

## **MULTI-OBJECTIVE ROAD PRICING: A Cooperative and Competitive Bilevel Optimization Approach**

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### **ABSTRACT**

Costs associated with traffic externalities such as congestion, air pollution, noise, safety, etcetera are becoming unbearable. The Braess paradox shows that combating congestion by adding infrastructure may not improve traffic conditions, and geographical and/or financial constraints may not allow infrastructure expansion. Road pricing presents an alternative to combat traffic externalities. The traditional way of road pricing, namely congestion charging, may create negative benefits for society. In this effect, we develop a flexible pricing scheme internalizing costs arising from all externalities. Using a game theoretical approach, we extend the single authority road pricing scheme to a pricing scheme with multiple authorities/regions (with likely contradicting objectives).

### **KEYWORDS**

Road pricing, multi-objective optimization, mathematical program with equilibrium constraints (MPEC), equilibrium problem with equilibrium constraint (EPEC), Nash equilibrium.

### **INTRODUCTION**

Over recent decades, vehicle ownership has increased tremendously. Many people now realize that the social cost of owning and driving a vehicle does not only include the purchase, fuel, and maintenance fees, but also the cost of time lost due to congestion and road

maintenance. The costs of accidents, inhalation of poisonous compounds emitted from vehicle exhaust pipes, and exposure to high noise levels from vehicles add to the welfare loss. Economists found that when a resource that is vital and scarce is either free or underpriced, then demand for such resource will outstrip supply, resulting in shortages. This phenomenon can be readily seen in the transportation sector. When the demand or number of vehicles using a certain road exceeds the road's capacity, then congestion begins to build up. In 2006, it was estimated that there were 41,118 traffic jams and approximately 60 million vehicle lost hours on Dutch highways [1]. As road space is a valuable and scarce resource, traffic engineers suggest that it ought to be rationed by a price mechanism. Road users should pay for using the road network to make optimum allocation decisions between transport and other activities. Traffic engineers argue that this will reduce travel time across the entire network. [2-4] propose the use of vehicle tax and parking charges to combat congestion. Nowadays, the benefits of owning a car generally still outweigh the taxes, and the subsidies received by employees from their employers for transportation fares cushion the effect of parking charges, thus rendering the taxes/fees less effective. Until now, researchers have mostly focused on congestion pricing [6, 10, 11] neglecting the overall effect of such practice.

Most real life optimization problems require the simultaneous optimization of more than one objective. This is because many real life problems are defined in many objectives [7]. In most cases, these objectives are in conflict with each other and may or may not be equally important [8]. A good road pricing model that maximizes social or economic welfare must involve not only the simultaneous minimization of travel time, road accident, road damage, noise and air pollution, but also maximization of user benefit. In this paper, we show that single objective road pricing can lead to bizarre network situations. We thus develop an optimization model which is capable of finding Pareto optimal flow patterns that optimize all the mentioned traffic externalities at once. To achieve this 'optimal' result in practice, we design a flexible tolling scheme capable of steering road users and optimally distributing them in the network. This leads to the most efficient use of the network.

All but few models assume that the transportation system is managed by a single decision maker, usually the government. They assume that only one body sets tolls on roads, and this they often relate to Stackelberg game. Using game theoretical approach, we extend this one leader Stackelberg game to a game of multiple actors (or stakeholders or leaders). The later represents a more general and realistic situation where different agencies or stakeholders or even regions set tolls on the road network to maximize their selfish interests. Specifically, for example, insurance companies may set tolls to minimize road accidents and have little or no interest in congestion, whereas the ministry of economics may be interested in minimizing man-hour loss in the traffic so as to boost productivity. Furthermore, one region of a country may set tolls to optimally distribute traffic on the regional network irrespective of the flow pattern and/or tolling scheme of other regions. The problem is formulated as equilibrium problem with equilibrium constraints (EPEC) [12].

## **METHODOLOGY, MATHEMATICAL AND ECONOMIC THEORY**

The single leader road pricing problem is often formulated as bi-level optimization [11]. The upper level solves the leader's interest and the lower level solves the so called user problem or user equilibrium. This problem is analogous to a Stackelberg game [9] where the leader moves first following by sequential move of other players. In the single leader road pricing game, the leader predicts users' behavior (that users will always minimize their travel cost) and thus, using tolls he steers them to his own interest. The user problem, namely minimizing individual travel cost, formulated as an optimization problem can be transformed to a set of

inequalities (called equilibrium constraints) such that the bi-level problem transforms to a single level optimization problem. This single level problem is called mathematical problem with equilibrium constraints (MPEC) [10]. If we assume that different stakeholders are allowed to toll the network, then, road users are influenced by decisions of multiple leaders or stakeholders. When leaders do not cooperate, we refer to it as a multi-leader-followers game or problem. When the followers (road users) perceive these (upper level) decisions, they react accordingly, and this reaction may cause the leaders to update their individual decisions which also results in lower level players updating their reactions. These updates continue until a stable situation is reached. In our road pricing problem, the lower level users are in equilibrium or stable state when no user can decrease his cost (or increase his benefit) further by unilaterally switching to another route or change their trip decision. In the same way, the upper level players with interests in one or more of the traffic externalities are in equilibrium state when no stakeholder can further improve his objective by unilaterally switching to another toll vector.

In the above scenario, each leader is continuously solving an MPEC which is influenced by other leaders' MPECs. A closer look reveals that we are confronted with an equilibrium problem subject to equilibrium constraints (EPEC). In game theory, a Nash equilibrium is defined as a state where no player can improve his or her outcome by altering his or her decision unilaterally. Hence, in classical game theory, our problem translates to solving two Nash problems. Our aim therefore is to find a toll vector (if it exists) for all leaders such that the upper level is at Nash equilibrium as well as the lower level.

When the stakeholders cooperate, then, they pursue one objective, which in our case is to keep the system cost as low as possible and the users' benefit as high as possible. This is achieved by aggregating their individual cost and searching for a point on the Pareto frontier that maximizes the economic benefit. Then, as seen in the model below (*equation (2)*), the multi-leader model is translated into a single leader model.

The non-cooperative road pricing model can be stated as:

*Maximize economic benefit [EB] with respect to stakeholder's system cost*

*s.t*

*\* flows are in user equilibrium*

*\* feasibility conditions are satisfied*

*Mathematically:*

$$\begin{aligned}
& \max_{v^k, d_w^k, \theta^k} EB = \text{User benefit} - \text{System cost} \\
& = \sum_{w \in W} \int_0^{d_w^k} B_w(\zeta) d\zeta - C_k(v^k) \\
& \quad s.t. \\
& \quad \Lambda^T \left( \beta t(v^k) + \theta^k + \sum_{j \in K \setminus k} \bar{\theta}^j \right) \geq \Gamma^T B(d^k) \\
& \quad \left( \beta t(v^k) + \theta^k + \sum_{j \in K \setminus k} \bar{\theta}^j \right) v^k = B(d^k)^T d^k \\
& \quad (v, d) \in \Omega
\end{aligned} \quad \left. \vphantom{\begin{aligned} \max \\ = \\ s.t. \\ \Lambda^T \\ \left( \beta t(v^k) + \theta^k + \sum_{j \in K \setminus k} \bar{\theta}^j \right) \\ \left( \beta t(v^k) + \theta^k + \sum_{j \in K \setminus k} \bar{\theta}^j \right) v^k \\ (v, d) \end{aligned}} \right\} \forall k \in K \quad (1)$$

$C_k$  ~ system cost for stakeholder  $k \in K$ ,  $v^k$  ~ link flow vector desired by stakeholder  $k$

$B_w$  ~ inverse demand function,  $d_w$  ~ demand of OD  $w \in W$

$t$  ~ link travel time function,  $\theta$  ~ link toll vector (variable)

$\bar{\theta}$  ~ link toll vector (fixed),  $\beta$  ~ monetary value of time [1]

$\Lambda$  ~ arc – path incident matrix,  $\Gamma$  ~ OD – path incident matrix.

The first two constraints ensure user equilibrium, and the last, feasibility conditions.

For the cooperative model, equation (1) becomes:

$$\begin{aligned}
& \max_{v, d_w, \theta} EB = \sum_{w \in W} \int_0^{d_w} B_w(\zeta) d\zeta - \sum_{k \in K} C_k(v) \\
& \quad s.t. \\
& \quad \Lambda^T (\beta t(v) + \theta) \geq \Gamma^T B(d) \\
& \quad (\beta t(v) + \theta) v = B(d)^T d \\
& \quad (v, d) \in \Omega
\end{aligned} \quad (2)$$

The model in equation (2) gives the maximum possible societal benefit. The system achieves such high societal welfare on the cost of some objectives, and as such, some stakeholders will see such coalition to be detrimental. Thus, such model is not stable since some stakeholders will be better off if they pull out of the coalition. To stabilize the model, the benefit derived from (2) is distributed such that each stakeholder is guaranteed his outcome in (1) and the remaining benefit distributed equally or as agreed upon.

## CONTRIBUTIONS

We designed a flexible tolling scheme that can be used to distribute traffic on the network such that traffic externalities such as noise, emission, congestion, accidents, e.t.c. are minimized. Our model, formulated as an MPEC is capable of finding a single point on the Pareto frontier that optimizes the entire system performance when all links are allowed to be tolled. We showed that this result is analogous to the grand coalition game of the actors. When stakeholders do not cooperate, we proved that Nash equilibrium can never be reached with unbounded link tolls. We also showed that if the tolls are bounded and some conditions satisfied, Nash equilibrium always exists. Under non-cooperative game, we proposed a benefit sharing strategy that is beneficial to all actors, and thus showing that grand coalition is always possible among the stakeholders.

## CONCLUSIONS

Since road pricing schemes centred around congestion alone may create negative benefits for society, thus defeating its objective (increasing transportation efficiency and social welfare), we have designed a flexible framework to determine tolling schemes that will help reduce the impacts of most important traffic externalities. The well-structured mathematical models can be easily implemented in practice through the flexible tolling scheme. When tolling all links is not feasible, we established that the most efficient use of the network may still be achieved. Furthermore, due to political and equity reasons, many stakeholders and/or regions may partake in toll setting. We studied the system stability under the existence of Nash equilibrium among conflicting stakeholders. The paper described an egalitarian sharing rule that lures leaders to form grand coalition, and thus achieving system optimal flow pattern.

Since the models used in this paper are centred on classical optimization formulations, the number of variables can grow uncontrollably large for real life networks. This calls for an efficient optimisation heuristic which can transform the analytical models into heuristic algorithms capable of handling large networks. A time dependent model will be an extension.

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