`OrdinalPatterns.jl`: A Julia package for the statistical analysis of (spatial) time series using ordinal patterns

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julia; software; ordinal patterns; (spatial) time series

`OrdinalPatterns.jl` is a `Julia` (Bezanson et al., 2017) package designed for the statistical analysis of time series and spatial-temporal data using ordinal patterns based on the seminal study by Bandt and Pompe (2002). The package implements a set of statistical tools for using ordinal patterns to investigate things like serial dependence, as proposed by Schnurr and Dehling (2017), Weiß (2022), Weiß and Testik (2023) and Adämmer et al. (2024). The package is designed to be user-friendly and efficient, with an emphasis on high performance.

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ON ALGEBRAIC REPRESENTATIONS OF TIME SERIES, TRANSCRIPTS, AND APPLICATIONS

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Time series; symbolic representations; ordinal patterns; transcripts

Symbolic representations are very helpful in time series analysis. In addition to the dynamical partitions used in symbolic dynamics and ergodic theory, other, more practical methods have been proposed in the last two decades or so. The symbols employed in these methods include, for example, ordinal patterns (or permutations) [1], graphs [2] and homology groups [3], so we can speak, respectively, of algebraic, graph-theoretical and topological representations in those cases. To the best of our knowledge, the concept of transcript [4] has been the first proposal to harness the algebraic structure of the ordinal patterns (i.e., of the symmetric groups). In this talk, we will revisit the concept of transcript, its basic properties and some applications. If possible, we will also report on work in progress.

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CHARACTERIZING TEXTURES USING INFORMATION FROM HILBERT CURVE PATHS

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image texture; Hilbert curve; rotation invariance; permutation entropy; Fisher Information Measure

This research explores a novel approach to analyzing image textures. It utilizes the Hilbert curve, a space-filling curve, to extract data from images. This data is then processed using three information theory measures: permutation entropy, permutation complexity, and Fisher information. The proposed method demonstrates several key advantages: (a) It can effectively distinguish between textures with different levels of correlation. (b) It remains unaffected by rotations and symmetries within the image. (c) Its results are consistent regardless of the image size. (d) It is applicable to both grayscale and color images. The method's effectiveness is validated using both synthetic and real-world images from the well-known Brodatz database.

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UNVEILING SPIKE TIMING PATTERNS IN CHAOTIC LASERS: AN APPROACH USING ORDINAL ANALYSIS AND MACHINE LEARNING

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ordinal patterns; laser data; inter-spike intervals (ISIs); machine learning;

Semiconductor lasers subject to optical feedback exhibit rich nonlinear dynamics, often producing optical pulses reminiscent of neural spikes. The interplay of feedback delay and noise sources, including quantum spontaneous emission noise, introduces significant stochasticity into the spike timing [1]. A precise characterization of these timing statistics is crucial for advancing photonic applications such as information processing and random number generation. In this study, we analyze experimental inter-spike interval (ISI) sequences obtained when a sinusoidal modulation was applied to the laser current. Varying the DC level and modulation frequency results in ISI sequences with distinct statistical behaviors, motivating an exploration of their underlying patterns. This investigation aims to uncover encoding mechanisms within optical spike sequences, offering new perspectives on information transmission. Using ordinal analysis and machine learning [2], we demonstrate that the statistical properties of ISI sequences resemble those of flicker noise, characterized by a tunable parameter α. We also find that, in this dataset, the (α, permutation entropy) plane provides more discriminative power than the (complexity, permutation entropy) representation, facilitating improved differentiation of ISI sequences recorded under distinct experimental conditions and distinguishing original data from surrogate sequences [3].

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Large sample behavior of ordinal patterns in time series

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Ordinal patterns; time series; long-range dependence; short-range dependence; limit theorems

We will review some results on ordinal patterns in time series, both under long- and short-range dependence. Schnurr (2014) introduced the concept of ordinal pattern dependence as a measure of the degree of dependence between two stationary time series. Schnurr and Dehling (2017) analyzed the large sample behavior of empirical versions of ordinal pattern dependence in the case of short-range dependent time series. In addition, they proposed a test for structural breaks based on ordinal patterns. Betken et al. (2021) investigate ordinal patterns in long-range dependent time series, extending earlier work by Sinn and Keller (2011) to the case of non-Gaussian limit distributions.

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STATISTICAL PROPERTIES OF FEATURES FROM ORDINAL PATTERNS

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ordinal patterns; time series; statistical models

We have been intrigued by the statistical properties of features computed from ordinal patterns. We first obtained confidence regions for points in the Entropy-Statistical Complexity plane [1]. We used an empirical approach based on true random series from a quantum device, and obtained confidence regions for several finite-size time series. These results allowed us to see how contamination affects a point, deviating from the expected behavior.

Although valuable and insightful, those confidence boxes lack generalization. Then, we obtained the asymptotic distribution of the Shannon, Rényi, and Tsallis forms of entropy and the Fisher information measure. We presented those results in Refs. [2], [3] under the Multinomial model, i.e., when the patterns are independent. More recently, we incorporated the serial dependence stemming from the Bandt & Pompe symbolization scheme and generalized previous results [4]. Still unpublished, we also have the asymptotic distribution of the Statistical Complexity under the Multinomial model. We show that discarding the serial correlation does not significantly affect the validity of these results for ordinal patterns.

We will present those results in a unified manner while stressing the tools, restrictions, and applications to the data analysis.

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Permutation entropy–based early warning indicators for explosive synchronization in networks of Kuramoto oscillators

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ordinal patterns; phase oscillators; explosive synchronization

Many complex systems experience sudden changes in their dynamics as they transition between different states when a control parameter is varied. The threshold value of the parameter for which an abrupt transition occurs is know as "tipping point". For real–world systems, such as animal populations, vegetation fields, and the climate, there is a particular interest in being able to detect if a tipping point is approaching, since once crossed the systems may not recover the previous stable state [1]. An example of a sudden change is explosive synchronization (ES), an abrupt transition between a disorder state and a synchronous one, characterized by being of first–order [2].

In this work, we propose several quantifiers based on the permutation entropy (PE) that present a particular behavior when the network approaches a synchronous state. PE is a time series analysis technique that uses symbols (ordinal patterns) defined by the ordering of the data points, disregarding the actual values [3], and has recently been proposed as a new indicator for critical transitions [4], [5]. We test this new method in different networks of Kuramoto phase oscillators linked in such a way that they present ES. We observe substantial and characteristic changes in these quantities as the networks approach synchronization, even when the synchronization is explosive. Therefore, PE-based quantities can serve as early warning signals (EWS) of this class of transitions. Our results complement previous findings regarding PE methods for EWS of critical transitions, expanding their applications to systems characterized by complex temporal behaviors.

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Specification Tests based on Ordinal Patterns

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ordinal patterns; time series; cross-sections; specification tests

The vast majority of applications of ordinal patterns and, more generally, the approximation of observable phenomena through symbolic analysis have focused on the analysis of temporal processes. In this research, we aim to broaden our scope by exploring to what extent the design of statistical tools based on ordinal patterns can contribute to the development of statistical tests that support the process of statistical and econometric modeling. Our focus will primarily be on the potential use of ordinal patterns as a central element for specifying models based on cross-sectional data.

ADVANCING OUR UNDERSTANDING OF CARDIOVASCULAR DISEASES: THE ROLE OF NON-INVASIVE METHODS AND ORDINAL PATTERNS IN ASSESSING CARDIOVASCULAR REGULATION

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ordinal patterns; cardiovascular disease; cardiovascular regulation; cardiorespiratory interactions

Cardiovascular disease (CVD) remains the leading cause of death worldwide, with 1.71 million deaths in the European Union alone in 2021 (Eurostat). This highlights the urgent need for research into cardiovascular alterations and improved diagnostic strategies.

The cardiovascular and respiratory systems operate dynamically, adapting to internal and external conditions to maintain homeostasis. Their regulation involves complex interactions between heart rate, blood pressure, and respiratory processes, controlled by several mechanisms, including the autonomic nervous system. This intricate network ensures physiological stability by responding to metabolic demands, environmental changes, and stressors. Non-invasive methods are crucial for assessing cardiovascular regulation, providing insights through heart rate variability (HRV), blood pressure variability (BPV), and respiratory pattern variability (RPV). Understanding the interplay between these factors is essential, as disruptions may signal early cardiovascular dysfunctions. However, while studies often focus on healthy individuals or those with severe disease, data on early-stage alterations remain limited.

Our research investigates these regulatory mechanisms across different populations, including healthy individuals, patients with vasovagal syncope, those with hypertension and its complications, and individuals experiencing disordered breathing during sleep. To enhance cardiovascular assessment, we employ ordinal patterns - a method for analyzing nonlinear and dynamic physiological changes. This approach refines the characterization of cardiorespiratory interactions, aiding in early detection and risk stratification of cardiovascular dysfunctions.

This presentation will focus on clinical aspects of cardiovascular regulation, emphasizing non-invasive methods and ordinal patterns (a separate presentation by Grzegorz Graff will cover the methodological details). Key findings across patient groups will be discussed, highlighting their diagnostic relevance. Finally, challenges in interpreting ordinal pattern-based parameters along with potential future directions for clinical applications will be outlined.

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INVESTIGATING CARDIOVASCULAR INTERACTIONS USING ORDINAL PATTERNS

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ordinal patterns; heart rate variability; cardiovascular regulation; cardiorespiratory coupling

Modern mathematical tools, particularly ordinal patterns, offer a novel approach to studying the dynamics of complex physiological signals. Ordinal patterns allow for the detection of subtle changes in signal structure and provide a robust framework for assessing nonlinear dependencies between heart rate, blood pressure and respiration.

In our previous studies (cf. [1-3]), we applied a diverse set of indices based on ordinal patterns (permutation entropy, transcript entropy, mutual information and others), which demonstrated effectiveness in analyzing ECG data and evaluating the relationships between data series consisting of RR intervals and blood pressure values.

In this study, we employ the ordinal pattern methodology to analyze heart rate and blood pressure, but distinguishing between the phases of inspiration and expiration.

In particular, our earlier study [4] has shown that the distribution of ordinal patterns in healthy individuals is non-uniform, with specific groups of patterns appearing at distinct frequencies. Our objective is to analyze how these distributions change depending on the phase of respiration.

We examine both healthy individuals and patients with hypertension to evaluate how the respiratory phase influences the statistical properties of ordinal patterns, identifying potential distinctions between normal and pathological conditions.

Our findings contribute to a deeper understanding of cardiovascular regulation and might provide new perspectives for non-invasive cardiovascular assessment using ordinal pattern analysis.

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An Extension of Ordinal Persistent Homology for Coupling Complexity Analysis in terms of Subpattern Matching

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ordinal patterns; multivariate time series; persistent homology; random Boolean networks

Ordinal persistent homology proposed in [1] aims to measure the complexity of the relations among components of a given real-valued multivariate time series. In its original formulation [1] and its generalization [2], a filtered simplicial complex reflecting similarity among the ordinal patterns of individual time series is constructed using the intersections of the ordinal patterns as binary relations. In this presentation, we reformulate the construction of the filtered simplicial complex in terms of subpattern matching among the ordinal patterns. Although this viewpoint has already appeared in [1], it has only played an auxiliary role in proving a mathematical property of ordinal persistent homology. The reformulation makes it possible to extend the idea of ordinal persistent homology to finite-alphabet multivariate time series: subpattern matching is performed directly on the symbol sequences themselves rather than on their ordinal patterns induced by an arbitrary order relation on the alphabet. We apply the extended method to binary multivariate time series generated by random Boolean networks. We show that the original ordinal persistent homology cannot capture the transition of the dynamic phase of random Boolean networks well, but the extended method can do so.

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Complexity from ordinal pattern positioned slopes

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Measuring complexity is a general approach to characterize complex systems. However, existing techniques are limited to simultaneously quantify complexity from short length data sets, detect phase transitions and identify periodic dynamics. This contribution presents an innovative approach based on ordinal pattern positioned slopes (OPPS). It considers OPPS group occurrences to compute complexity from OPPS (COPPS) as the average number of patterns and discusses applications even to short data series. This new measure of complexity, COPPS, is then successfully applied to simulated data for detecting phase transitions and regular dynamics, distinguishing between chaotic and stochastic dynamics, and to real-world data for identifying arrhythmia from ECG data.

UNSUPERVISED TIME-EVENT PROBABILISTIC CLASSIFICATION USING LARGE PANELS OF TIME SERIES AND ORDINAL PATTERNS

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Markov-switching models; event detection; change-point detection; symbolic dynamics; compositional data.

This work presents a general framework for partitioning a time span into meaningful, non-overlapping segments using time series datasets characterized by a large cross-sectional dimension. These datasets commonly exhibit complexities and challenges such as non-linearities, structural breaks, asynchronicity, missing data or significant outliers that hamper data analysis and modelling. Aiming at accurate time-event classification and change or breakpoint detection in this setting, our proposal integrates three distinct components into a unified approach: symbolic analysis, compositional data analysis, and Markov-switching time series modeling. A comprehensive Monte Carlo simulation study is conducted to assess the performance of the method, demonstrating exceptional robustness across diverse scenarios. Moreover, its use in real-world applications is illustrated through two economic examples: (i) identifying recurrent recession and expansion regimes in the US economic cycle; and (ii) dating change points to systemic risk episodes in the US stock market.

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Characterizing and detecting regime transitions by using ordinal analysis

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Complex systems often exhibit sudden and potentially dangerous regime transitions. Several data-based diagnostic tools have been proposed to anticipate these changes, but their reliability varies with the system and the characteristics of the observed data. In this talk, I will discuss the performance of the spatial permutation entropy, using real data (high and low resolution vegetation images that reveal desertification transitions), as well as experimental data (the noisy output of a semiconductor laser during the turn-on transition, under a controlled variation of the bifurcation parameter).

Change-point detection via turning rate analysis

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Ordinal patterns have recently been applied to sleep stage classification in EEG data. Sinn and Keller [1] classified sleep stages by analyzing the distribution of ordinal patterns in different segments of a time series detecting breakpoints using the Maximum Mean Discrepancy statistic. Another methodology that has been receiving great attention in the field is the analysis of EEG via the turning rate, introduced in [2].

In a time series, the turning rate corresponds to the relative number of local maxima and minima in a fixed epoch of the series. More precisely, the turning rate corresponds to the frequency of observing one of the ordinal patterns $(0, 2, 1), (1, 0, 2), (1, 2, 0), (2, 0, 1)$, each representing a local minimum or maximum.

We propose a hypothesis test for detecting changes in the distribution of a time series based on variations in the turning rate sequence. To achieve this, we establish limit theorems for estimators of ordinal patterns under the assumption that the increments of the time series are generated by a linear process.

As theoretical background, we establish empirical process limit theory for short- and long-range dependent multivariate linear time series. These general results, which are of independent interest, then serve as the basis for deriving the asymptotic properties of estimators for ordinal pattern probabilities.

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Patterns in Spatio-Temporal Extremes

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cluster of extremes; extreme events; functional time series; ordinal patterns; regular variation; sea surface temperature data; spatio-temporal extremes

In environmental science applications, extreme events frequently exhibit a complex spatio-temporal structure, which is difficult to describe flexibly and estimate in a computationally efficient way using state-of-art parametric extreme-value models. In this talk, we propose a computationally-cheap non-parametric approach to investigate the probability distribution of temporal clusters of spatial extremes generalizing the univariate approach in Oesting and Schnurr (2020), and study within-cluster patterns with respect to various characteristics. These include risk functionals describing the overall event magnitude, spatial risk measures such as the size of the affected area, and measures representing the location of the extreme event. Under the framework of functional regular variation, we verify the existence of the corresponding limit distributions as the considered events become increasingly extreme.

Furthermore, we develop non-parametric estimators for the limiting expressions of interest and show their asymptotic normality under appropriate mixing conditions. Uncertainty is assessed using a multiplier block bootstrap. The finite-sample behaviour of our estimators and the bootstrap scheme is demonstrated in a spatio-temporal simulated example. Our methodology is then applied to study the spatio-temporal dependence structure of high-dimensional sea surface temperature data for the southern Red Sea. Our analysis reveals new insights into the temporal persistence, and the complex hydrodynamic patterns of extreme sea temperature events in this region.

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COMPLEX DYNAMICS AND CAUSALITY: PHASE-SPACE PARTITION OR ORDINAL PATTERNS?

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causality; dynamics; information; ordinal patterns; time series

Information theory provides valuable tools for study and understanding of dynamics and interactions in complex systems.Measures such as entropy and information rates characterize the complexity of dynamics, as well as the memory and information exchange within and between systems. Causal relationships or directed information transfer between different (sub)systems can be inferred using conditional mutual information (CMI), also known as transfer entropy. However, practical evaluation of these measures requires estimating probability distributions, which is often challenging due to data limitations and the curse of dimensionality. To address this, coding time series of continuous variables into sequences of discrete symbols using ordinal patterns (OP) has become a popular alternative to traditional methods like binning or distance-based (e.g., k-nearest neighbors) estimation algorithms.

New avenues of causality research have emerged due to the specific properties of complex systems, such as their evolution on multiple time scales or the occurrence of extreme behavior driven by heavy-tailed probability distributions. The application of CMI to the phases and amplitudes of oscillatory components in complex systems has proven effective in uncovering cross-scale interactions in Earth climate dynamics [1] and cross-frequency causality in brain dynamics [2]. Additionally, reformulating CMI in terms of Rényi entropy has provided a valuable tool for identifying the causes of extreme events [3].

OP coding can be effective for CMI estimations within certain ranges of system dimensions. While some authors have recommended OP coding for inferring causality in phase dynamics, we explain why this approach is not appropriate. Instead, we introduce hybrid algorithms that combine binning and OP coding, discussing their potential usefulness in specific applications.

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ORDINAL PATTERN ANALYSIS OF HIGH-DIMENSIONAL TIME SERIES

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ordinal patterns; chaotic time series; complexity-entropy plane

Ordinal pattern statistics provides useful features for classifying complex data such as multichannel EEG-recordings [1]. It has also been shown that time series from low-dimensional discrete or continuous dynamical systems can be distinguished from stochastic signals (noise) by their position in the ordinal pattern-based complexity–entropy plane [2]. For data representing high dimensional dynamics, however, this approach may fail, because high-dimensional deterministic time series and (their) stochastic surrogate data may be located in the same region of the complexity–entropy plane, with a very similar dependence on the lag used to sample the patterns [3]. This challenge will be illustrated using time series generated by the Lorenz-96 system, the generalized Hénon map, the Mackey–Glass equation, the Kuramoto–Sivashinsky equation, and by phase-randomized surrogates of these data. One possibility to cope with these potential limitations of ordinal pattern-based complexity analysis is to apply a surrogate data test. If the null hypothesis that the given time series deviates from a linear stochastic process cannot be rejected using a combination of permutation entropy and statistical complexity as a discriminating statistic, any resulting position in the complexity-entropy plane should be interpreted with caution. Since for very high-dimensional data, large pattern lengths would be needed to properly separate chaos from noise, the often limited amount of data makes the estimation of finely partitioned distributions in such cases difficult to impossible.

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Synchronisation Detection Using Spatial Ordinal Partitions in Networked Dynamical Systems

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Ordinal Partitions; Dynamical Networks; Synchronisation; Permutation Entropy

Numerous real-world systems can be represented as complex networks with individual units interacting via a graph of connections [1]. Both the connection graph and the characteristics of the individual dynamical systems affect the collective behaviour of the network [2]. Even in situations where the behaviour of individual oscillators is relatively simple, coupled oscillator systems can exhibit complex and varied dynamics [3].

A better understanding of how collective behaviour arises and changes within networks of dynamical systems can be beneficial to a wide range of fields. In global or nonlocally coupled chaotic systems, collective behaviours vary from complete chaotic synchronisation through chimera state and clustering to complete desynchronisation as coupling strength between oscillators decreases. Ordinal partitioning is a low-cost description of a system's dynamics that can be beneficial for identifying the system's collective behaviour and estimating boundaries between synchronous groups of oscillators in networks.

Our method employs ordinal patterns of the spatial configuration of neighbour oscillators at each time point to ascertain whether or not neighbouring nodes in a network are synchronised. We then use permutation entropy and forbidden sequence cardinality to classify collective behaviour. We demonstrate our method's effectiveness on the time series of a ring of coupled Logistic maps.

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LIMIT THEOREMS FOR THE SYMBOLIC CORRELATION INTEGRAL UNDER SHORT-RANGE DEPENDENCE

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ordinal patterns; time series; Rényi-2 entropy, limit theorems

Inspired by the widely used classical correlation integral defined by Grassberger and Procaccia [2], Caballero-Pintado et al. [1] introduced the so-called symbolic correlation integral which can be interpreted as the degree of recurrence of ordinal patterns in a time series. It is strongly related to the Rènyi-2 entropy of the ordinal pattern distribution and can, therefore, be understood as a measure of complexity.

We investigate the symbolic correlation integral further by deriving limit theorems for an estimator based on U-statistics under the assumption of short-range dependence and stationarity.

Hence, we complement the results by Caballero-Pintado et al. [1] and Weiß [4] who considered the i.i.d.-case.

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AN ORDINAL PARTITION OF STATE SPACE PROVIDES GOOD PRESERVATION OF INFORMATION

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ordinal patterns; time series; mutual information; ordinal partition

Ordinal partitions are commonly used for time series analysis to provide a quantisation of an otherwise continuous signal. Such quantisation provides a nice robustness to noise and can be performed in a way that avoids selection of many parameters. While apparently (almost) "parameter-free" it is still an arbitrary choice. Using mutual information and an appeal to the Shadowing Lemma [1] we propose a measure of the information encoded in an arbitrary partition of a dynamical system. Partitions based on trajectory history, such as the ordinal partition, perform best. Hence, we are able to propose a generalisation of the ordinal partition, a *weighted ordinal partition*, and use this measure to select optimal weights [2].

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NON-PARAMETRIC MONITORING OF SPATIAL DEPENDENCE

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non-parametric control charts; ordinal patterns; regular lattice data; spatial dependence; spatial processes

In process monitoring applications, measurements are often taken regularly or randomly from different spatial locations in two or three dimensions. In this research, see [1] for details, we consider streams of regular, rectangular data sets and use spatial ordinal patterns (SOPs) as a non-parametric approach to detect spatial dependencies. Such SOPs were recently introduced by [2] in the context of image analysis, and [3] used SOPs to define non-parametric hypothesis tests for spatial dependence. But instead of a *retrospective* test like in [3], our aim is to develop control charts for the *prospective* online monitoring (sequential testing) of spatial dependence. Our idea is to combine the exponentially weighted moving average (EWMA) approach for univariate ordinal patterns developed by [4] with the test statistics considered by [3]. The proposed SOP-EWMA charts possess an inherent memory (the strength of which is adjusted by the EWMA's smoothing parameter) such that they are sensitive already to small process changes. Another key feature of our proposed SOP-EWMA charts is that they are distribution-free and do not require prior Phase-I analysis. We conduct an extensive simulation study, where the average run length (ARL) performance of the proposed SOP-EWMA charts is examined with respect to various out-of-control scenarios. We demonstrate the superiority and effectiveness of the proposed charts compared to traditional parametric approaches such as control charts relying on the spatial autocorrelation function. We apply the SOP-based control charts to detect heavy rainfall in Germany, war-related fires in (eastern) Ukraine, and manufacturing defects in textile production. The wide range of applications and insights illustrate the broad utility of our non-parametric approach.

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APPLY ORDINAL PATTERN TRANSITION NETWORKS TO CHARACTERIZE STATISTICAL COMPLEXITY AND PERIODIC WINDOWS

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ordinal patterns; time series; transition entropy; shrimp

In this talk, we show two successful applications of ordinal pattern transition networks to characterize statistical complexity measure (SCM) and to identify complex periodic windows surrounded by chaos in the two dimensional parameter space of certain dynamical systems. More specifically, we explicitly focus on heterogeneous frequencies of ordinal pattern transitions. Both artificial and experimental data demonstrate that the consideration of transition frequencies between different ordinal patterns leads to dynamically meaningful estimates of SCMs. In the second example, we compare three measures based on ordinal patterns: traditional permutation entropy ε_O, average amplitude fluctuations of ordinal patterns $\langle\sigma\rangle$, and OPTN out-link transition entropy ε_E. Our results demonstrate that among those three measures, ε_E performs best in distinguishing chaotic from periodic time series in terms of classification accuracy. Therefore, we conclude that transition frequencies between ordinal patterns encoded in the OPTN link weights provide complementary perspectives going beyond traditional methods of ordinal time series analysis that are solely based on pattern occurrence frequencies.

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