generate small set of ORDSs with LPT heuristic
  - Solve restricted LP, obtain shadow prices
  - Pricing problem:
    - does there exist an ORDS *u* for OR-day of type *r* with negative reduced costs?
    - → another ILP

Column generation

$$\min \sum_{r\in R} \sum_{u\in U_r} d_r \cdot X_u$$

$$\sum_{r\in R} \sum_{u\in U_r} a_{iu} \cdot X_u \geq s_i \quad (\forall i)$$

$$\sum_{u\in U_r} X_u \leq m_r \quad (\forall r \in R)$$

$$X_u \in \mathbb{N} \quad (\forall u \in U)$$

Dual problem:

$$\sum_{i=1}^{l} \lambda_i \cdot a_{iu} + \pi_r \leq d_r \quad (\forall r)$$

$$\lambda_i \geq 0 \quad (\forall i)$$

$$\pi_r \leq 0 \quad (\forall r)$$

Reduced costs of ORDS $u \in U_r$

$$d_r - \sum_{i=1}^{l} \lambda_i \cdot a_{iu} - \pi_r$$

Pricing problem:

∃ ORDS *u* for OR-day with capacity size *r* with negative reduced costs?

- Reduced costs: $d_r - \sum_{i=1}^{l} \lambda_i \cdot a_{iu} - \pi_r$
- Pricing problem thus becomes:

$$\max \sum_{i=1}^{l} \lambda_i \cdot Z_{iu}$$

$$\sum_i p_i \cdot Z_{iu} \leq d_r \quad (\forall u \in U_r)$$

$$Z_{iu} \in \mathbf{N}$$

Frequency of surgery type *i* in ORDS *u*

Deterministic OR-capacity constraint

Y_{ujt} : assign ORDS u to OR (j,t)

HB_b: max usage of bed type b

Phase 2: Hospital bed leveling

- Assignment of all ORDSs to an operating room and day
- Minimize the max. number of required hospital beds per day
- ILP solved with CPLEX:

$$\min \sum_{b=1}^{B} \left\lceil \frac{c_b}{\left\lceil \sum_{i=1}^{I} l_{ib} \cdot s_i \right\rceil / T} \right\rceil \cdot HB_b \quad (12)$$

Max. bed usage ⟶

$$\sum_{r=1}^{R} \sum_{u \in \bar{U}_r} \sum_{(j,t) \in \varphi_r} \sum_{i=1}^{I} \sum_{t=1}^{T} \psi_{t\tau ib} \cdot a_{iu} \cdot Y_{ujt} \leq HB_b \qquad \tau = 1, \ldots, T,\ b = 1, \ldots, B \quad (13)$$

All ORDSs assigned ⟶

$$\sum_{(j,t) \in \varphi_r} Y_{ujt} = X_u \qquad r = 1, \ldots, R;\ u \in \bar{U}_r \quad (14)$$

$$\sum_{u \in \bar{U}_r} Y_{ujt} \leq 1 \qquad r = 1, \ldots, R;\ (j,t) \in \varphi \quad (15)$$

$$Y_{ujt} \in \{0, 1\} \qquad u \in \bar{U}_r;\ (j,t) \in \varphi_r$$

$$z_b \geq 0 \qquad b = 1, \ldots, B$$

Master surgical scheduling: approach

PHASE 1:
Generation of
"OR-day schedules"

Goal: capacity utilization

→ ILP, solved by column generation and then rounding

Constraints:
- All surgeries must be planned
- OR-capacity (probabilistic)

PHASE 2:
Assignment of
"OR-day schedules"

Goal: bed usage leveling

→ ILP, solved using CPLEX in AIMMS modeling language

MSS test approach

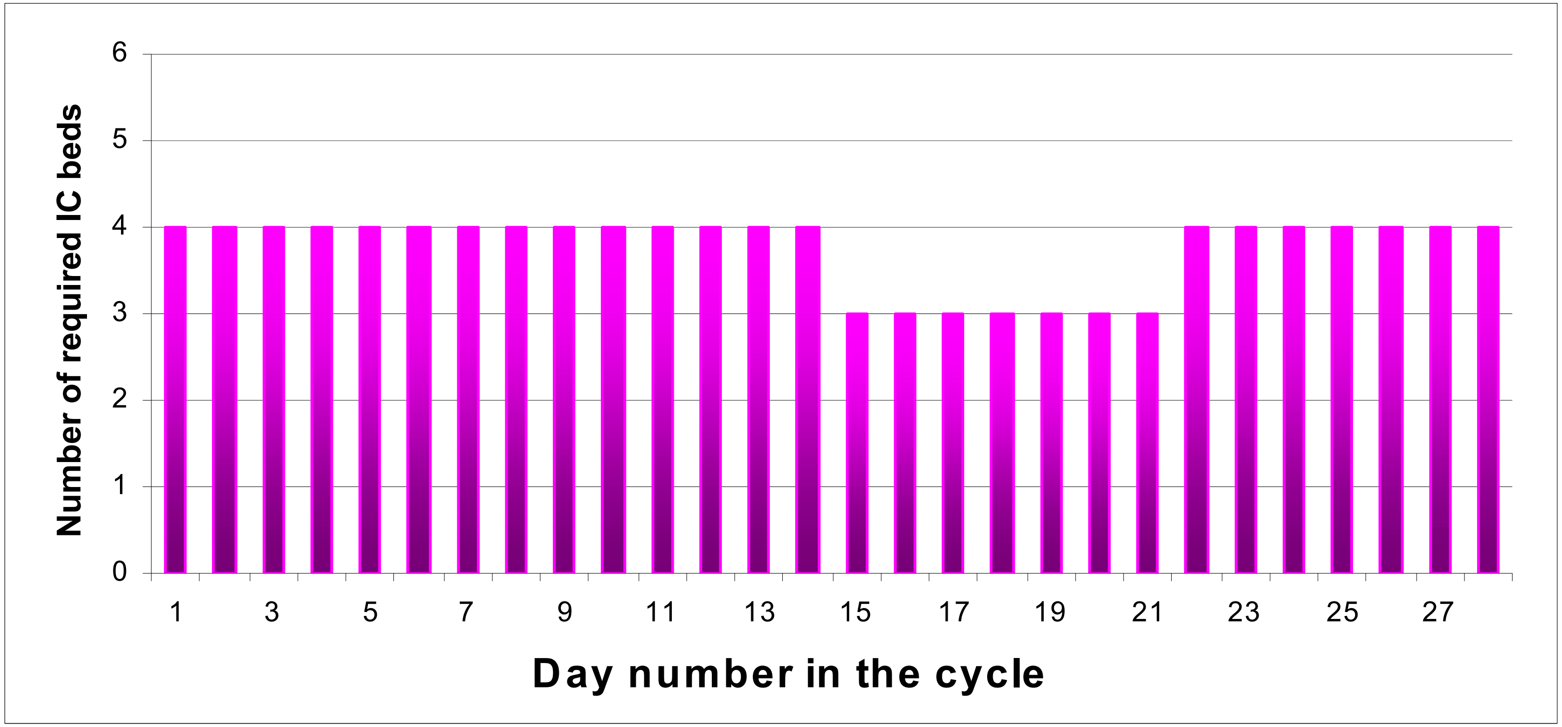
1. Statistical analysis of surgery frequencies
2. Select a cycle length (1, 2, or 4 weeks)
3. Construct an MSS (2-phase approach)
   - Tools: AIMMS modeling language, with CPLEX solver
4. Discrete event simulation
   - Schedule rare elective procedures in reserved capacity
   - Admission of emergency surgeries (add-on and online planning)

Data: historical data from 3 types of hospitals; academic hospital, regional hospital, and clinic

Master surgical scheduling: results

Req. number of ICU-beds <u>without MSS</u>: between 0 and 12 p.day

Req. number of ICU-beds <u>with MSS</u> (4 week cycle):



74.3% of the total ICU bed requirement is planned in an MSS of four weeks.

Master surgical scheduling: results

Reduction OR-capacity usage (portfolio effect):

Cycle length	**1 week**	**2 weeks**	**4 weeks**
Academic hospital	1.1 %	2.7 %	4.2 %
Regional hospital	2.8 %	5.7 %	6.3 %
Clinic	4.9 %	7.3 %	8.6 %

Master surgical scheduling conclusions

Advantages:

- Easy to implement
- Allows personnel coordination in early stage
- Less overtime, higher utilization (up to 8.6%)
- Less surgery cancellations → shorter lead-times
- Improved coordination between departments

Disadvantage:

- Does not cover all surgeries