

THE NUMERICAL SOLUTION OF THE ELASTOHYDRODYNAMICALLY LUBRICATED  
LINE- AND POINT CONTACT PROBLEM, USING MULTIGRID TECHNIQUES

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LINE- AND POINT CONTACT PROBLEM, USING MULTIGRID TECHNIQUES

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## ABSTRACT

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The main topic of this thesis is the numerical solution of the problem of Elasto-Hydrodynamic Lubrication (E.H.L.) for line- and point contact conditions. As an introduction to the subject, a historical review of the field of E.H.L. is given and the presently used numerical methods are discussed on their merits.

Next, the equations which describe the problem are derived together with the dimensionless groups of parameters for the line- and point contact problem. As a first attempt to solve the line contact problem, the Newton-Raphson method is applied and its advantages and drawbacks are highlighted. Though this method works satisfactorily for the line contact case, it is useless for the point contact problem because of the required computing time.

Therefore, an alternative solution technique called Multigrid is discussed with respect to the E.H.L. problem and the topic of stability and convergence is treated.

Applying the Multigrid technique to the E.H.L. line contact equations results in a considerable reduction of the computing time. A number of solutions is presented together with a new film thickness formula, based on these numerical calculations and on analytical asymptotes. In a number of these solutions the so-called "pressure spike" can be detected, and the nature of this spike is discussed. Making use of the reduction in computing time, the point contact problem is tackled and a number of solutions for different lubricating conditions is given. The specific problems, and their solutions, of this two-dimensional case are outlined.

The results of both the line- and point contact calculations are then used to compute the so-called "rolling traction". Next, the influence of different types of surface roughness geometries is investigated, and a number of solutions is presented. Finally, recommendations for future research are given.

## SAMENVATTING

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Het onderwerp van dit proefschrift is de numerieke oplossing van de Elasto-Hydrodynamische Smeringsvergelijkingen (E.H.S.) voor lijn- en puntcontact. Als inleiding wordt een historisch overzicht gegeven van het werkgebied en de huidige oplosmethoden worden beoordeeld op hun merites. Vervolgens worden de vergelijkingen afgeleid die het E.H.S. probleem beschrijven, als ook de dimensieloze groepen die van toepassing zijn op het lijn- en puntcontact E.H.S. probleem. Als eerste aanzet, om tot een numerieke oplossing van het lijn contact probleem te komen, is de Newton-Raphson oplosmethode toegepast. Een aantal resultaten wordt gepresenteerd en de tekortkomingen van deze aanpak worden aangegeven. Hoewel de methode bevredigend werkt voor het één-dimensionale probleem is zij niet bruikbaar voor het twee-dimensionale probleem vanwege de vereiste rekentijd. Daarom is een snelle oplosmethode genaamd Multigrid gebruikt. De methode wordt evenals de stabiliteit en convergentie besproken. Vervolgens wordt deze oplosteknik op het E.H.S. lijncontact probleem toegepast en worden de specifieke moeilijkheden en hun oplossingen belicht en de tijdwinst besproken. In een aantal gepresenteerde oplossingen is de zogenaamde "drukpiek" zichtbaar, en de fysische achtergrond van dit verschijnsel wordt besproken. Ook wordt een filmdikteformule gegeven die gebaseerd is op deze numerieke berekeningen en op analytische asymptoten. Gebruikmakend van deze winst in rekentijd wordt het puntcontact E.H.S. probleem aangepakt en worden een aantal oplossingen gepresenteerd, samen met de specifieke moeilijkheden van dit probleem. De resultaten van de numerieke berekeningen voor lijn- en puntcontact worden gebruikt voor de berekening van de zogenaamde "rollende wrijving". Tenslotte wordt de invloed van verschillende oppervlakteruwheids-geometriën besproken en een aantal oplossingen worden gegeven. Als afsluiting wordt een aantal aanbevelingen gedaan voor toekomstig onderzoek.



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LIST OF SYMBOLS FOR E.H.L.

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Symbols may have different meanings for the one-dimensional (1d) and the two-dimensional case (2d).

$a$  = amplitude of the asperity  
 $A$  = dimensionless amplitude of the asperity,  $A = aR_x/b^2$   
 $b$  = half width of the Hertzian contact (1d),  $b = \sqrt{((8wR)/(\pi E'))}$   
 $b$  = radius of the Hertzian contact (2d),  $b = ((3wR_x)/(2E'))^{1/3}$   
 $E$  = elasticity modulus  
 $E'$  = reduced elasticity modulus,  $2/E' = (1-\nu_a^2)/E_a + (1-\nu_b^2)/E_b$   
 $f$  = coefficient of rolling traction,  $f = F_r/w$   
 $F_r$  = rolling traction force  
 $\underline{F}$  = external force field  
 $G$  = material parameter,  $G = \alpha E'$   
 $h$  = film thickness  
 $h_c$  = integration constant  
 $h_{00}$  = constant in the film thickness equation  
 $H$  = dimensionless film thickness, (1d)  $H = hR/b^2$   
 $H$  = dimensionless film thickness, (2d)  $H = hR_x/b^2$   
 $H_{dh}$  = dimensionless film thickness according to Dowson and Higginson (1d),  $H_{dh} = h/R$   
 $H_{hd}$  = dimensionless film thickness according to Hamrock and Dowson (2d),  $H_{hd} = h/R_x$   
 $H'$  = dimensionless minimum film thickness according to Moes (1d),  $H' = H_{dh}(2U)^{-1/2}$   
 $H'$  = dimensionless minimum film thickness according to Moes (2d),  $H' = H_{hd}(2U)^{-1/2}$   
 $H_{00}$  = dimensionless constant in the film thickness equation  
 $\Delta H$  = distance between two lines of constant height  
 $L$  = dimensionless material parameter (Moes) (1d),  $L = G(2U)^{1/4}$   
 $L$  = dimensionless material parameter (Moes) (2d),  $L = G(2U)^{1/4}$

$M =$  dimensionless load parameter (Moes) (1d),  $M = W(2U)^{-1/2}$   
 $M =$  dimensionless load parameter (Moes) (2d),  $M = W(2U)^{-3/4}$   
 $\Delta mf =$  relative mass flux defect  
 $p =$  pressure  
 $P =$  dimensionless pressure,  $P = p/p_h$   
 $p_h =$  maximum Hertzian pressure (1d),  $p_h = 2w/(\pi b)$   
 $p_h =$  maximum Hertzian pressure (2d),  $p_h = 3w/(2\pi b^2)$   
 $q =$  dimensionless reduced pressure,  $q = \exp(-\bar{\alpha}P)$   
 $R =$  reduced radius of curvature (1d)  
 $R_x =$  reduced radius of curvature in X-direction (2d)  
 $R_y =$  reduced radius of curvature in Y-direction (2d)  
 $r_a =$  radius of the asperity  
 $R_a =$  dimensionless radius of the asperity,  $R_a = r_a/b$   
 $t =$  time  
 $T =$  dimensionless time,  $T = u_m t/b$   
 $U =$  dimensionless speed parameter (1d),  $U = \eta_0 U_m / (E'R)$   
 $U =$  dimensionless speed parameter (2d),  $U = \eta_0 U_m / (E'R_x)$   
 $\underline{U} =$  velocity vector  
 $U_m =$  mean velocity  
 $U_\Sigma =$  sum velocity,  $U_\Sigma = 2U_m$   
 $v =$  deformation  
 $w =$  load per meter (1d)  
 $w =$  load (2d)  
 $W =$  dimensionless load (1d),  $W = w/(E'R)$   
 $W =$  dimensionless load (2d),  $W = w/(E'R_x^2)$   
 $x, y =$  coordinate  
 $X, Y =$  dimensionless coordinate,  $X = x/b, Y = y/b$   
 $z =$  pressure viscosity parameter (Roelands)

$\alpha =$  pressure viscosity index (Barus)  
 $\bar{\alpha} =$  dimensionless viscosity index,  $\bar{\alpha} = \alpha p_h$   
 $\Delta =$  distance between two neighboring gridpoints