Multi-objective Optimization of traffic externalities using tolls

A comparison of genetic algorithm and game theoretical approach

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Abstract—Genetic algorithms (GAs) are widely accepted by researchers as a method of solving multi-objective optimization problems (MOPs), at least for listing a high quality approximation of the Pareto front of a MOP. In traffic management, it has been long established that tolls can be used to optimally distribute traffic in a network with aim of combating some traffic externalities such as congestion, emission, noise, safety issues. Formulating the multi-objective toll problem as a one point solution problem fails to give the general overview of the objective space of the MOP. Therefore, in this paper we develop a game theoretic approach that gives the general overview of the objective space of the multi-objective problem and compare the results with those of the well-known genetic algorithm *non-dominated sorting genetic algorithm II* (*NSGA-II*). Results show that the game theoretic approach presents a promising tool for solving multi-objective problems, since it produces similar non-dominated solutions as *NSGA-II*, indicating that competing objectives (or stakeholders in the game setting) can still produce Pareto optimal solutions. Most fascinating is that a range of non-dominated solutions is generated during the game, and almost all generated solutions are in the neighborhood of the Pareto set. This indicates that good solutions are generated very fast during the game.

Keywords: Mult-objective problems, Game theory, Genetic algorithm, NSGA-II, Toll setting problem, Transportation network design.

I. INTRODUCTION

Road tolling/pricing is a well-accepted technique in transportation economics to combat traffic externalities such as congestion, emission, noise, safety issues. The problem is how much toll to place and on which road segment such that traffic is efficiently distributed in a given network. Efficiency here means a traffic pattern that optimizes the externalities of interest. Since the mentioned traffic externalities may very well be in conflict with each other, a toll pattern and hence a traffic pattern that optimizes one externality may be to the detriment of the other externalities. Consequently, there is no specific toll pattern that is best for all objectives. For this reason, it is important to enlist all possibly optimal or non-dominated solutions. These non-dominated solutions can then be presented to the decision or policy makers for possible candidate solution(s).

Genetic algorithms (GAs) are widely accepted by researchers as a method of solving multi-objective optimization problems, at least for listing a high quality approximation of the Pareto front

of a MOP. Many researchers have turned attention to solving multi-objective problems using genetic algorithms (GAs) over the recent years. This is mostly because of their robustness in listing layers of Pareto fronts using the so called Pareto ranking. An interested reader should see [1] for a general review of the field of GAs in multi-objective optimization and see [2] for extensive description of the field. The articles discuss some of the most representative algorithms that have been developed so far, as well as some of their applications. Methodological issues related to the use of multi-objective evolutionary algorithms, as well as some of the current and future research trends in the area are discussed in [1]. Our motivation for this paper stems from the recommendation in [1] to seek for "*alternative mechanisms into an evolutionary algorithm to generate non-dominated solutions without relying on Pareto ranking (e.g., adopting concepts from game theory)*". There have been efforts to incorporate game theory to enhance the performance of GAs. In order to force GA to enlist Nash equilibrium points, [3] developed an algorithm that merges GAs and Nash strategy. Application of such merge to domain decomposition method (DDM) - nozzle optimization problems is studied in [4]. Other applications can be found in [5], [6]. GAs have been used to find solutions to some game theoretic problem [7]. In their paper [7], they used a GA to find the optimal strategy of players in a given game. On the other hand, game theorists have incorporated the idea of evolution into game theory in what is now known as evolutionary game theory. These efforts to merge the two disciplines however fail to look at the results distinctly. Therefore, in this paper we address the following: firstly, we use a game theoretic approach to construct an approximation of the Pareto front of a multi-objective problem, and secondly, we compare this Pareto front with a Pareto front that is constructed by the well-known genetic algorithm, *non-dominated sorting genetic algorithm II* (*NSGA-II*). *NSGA-II* is a widely accepted GA that has been used by researchers and is developed in [8].

The remainder of the paper is organized as follows: section 2 gives the general overview of traffic externalities and road pricing. Section 3 describes the problem and the solution methods employed: *NSGA-II* and game theoretical approaches. In section 4, we demonstrate our models using a numerical example, and finally, section 5 concludes the paper.

II. TRAFFIC EXTERNALITIES AND ROAD PRICING

Over the past years, vehicle ownership has increased tremendously. It has been realized that the social cost of owning and driving a vehicle does not only include the purchase, fuel, and maintenance fees, but also the cost of man hour loss to congestion and road maintenance, costs of health issues resulting from accidents, exposure to poisonous compounds from car exhaust pipes, and high noise level from vehicles. So, optimizing traffic flow requires a model that optimizes more than one objective which may conflict with each other. Optimization of more than one traffic externality is not a novel idea, but what is novel is that we are using game theoretical approach to enlist solutions in the solution space. The motivation for solving the multi-objective problem using game theoretical approach stems from the limitations and critics arising from the traditional way of modelling road pricing. Traditional road pricing models assume a Stackelberg game where there is only one leader (e.g. the government or the toll operators) and road users as followers. The leader sets the road toll and the road users react to these tolls re-routing themselves according to user or Wardrop's equilibrium - a traffic condition where no user has any incentive to switch routes. The limitation/critic is that when a single actor or stakeholder or one decision maker (*dm*), (e.g. the government or a private road owner) controls the traffic flow of a transportation system through road pricing, then it is likely that some other stakeholders/regions affected by activities of transportation may not be happy with the effects of the decisions made by this *dm*. This is because when the *dm* models the multi- (or single) objective road pricing problem, all traffic externalities are simultaneously considered (usually using the method of weighted sum [9]) with or without preference for any externality. When preference is given, say, to congestion, then the effect of the preferred externality subdues the effect of other externalities, and this may translate to huge costs for some stakeholders, regions or even road users in terms of noise or emission or other externalities that conflict with travel time. For example, lower travel time (say high speeds) may translate to more accidents (costs for insurance companies). Even without preference to any externality, it is intuitive that stakeholders still will prefer to partake in toll setting discussions to protect their interests. The main problem of a classical approach from multi-objective optimization is the following: supposing that each stakeholder can influence the toll setting, why should an independent player accept a situation which he can improve by changing the (or at least suggesting another) toll patterns?

In such a situation the classical concept of Nash equilibrium in game theory gives an appropriate alternative model. Such models are accepted in economics in situations where independent players may influence the market with their strategies in order to optimize their specific objective.

The question we would like to address from a game theoretical/economic point of view is; what happens when each stakeholder optimizes his objective by tolling the same network, given that other stakeholders are doing the same? How would the results of the dynamic game look like? And finally,

how would the results of the game look like as compared to the results of the multi-objective optimization using genetic algorithms? In what follows, we give brief explanation of the solution methods we have used in this paper.

1) Genaral traffic model and Pareto optimality: Let $G = (N, A)$ be a network, with N the set of all nodes and A the set of (directed) arcs or links in G. We use the following notations:

Table I
NOTATIONS

A	set of all arcs (links or road segments) in G
a	index for links in G
R	set of all paths
r	index for paths (routes)
W	set of all origin-destination (OD) pairs
w	index for OD pairs
f	path flow vector
f_r	flow on path r
θ	link toll vector
θ_a	toll on link a
$v(\theta)$	toll dependent vector of link flows
$v_a(\theta)$	toll dependent flow on link a
$\bar{d}$	travel demand vector
$\bar{d}_w$	demand for the wth OD pair
Γ	OD-path incident matrix
Λ	arc-path incident matrix
$t(v)$	vector of link travel time functions
K	set of all actors in the road pricing game
$C^k(v)$	network cost function for the kth objective
β	monetary value of time per minute (VOT)

We first state the flow feasibility conditions for a fixed demand static network assignment (STA). The term flow is used in transportation engineering to mean the number of vehicles. The following describes a feasible flow for a fixed demand STA:

$$
\begin{aligned}
v &= \Lambda f \\
\Gamma f &= \bar{d} \qquad FeC\_FD \\
f &\geq 0
\end{aligned}
\tag{1}
$$

The first constraint states that the flow v on a link or road is equal to the sum of all path flows f that passes through this link. The second equation is the flow-OD balance constraint, it preserves flow for each origin-destination (OD) pair. It states that the sum of flows on all paths originating from origin node p and ending at destination node q for an OD pair pq equals the demand $\bar{d}$ for this OD pair. The third inequality simply states that the path flows are non-negative. The non-negativity of link flows follows directly from this third inequality. Henceforth we will refer to Eqn (1) as *feasibility condition for fixed demand (FeC_FD)*.

Pareto optimality: If for objective $k \in K$, $C^k(v(\theta))$ denotes the cost or objective function (to be minimized), then a solution vector $\bar{v}(\theta) \in V$ dominates a solution vector $v(\theta) \in V$ if and only if the following holds:

$$
\begin{aligned}
C^k(\bar{v}(\theta)) &\leq C^k(v(\theta)) && \forall k \in K \quad \text{and} \\
C^j(\bar{v}(\theta)) &< C^j(v(\theta)) && \text{for at least one } j \in K
\end{aligned}
$$

the solution $\bar{v}(\theta) \in V$ is Pareto optimal if there does not exist any other solution vector $v(\theta) \in V$ that dominates $\bar{v}(\theta)$.

The line that connects the set of all Pareto points (sometimes called efficient points) to a multi-objective optimization problem is called the Pareto or efficient frontier [10]: these solutions points form the Pareto-optimal set P.

III. SOLUTION METHODS

A. The Game Theoretical Approach

The road pricing game is always formulated as a Stackelberg game where a leader (system controller) moves first followed by sequential move of other players (road users) [11], [12], [9]. When we have just one (or a weighted sum of distinct) objective, then it is assumed that only one leader stays at the upper level of the road pricing bi-level game. In practice it always a difficult question to know when a trade-off between conflicting objectives is beneficial for multi-objective problems. Moreover, actors (stakeholders, leaders and actors are used interchangeably) have preferred objectives, and would want their preferred objectives to have more weights in the weighted sum optimization. So, a solution that favours one stakeholder may be to the detriment of another player. In this paper, we adopt the game theoretic model in [9] where each actor is modelled to control/optimize one externality. In the paper, they assume that various stakeholders can influence (or at least propose) the network tolls. In that case, road users are influenced not only by just one leader as in Stackelberg game, but by more than one decision maker. In the multi-leader-multi-follower game/problem, the leaders, turn by turn, make decisions (search for toll vectors that optimize their respective objectives of interest) at the upper level which influence the followers (users) at the lower level. A toll decision from the upper level is added to the network in the form road tolls, thereby adding to the travel costs for these roads. The followers then react according to user or Wardrop's equilibrium - a traffic condition where no user has any incentive to switch routes. This in turn may cause the leaders to update their individual decisions (that is, changing their toll patterns) leading to lower level players reactions again. Note that given a system in Wardrop's equilibrium, the additional link tolls now distort the system which then triggers users to re-route themselves to be in Wardrop's equilibrium again. So when an actor tolls the network in a manner that optimizes his concerned externality, the users perceive these tolls (as added travel costs) and re-route themselves to satisfy Wardrop's equilibrium, then the next actor in turn seeing the new state of the system and the level of tolls set by previous actors, updates his toll (decision/strategy) to ensure that given the current situation, his current toll level is the best he can do to optimize his objective. These updates in the upper and lower level continue until a stable situation or maximum number of assigned iteration is reached. A stable (Nash equilibrium) state is reached if no stakeholder can improve his objective by unilaterally changing his proposed toll. Note however, that given the stable state decision tolls of leaders, the lower level stable situation is given by the Wardrop's equilibrium. Therefore, the tolling game is

now seen as a bi-level problem, with the stakeholders in the upper level and the travellers in the lower level. The lower level is a constraint to the upper level. In the above dynamic non-cooperative scenario, each actor continuously solves a *program with equilibrium conditions* which is influenced by other actors' *programs with equilibrium conditions*, and these translate to an *equilibrium problem subject to equilibrium condition*s. Note that the push by actors to optimize their objectives in every turn gives a potential to enlisting non-dominated solutions or points in every play. Our aim in this paper is to keep track of the attained solution during this dynamic game, construct a Pareto front from these attained solutions and compare it with the solution of the same problem solved using genetic algorithm *NSGA-II*. For the analysis of Nash equilibrium solution of the road pricing game, see [13].
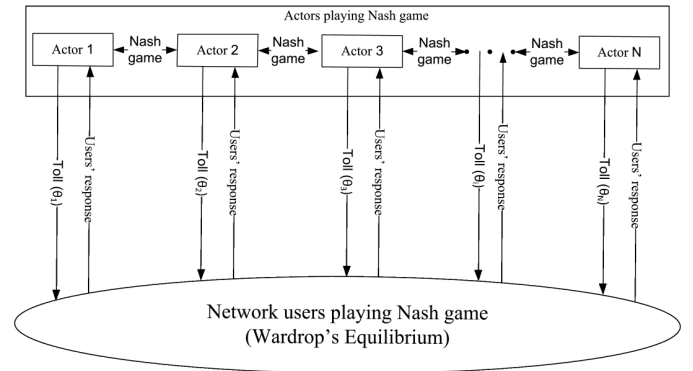

Figure 1. Diagrammatic representation of the dynamic game model

1) Mathematical Formulation of The Game Theoretical Approach: Adopting the game model in [9], and using the Beckmann's convex formulation of Wardrop's user equilibrium (UE) [14], each actor $k \in K$ now solves the following bi-level problem:

$$\min_{\theta^k} C^k(v(\theta^k))$$

$$s.t$$

$$FeC\_FD \tag{2}$$

$$\min_{v_a^k} \sum_{a \in A} \int_0^{v_a^k} \left(\beta t_a(u) + \theta^k + \sum_{j \in K \setminus k} \bar{\theta}^j \right) du$$

Where $C^k(v(\theta^k))$ is player k's objective of interest which depends on the network flow pattern $v(\theta^k)$, θ^k is the link toll vector of player $k \in K$, and $\sum_{j \in K \setminus k} \bar{\theta}^j$ denote cumulative toll vectors in $K \setminus k$. Note that player k cannot change this sum, instead, given this sum, he optimizes his objective using θ^k. The first constraint ensures that the resulting flow is feasible, while the second (also called the lower level problem) ensures that the feasible flow is in user (or Wardrop's) equilibrium [14]. To avoid ambiguity in the use of terms, we will mostly write v to mean $v(\theta)$.

Since the outcome of the lower level problem of Eqn (2) determines the input vector v^k for the objective $C^k(v^k)$ and knowing that this determinant (lower level problem) is given by the Beckmann's formulation in Eqn (2), player $k \in K$ thus

chooses his toll θ^k in a way that optimizes his objective $C^k(v^k)$. In fact, Eqn (2) yields a feasible link flow vector v for every vector sum $\sum_{k \varepsilon K} \bar{\theta}^k$.

During this game, we expect that every move by a player leads to a solution that is (or close to) Pareto efficient (at least for the playing actor) given the current toll level of other actors. For every play and for every turn, the corresponding objective values for all considered objectives are saved during the game.

B. Genetic Algorithmic Approach

The *NSGA-II* algorithm, developed in [8], is a multi-objective optimization algorithm that optimizes several objectives simultaneously, searching for a set of non-dominated solutions, or the Pareto optimal set. It is a genetic algorithm, so based on the principles of natural selection within evolution, it combines solutions to new solutions (*crossover*), where the solutions with a higher fitness value have a higher chance to survive over worse solutions. In the next generation, these enhanced solutions are recombined again, until no progress is made any more or until the maximum number of iterations H is reached. Within *NSGA-II*, the mating selection is done by binary tournament selection with replacement. All selected parents mate using uniform crossover as crossover operator. In addition to this mating process, a random mutation operator is applied to a limited number of solutions from each generation, to promote the exploration of different regions in the solution space. In our case, mutation rate was set to 0.03, so for every design variable there is a 0.03 chance that it is mutated. If a design variable is mutated, it is randomly set to a new feasible value. Evolutionary algorithms are often used to solve multi-objective problems, because they do not end up in a local minimum, and do not require the calculation of a gradient, and still are able to produce a diverse Pareto set. More information on genetic algorithms in a multi-objective context can be found in [2].

Within the algorithm, the fitness value is calculated in two steps. In the first step (non-dominated sorting), the solutions are ranked based on Pareto dominance. All solutions in the Pareto front receive rank 1. These solutions are then extracted from the set and all Pareto solutions in the remaining set receive rank 2, etcetera. In the second step, the solutions are sorted within these ranks based on their crowding distance. Crowding distance calculation requires sorting of the population according to each objective value. The extreme values for each objective are assigned an infinite value, assuring that these values survive. All intermediate solutions are assigned a value equal to the absolute difference in the function values of two adjacent solutions. The crowding distance value (and thus the fitness value) is higher if a solution is more isolated, promoting a more diverse Pareto optimal set. *NSGA-II* contains elitism, to preserve good solutions in an archive φ. The archive only contains the best solutions based on the defined fitness value. This implies that in case the number of non-dominated solutions grows bigger than the archive size, solutions are selected based on crowding distance instead of dominance. For details on the algorithm, the reader is referred to [8].

The objectives optimized in system (2) are all system objectives, for which the Pareto set is constructed. *NSGA-II* is designed to construct a diverse set, so containing solutions with low (assuming that objectives corresponds to costs) values for the first objective, but also solutions with low values for the other objectives. It aims to show the complete spectrum of possible solutions, giving attention to all objectives (or players in the game approach). However, the travellers in the traffic system optimize their own benefits (costs in the form of tolls and travel time) in a similar way as in the game approach: they achieve user equilibrium. Therefore, the toll design problem is now seen as a bi-level problem, with the road authority in the upper level and the travellers in the lower level. The lower level is a constraint to the upper level. For every solution the genetic algorithm comes up with, a lower level user equilibrium problem is solved, resulting in network flows and costs, from which the objective functions can be calculated. This process is then repeated over and over again until no progress is made any more or until the maximum number of iterations is reached. Using *NSGA-II* as a yard stick, results of our game model are then compared to those of the *NSGA-II*.

1) Mathematical Formulation of Genetic Algorithm Approach: Mathematically, the toll optimization problem for the *NSGA-II* is different from the game theoretic approach given in Eqn (2) in sense that the tolls are not differentiated among the objectives. *NSGA-II* selects one generic toll θ per link to optimize the objectives simultaneously. For *NSGA-II*, the modified version of Eqn (2) is formulated as follows:

$$\min_{\theta} \left(C^1(v(\theta)), C^2(v(\theta)), \cdots, C^{|K|}(v(\theta)) \right)$$
$$s.t$$
$$FeC\_FD$$
$$\min_{v_a} \sum_{a \in A} \int_0^{v_a} (\beta t_a(u) + \theta) \, du \tag{3}$$

FeC_FD is as given in Eqn (1), and $|K|$ denotes the total number of objectives (corresponding to players in the game approach). Again, the second constraint (the lower level problem) is the Beckmann's convex formulation of Wardrop's user equilibrium (UE). It ensures that any feasible solution flow v resulting from system (3) is in user equilibrium: a condition where no individual road user can reduce his/her travel cost by unilaterally switching routes.

Note that *NSGA-II* has been applied successfully by researchers to solve a multi-objective optimization problem in traffic engineering, e.g. [15], [16]

IV. NUMERICAL RESULTS

A. Five-Node Network

1) Link Attributes and Input: We will use a five-node network used in [9] to compare the two models described in the preceding sections. We demonstrate first-best pricing scheme - where tolls are allowed on all links. For the second-best scheme - where some links are not allowed to be tolled, one only need

to add the additional toll constraints on links.

The origin-destination demand for the example network is 1000 users.

Table II
NETWORK ATTRIBUTES

1a. Link Attributes | | | | Vehicle Class = Private cars | | |

Link	Length(km)	Free Speed(km/hr)	Capacity	Air Emission Cost for NOx (€/gram)	Air Emission Cost for PM10 (€/gram)	Safety factor (Injury per Veh-km)
1	10	100	400	10	5	0.008
2	7	70	300	10	5	0.08
3	10.5	100	350	45	40	0.008
4	5	70	200	60	60	0.00001
5	4	70	250	45	40	0.00001
6	10	90	250	10	5	0.09
7	5	80	250	10	5	0.009
8	8.5	90	300	45	40	0.009

1b

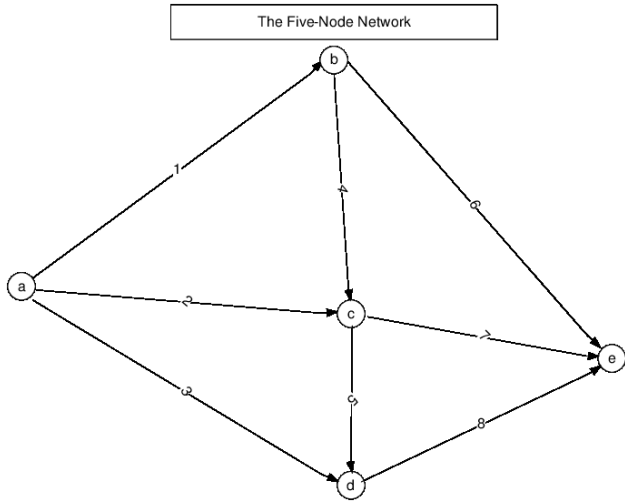| Emission Factor (g/km/veh)		
Speed (km/hr)	NOx	PM10
< 15	0.702	0.061
<= 30	0.456	0.059
<= 45	0.48	0.059
< 65	0.227	0.035
>= 65	0.236	0.043

Figure 2. The five-node network with eight links

To demonstrate our models, we have chosen three externalities / objective, namely: (1) *minimizing total system travel cost*, (2) *minimizing total system emission cost*, (3) *maximizing the network safety* (or *minimizing cost related to safety issues*). We have used the following cost functions:

System Travel Time Cost:

$$C^t(v) = \sum_{a \in A} \beta v_a t_a(v_a) = \sum_{a \in A} \beta v_a T_a^{ff} \left(1 + \eta \left(\frac{v_a}{\hat{C}_a} \right)^{\phi} \right);$$

the so called *Bureau for Public Roads (BPR) function*, developed by Federal Highway Administration (FHWA), a division of the United States Department of Transportation that specializes in highway transportation.
T_a^{ff} - free flow travel time on link a,
v_a - total flow on link a,
$\hat{C}_a$ - practical capacity of link a, and
$\eta \, and \, \phi$ - *BPR* scaling parameters, with $\eta = 0.15$, $\phi = 4$.
β is the value of time (VOT) with the value *0.167EUR / minute* [17], see Table II for other parameters.

Emission Cost:
$$C^e(v) = \sum_{a \in A} e_a(v_a) = \sum_{a \in A} v_a \alpha_a \varkappa_a l_a;$$ where
$\varkappa_a$ - emission factor for link a (depending on the emission type and the vehicle speed on link a given in g/vehicle-kilometre).
l_a - length of link a. In this case study, we only consider two emission types; NO_x and PM_{10}.
See Table II for the emission costs α_a and emission factor $\varkappa_a$.

Safety Cost:
$$C^s(v) = \sum_{a \in A} s_a(v_a) = \sum_{a \in A} \rho \kappa_a E_a = \sum_{a \in A} v_a \rho \kappa_a l_a \, ;$$ where
κ_a - risk factor for link a, measured in number of injury-crashes/vehicle-kilometre (see Table II).
$E_a = l_a * v_a$ - measure of level of exposure on link a.
We set the cost of one injury ρ to *300EUR / injury*.
Emission factors are from the CAR-model [18], emission and injury costs are chosen in a reasonable way. For more on the formulation of the objective functions, interested reader is referred to [19].

MATLAB is used to solve all programs. We solve the non-cooperative (Nash) game among the actors using the *NIRA-3* [20]. *NIRA-3* is a MATLAB package that uses the Nikaido-Isoda function and relaxation algorithm to find unique Nash equilibria in infinite games. An interested reader may also wish to see [21] for an evolutionary algorithm for equilibrium problem with equilibrium constraints (EPECs). In *NIRA-3*, we set *alphamethod = 0.5, precision = [1e-3, 1e-3], and TolCon = TolFun = TolX = 1e-3*. For more on the *NIRA-3* see [20].

For the game, we place a toll bound condition of [0,5]EUR per link per player to limit the solution space. Since *NSGA-II* has discrete design variables as input, the tolls are discretized with steps of 0.1 EUR. In the game problem each of the 3 players could vary the toll within the interval [0,5], making the total toll of the 3 players to vary within 0 EUR and 15 EUR per link. In *NSGA-II* application, this translates to 8 design variables (one for each link) with 151 different possible toll values from the set {0.0,0.1, ... ,14.9,15.0} for each link. Note that in *NSGA-II* application, the tolls are not differentiated among the objectives unlike in the game approach where each player has control over a specific toll range, i.e [0,5] per link. To search for non-dominant solutions, the *NSGA-II* application uses a whole (discretized) toll range of [0,15] per link which corresponds to three players total toll range per link in the game approach.

Within this application of *NSGA-II*, every solution in the parent generation will combine to new solutions, so the crossover parameter is set to 1 (a chance of 1 to crossover). The initial chance for a toll value to mutate is set to 0.03. For every generation, this chance is reduced by 5%, in order to achieve convergence.

All three objectives are simultaneously optimized in *NSGA-II*, and all the three players compete in turns in the non-cooperative game. All calculations were conducted on MATLAB version 9 running on a 64-bit Windows 7 machine with 4 GB of RAM.

B. Results

Defnitions

Output definitions: The set Θ is defined as all decision vectors (or solutions) that are calculated during one optimization process, so $|\Theta| = \varphi H$. Where φ is the size of the archive in one optimization process, and H is the maximum number of generations. N is the cardinality of the Pareto set. The set of N solutions $P \in \Theta$ with $P = \{v_1, v_2, \cdots, v_N\}$ is defined as the Pareto set resulting from one optimization process, which includes all non-dominated solutions with respect to all solutions in Θ, there is no $v_i \in P$ such that $v_j \in \Theta$ dominates v_i. P is the outcome of our MOP.

Hypervolume indicator: This is the space coverage of the Pareto set as implemented in [22], also known as S-metric or hypervolume. In the 2-dimensional case it determines the area that is covered by the Pareto set with respect to a reference point (the star in Figure 3). The reference point represents the upper bound of all objectives: the reference point is defined such that it is dominated by all solutions in the Pareto set. Because the true maximum values of the objective functions are not known, we choose a conservative point, based on the evaluated solutions. In the 3 dimensional case area is replaced by volume, and in the more dimensional case by hypervolume. The area or the hypervolume covered by the Pareto set P is denoted by *SSC(P)* in the figure below.
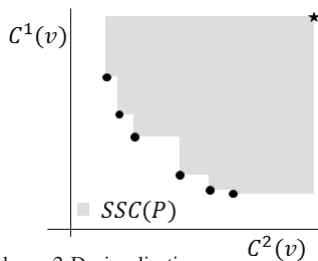

Figure 3.   Hypervolume 2-D visualisation

In the non-cooperative game model, for every play and for every turn, the corresponding objective values for all considered objectives are saved. Similarly the multi-objective optimization results from the *NSGA-II* are saved for every iteration. For easy visualization, we have displayed the results of the 3-dimensional optimization process for only two objectives per plot. For the *NSGA-II*, we allowed 60 solutions to be generated within 1 generation for 100 generations. For comparison reasons, we also allowed a maximum of 2000 play turns for each of the three players in the non-cooperative game model. On the graphs that follow, we have displayed and compared non-dominated solutions resulting from the two distinct approaches.

Figure 4 shows the Pareto set (or non-dominated solution) plot of the objectives; *total travel time cost* and *total safety cost* for the *NSGA-II* and game approach. See that the shapes of the two Pareto plots somewhat take the same U-shape.The plots show that   *NSGA-II* generated more points in Pareto set. Furthermore, *NSGA-II* achieves better values for the safety objective. Apparently the *Safety* player is not cable of achieving

much better values while competing with other players in the game approach. This may be due to the fact that *Travel time* and *Emission* objectives are more in line with the user equilibrium (lower level problem) cost function. The objectives of *System Travel time* and *Emission* have in them the travel cost function as giving in the user equilibrium problem, whereas the *Safety* objective has no such function incorporated in it. Therefore during the game, it is easier for the *Travel time* and *Emission players* to achieve better solutions for themselves as compared to those of the *Safety player*. This indicates that the objective *safety* can only be further minimized if all three players agree to cooperate so as to have free access to the complete range of feasible tolls as in *NSGA-II*. That notwithstanding, the game approach does not fail in producing non-dominated solutions as we can see from the Pareto set plots. Recall that we have displayed the results of the 3-dimensional optimization process for only two objectives per plot, so some points that seem dominated in one plot are non-dominated in another plot.
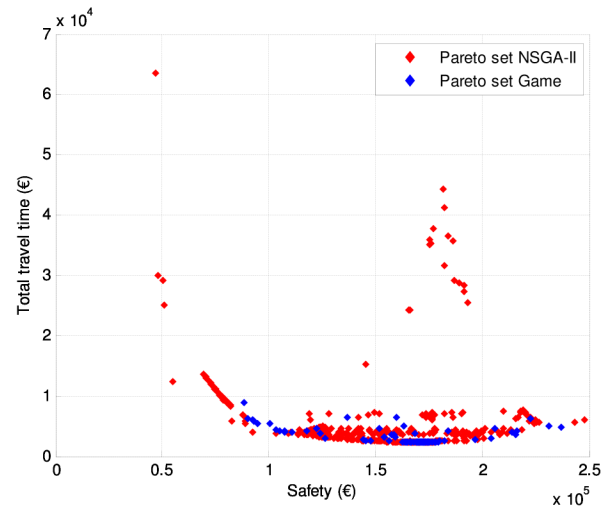

Figure 4.   Pareto set of *Travel time cost* vs *Safety cost* from NSGA-II and Non-cooperative game

Similar as in Figure 4, we have displayed again a Pareto set plot of *total travel time cost* and *total emission cost* in Figure 5. The figure shows again a more diverse plot by *NSGA-II*, note however that the differences seen in one figure are the same for all other figures, but displayed in different axis. Figure 6 displays the Pareto plots in the axis of *total emission cost* and *total safety cost*. What is interesting from the figures is that the game approach is almost able to discover all non-dominated plot clusters as displayed by the *NSGA-II*. We mention here that the game model is more constrained than the *NSGA-II* counterpart in the sense that *NSGA-II* has access to a whole toll spectrum [0,15] per link to optimize the three objectives simultaneously, whereas the game approach restricts a toll range of [0,5] per player per link. If we design the game as cooperative game instead of the non-cooperative game, then the three players will now have access to a whole toll spectrum [0,15] per link to optimize their three objectives simultaneously just as in *NSGA-II*. However, our aim in this paper is to demonstrate that non-cooperative game model presents a promising way of solving

multi-objective problems. In fact, the *NSGA-II* seems to be solving the cooperative game version of the game approach where all the players cooperate, use their combined toll ranges, and simultaneously do what is beneficial for all players.

Despite the "constrained" nature of the game approach, the non-cooperative game approach is capable of producing non-dominated solutions comparable to the *NSGA-II* results. This reveals that the game approach has a great potential in enlisting non-dominated solutions for multi-objective problems.
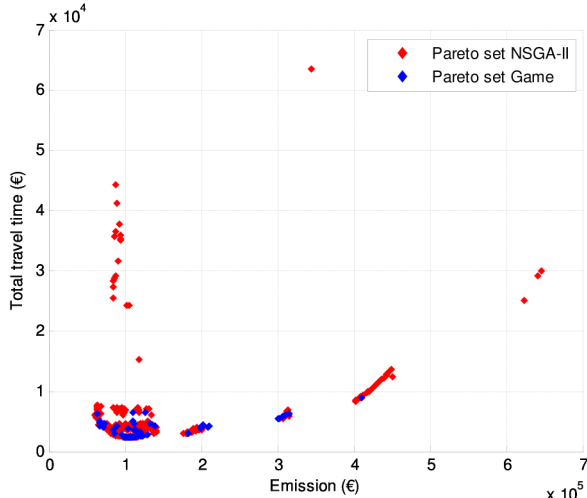

Figure 5.   Pareto set of *Travel time cost* vs *Emission cost* from NSGA-II and Non-cooperative game

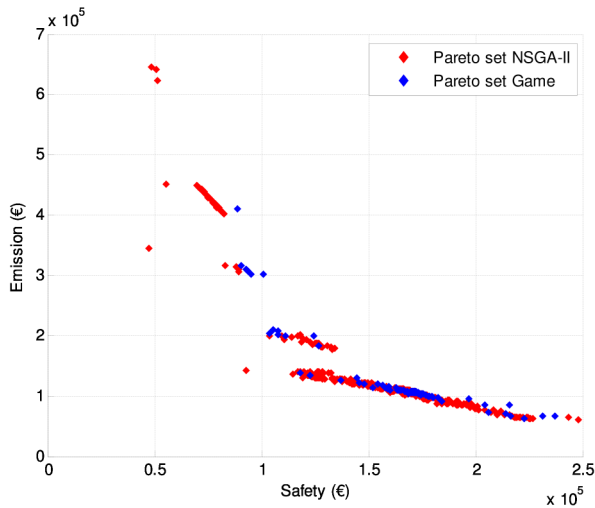Figure 6.   Pareto set of *Emission cost* vs *Safety cost* from NSGA-II and Non-cooperative game

Note that in general, with more solutions and iterations allowed, both the *NSGA-II* and the game approach have the potentials of improving on the Pareto fronts.

For the two approaches, we show below (Figure 7 and Figure 8) plots of all the generated solutions and a summary table (Table III). The plots show that a range of non-dominated solutions is generated during the game. Furthermore, Figure 7 shows that almost all generated solutions are in the neighborhood of the Pareto set, indicating that the non-dominated

solutions are generated early in the optimization process, and further asserts the consistency of the game approach. This is also underpinned by the notion that some of the solution points replicated themselves many times during the game, due to the nature of the game where after some moves a player will prefer to choose a set of tolls he had chosen earlier in the game. This further indicates that the game already reached convergence in less than 6000 iterations. As a result, the game approach generated a smaller number of Pareto solutions. In contrast, *NSGA-II* covers a larger solution area or hypervolume, some good portion of its generated solutions are very far from the Pareto front though. However, NSGA-II in the end achieves a more diverse and richer Pareto set as indicated by: the lower (and thus better) values for the minimum objective function values for all 3 objectives, the higher value for hypervolume covered and the comparison plots.
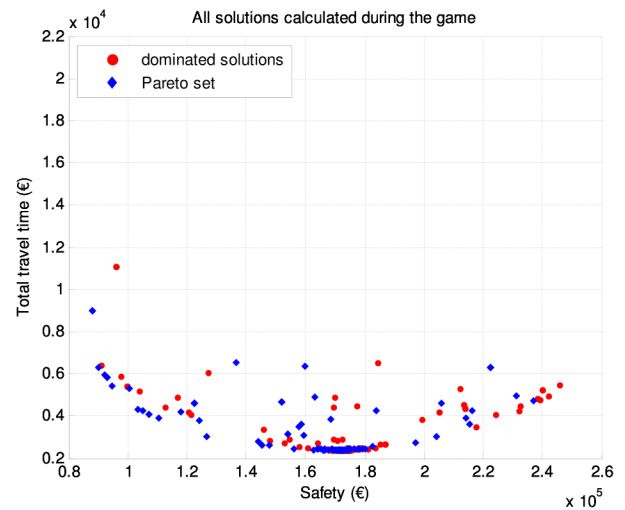

Figure 7.   All solutions from Non-cooperative game approach
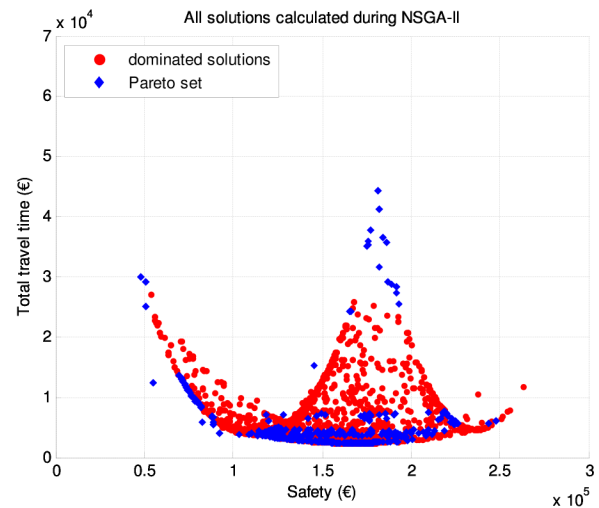

Figure 8.   All solutions from *NSGA-II* approach

The summary table further shows that the game approach generated fewer Pareto points. Note however that some of these

solution points (some of which are Pareto points) replicated themselves many times during the game. This is due to the nature of the game (as earlier mentioned) where after some moves a player will prefer to choose a set of tolls he had chosen earlier in the game.

Table III
SOLUTION SUMMARY

	Game	NSGA-II
Number of solutions in the Pareto set	101	794
Minimum value for travel time	2391	2389
Minimum value for emission	63367	60930
Minimum value for safety	88240	46894
Hypervolume covered by the set	6.22E+15	7.08E+15

V. CONCLUSION

In this paper, we compared the results of a multi-objective optimization using two distinct approaches, namely; the well-known genetic algorithm *NSGA-II* and a model from non-cooperative game theory. We applied these techniques to the problem of optimal toll design in a transportation network, with *total travel time*, *total emissions cost* and *total cost of safety* as objectives. In the game theoretic approach, every objective is optimized by one of the players, while the travelers aim for user equilibrium. The results show that the game approach has the potential of discovering non-dominant solutions. Though *NSGA-II* produces a more diverse Pareto set (seen in the plots and based on the hypervolume indicator), the game theoretic approach does somewhat approximate the *NSGA-II* solutions. The figures showed that similar clusters of Pareto points could be discovered by the game approach, except for the objective safety, because safety directly competes with the interests of the users. Further, plotting all solutions generated during the game showed that most dominated solutions still lie in the neighborhood of the Pareto front, asserting the consistency of the game approach. This implies that good solutions are generated fast during the game. Although the Nash game model does not ensure that all non-dominated solutions are generated, the competition among the actors (where each actor searches for the best solution given what other actors are doing) tends to draw the solution points near to the Pareto front. We therefore conclude that game theoretical approach described in this paper presents a promising method for enlisting non-dominated solution for multi-objective problems. We further acknowledge that Nash equilibrium solutions may not be Pareto efficient. Therefore, the next line of research will be to enhance the game model to ensure a diverse Pareto set.

VI. ACKNOWLEDGEMENT

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