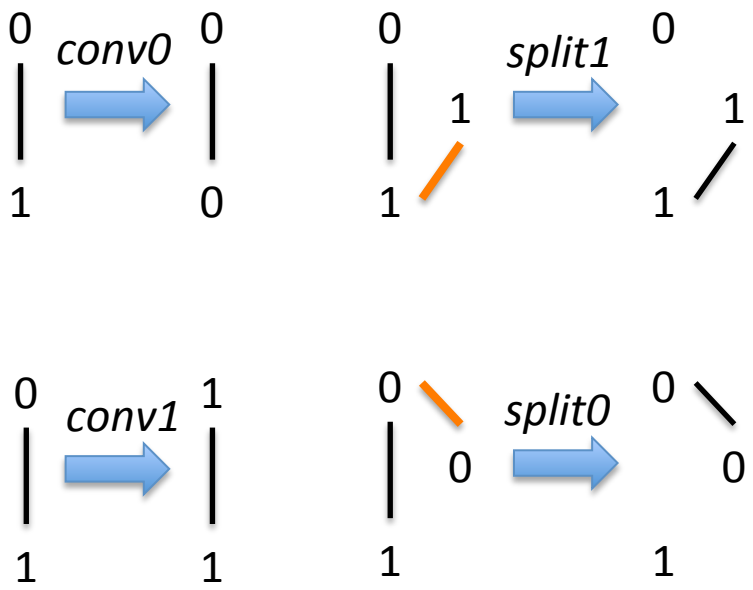
From GT to Differential Equations: Simulation or numerical integration

GaM Challenge

12 April 2015

Social Network Model



Patterns of interest

- # of 0, 1 - nodes
- # of 01, 00, 11 - edges

Agents hold one of two opinions (vote 0 or 1)

If two connected agents hold different opinions,

- one is converted to the opinion of the other, or
- their link is broken and one makes a new connection to a random individual of the same opinion

[Graph Fission in Evolving Voter Model, Durret et al., 2012]

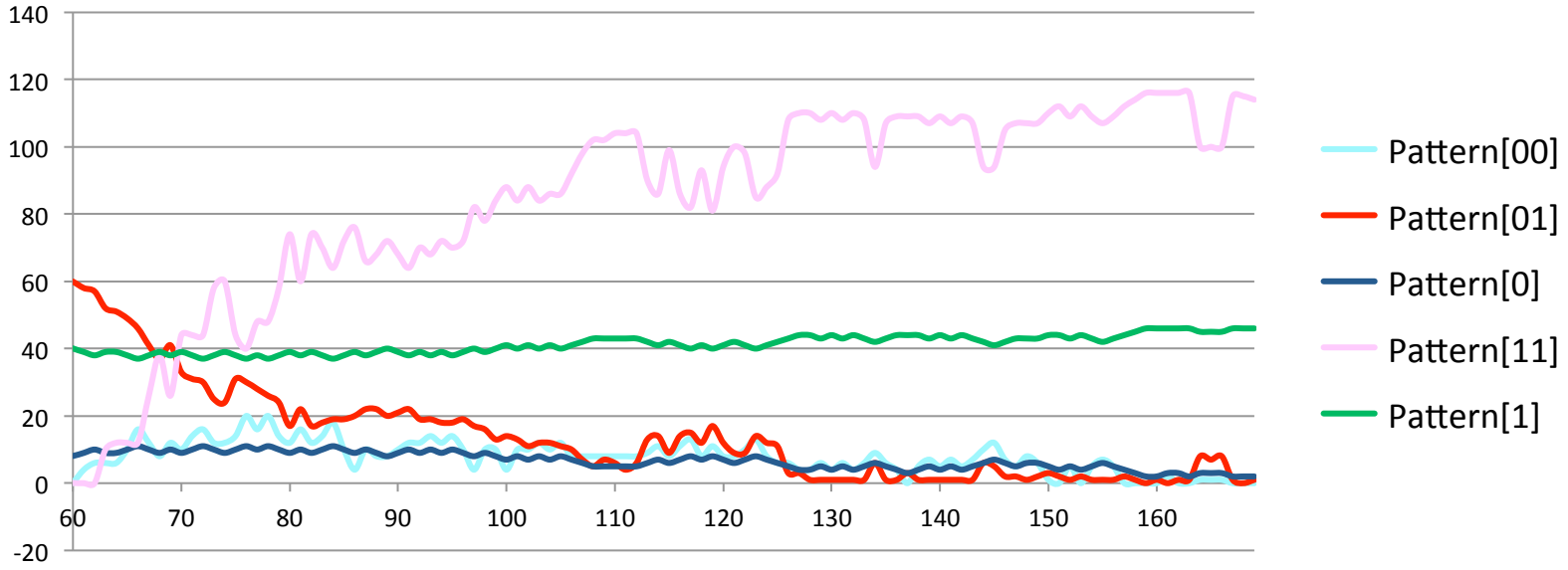
Questions

- How does the number of occurrences of patterns change over time?
- What is the resulting stable state of such a system?

Simulation: Monte Varlo-style stochastic simulation based on assumed rates of rules

Integration: Numerical analysis of ODEs derived

Direct Simulation



For rates of *conv* >> *split,* given 40 1-nodes, 8 0-nodes and 60 01-edges

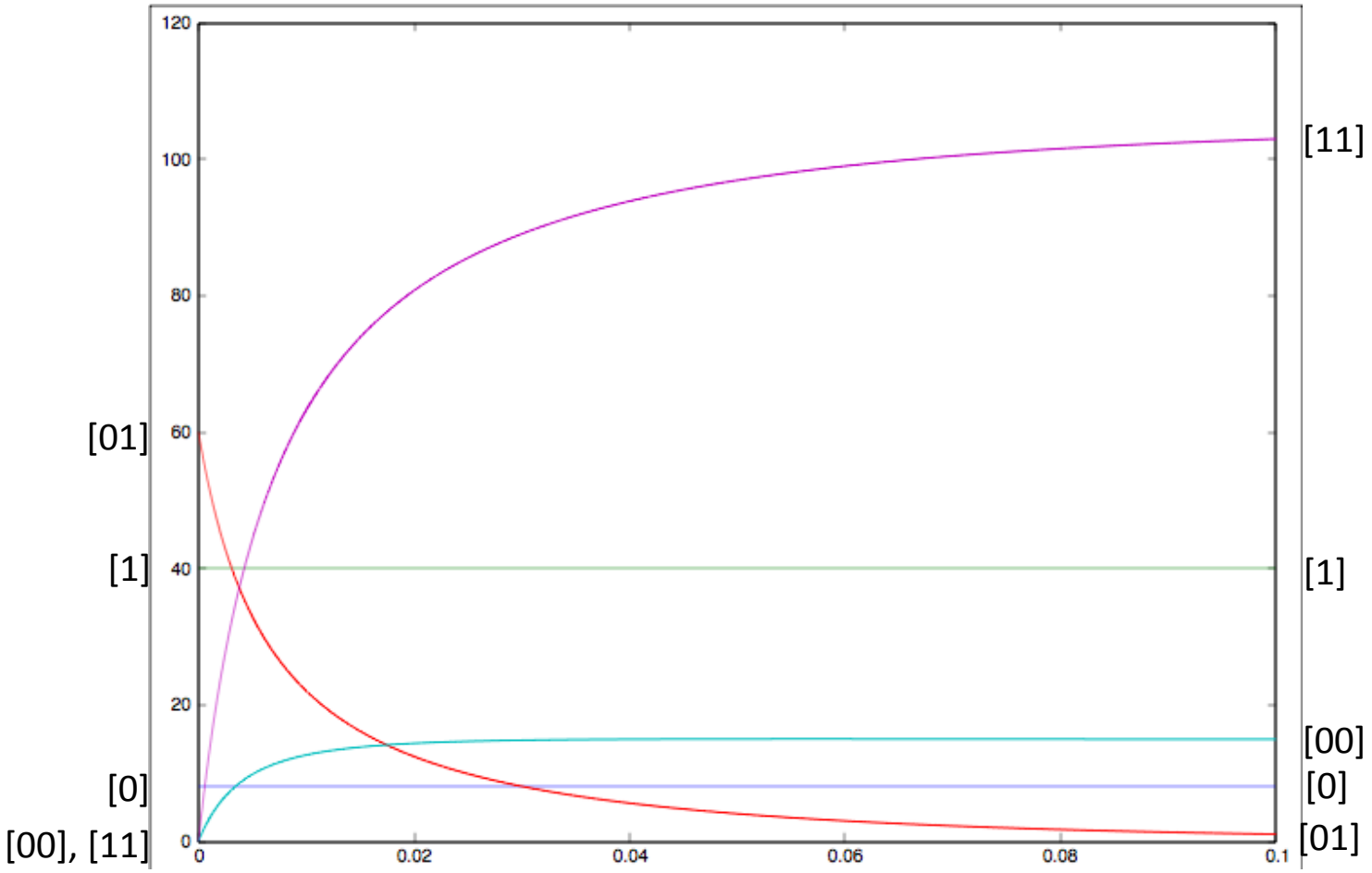
- 01-edges disappear
- Minority opinion 0 declines, along with 00-edges

Deriving ODEs

$$d[P]/dt = \sum_{r \text{ in } R} \#matches(r) * rate(r) * effect(r)$$

d[0]/dt	=	[01]	* conv0	*	1	
	+	[01]	* conv1	*	-1	
….						
d[01]/dt	=	([01]*[0] - [00]/([0]*[0]))	* split0	*	-1	
	+	([01]*[1] - [11]/([1]*[1]))	* split1	*	-1	
	+	[01]	* conv0	*	(([11]-[01])/[1] - 1)	
	+	[01]	* conv1	*	(([00]-[01])/[0] – 1)	
…						

Numerical Integration



Same input, different results: minority and majority constant at 8, 40.

Challenge

Would expect correctness in the limit of graph size towards infinity but ODEs don't account for topology, randomness.

- Better approximations, incorporating topology, randomness?
- More scalable simulation, separating graph into "regions" with no or little influence on each other ... see n-body simulation in physics.