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Fluid-particle simulation using Smoothed Particle Hydrodynamics and Discrete Element Method (SPH-DEM)

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1&2 PARDEM

Fluid-particle simulation using Smoothed Particle Hydrodynamics and Discrete Element Method (SPH-DEM)

1. **Fluid-particle systems** are ubiquitous in nature and industry
   - Geophysical – e.g. Sediment transport, erosion, avalanches, dam or levee failure.
   - Industry – e.g. food and powder mixing, fluidized bed reactors
2. Development of **meshfree** fluid-particle simulation method:
   - Smoothed Particle Hydrodynamics and Discrete Element Method (SPH-DEM)
3. Applied to:
   - **Validation test** cases using single/multi particle sedimentation
   - **Dispersion of granular bed** by liquid jet – collaboration with Nestle

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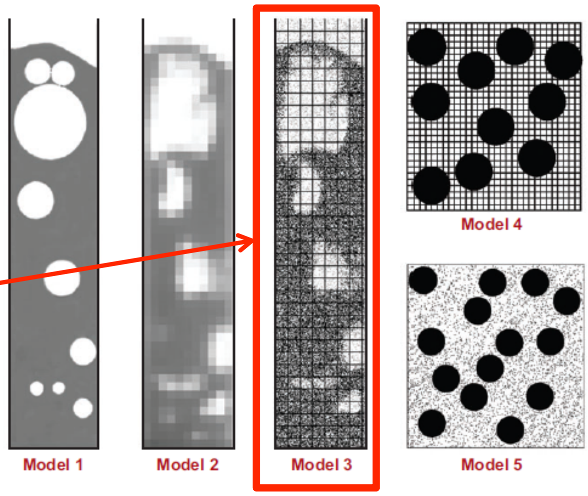
Fluid-particle simulation – which length scale?

Length scale of interest determines simulation method

Fluid resolution > particle diameter



Van der Hoef, M. A., van Sint Annaland, Deen, N. G., & Kuipers, J. A. M. (2008). Numerical simulation of dense gas-solid uidized beds: A multiscale modeling strategy. Annual Review of Fluid Mechanics, 40 (1), 47{70.

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Fluid Equations – Locally Averaged Navier Stokes Equations

- Anderson and Jacksons (1967) derived
  locally averaged Navier Stokes equations (AVNS)
- Solid particle distribution is converted to a smooth porosity field

$$\frac{\partial(\varepsilon\rho)}{\partial t} + \nabla\cdot(\varepsilon\rho\mathbf{u}) = 0$$

$$\varepsilon\rho\left(\frac{\partial\mathbf{u}}{\partial t} - \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla P + \nabla\cdot\tau - n\mathbf{f}_i + \varepsilon\rho\mathbf{g}$$

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$$\bar{\rho}\left(\frac{\partial \mathbf{u}}{\partial t}-\mathbf{u}\cdot\nabla\mathbf{u}\right)=-\nabla P+\nabla\cdot\tau-n\mathbf{f}_i+\bar{\rho}\mathbf{g}$$

Define a superficial density

$$\bar{\rho}_a=\varepsilon_a\rho_a=\sum_b m_b W_{ab}$$

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AVNS-SPH equations

- Substitute normal SPH density with superficial density

$$\frac{d\bar{\rho}_a}{dt}=\sum_b m_b \mathbf{v}_{ab}\cdot\nabla W_{ab}$$

$$\frac{d\mathbf{v}_a}{dt}=-\sum_b m_b\left(\frac{P_a}{\bar{\rho}_a^2}+\frac{P_b}{\bar{\rho}_b^2}+\Pi_{ab}\right)\nabla_a W_{ab}+\frac{\mathbf{f}_a}{m_a}$$

- Reference density for each particle scaled by porosity
- Smoothing length *h* varies with superficial density

$$P_a=B_a\left(\left(\frac{\bar{\rho}_a}{\varepsilon_a\rho_0}\right)^{\gamma}-1\right) \qquad B_a=\frac{100\varepsilon_a\rho_0 v_m^2}{\gamma} \qquad h_a=\sigma\left(\frac{m_a}{\bar{\rho}_a}\right)^{1/d}$$

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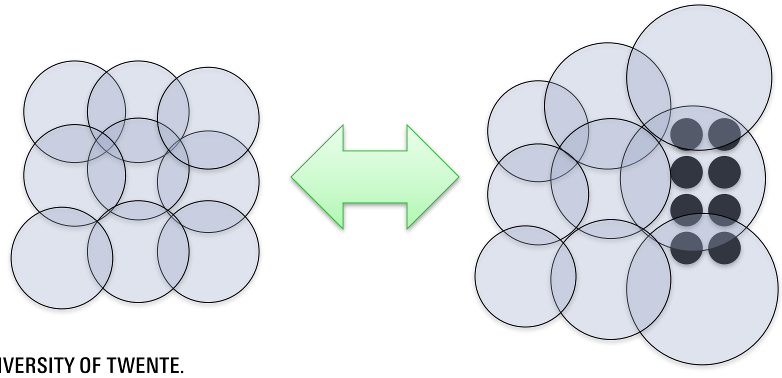
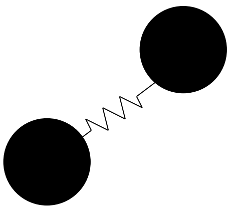
Variable resolution fluid solver

- SPH resolution (smoothing length h) depends on density
- Therefore, resolution coupled to porosity ($h \approx \varepsilon^{-1/3}$)
- Retains accuracy, as particles increase effective viscosity and inhibits turbulent flow



Solid phase - Discrete Element Method (DEM)



- Lagrangian method for simulating granular material
- Newton's equations of motion integrated for individual particles

$$m_i \frac{d^2\mathbf{r}_i}{dt^2} = \sum_j \mathbf{c}_{ij} + \mathbf{f}_i + m_i\mathbf{g}$$

- Contacts modeled using explicit forces $\mathbf{c}_{ij}$
  - We use a linear spring contact force (with dissipation)
- Coupled to fluid via drag force model $\mathbf{f}_i$

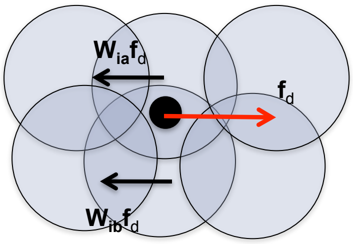
Fluid-Particle Drag Force

- Force on DEM particle due to fluid:

$$\mathbf{f}_i = V_p(-\nabla P + \nabla \cdot \tau) + \mathbf{f}_d$$

- Where $\mathbf{f}_d$ is the drag force model
  - Stokes drag (creeping flow, single particle)
  - Di Felice (1994) drag model (higher Re, multiple particles)

- Drag force calculated on each DEM particle
- Particle drag force then interpolated to surrounding SPH particles
  - Constructed so Newton's third law satisfied



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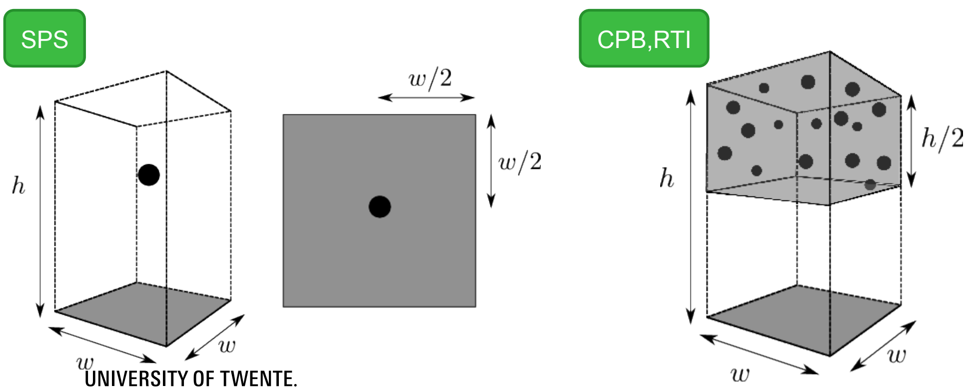
3D Sedimentation Test Cases

1. **Single Particle Sedimentation (SPS)**
2. **Sedimentation of a constant porosity block (CPB)**
3. **Rayleigh Taylor Instability (RTI)**



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Set of Realistic Fluid-Particle Parameters

- Particle properties chosen to match glass beads used in dispersion cell experiments
- Contact law – linear spring dashpot
  - Very low stiffness to speed up calculations – particle collisions not important here

Property	Value
Density	2500 kg/m^3
Diameter	1x10^{-4} m
Spring Stiffness	1x10^{-4} kg/s^2
Damping	0 kg/s

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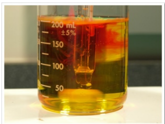
Set of Realistic Fluid-Particle Parameters

- Three different fluids used to provide a range of particle Reynolds Numbers
- Parameters based on air, water and 10% glycerol-water solution



Property	Air	Water	Glycerol-water
Density	1.18 kg/m^3	1000 kg/m^3	1150 kg/m^3
Viscosity	1.86x10^{-5} Pa·s	8.9x10^{-4} Pa·s	8.9x10^{-3} Pa·s
Re_p	0.65 – 3.19	0.15 – 0.85	0.002 – 0.011

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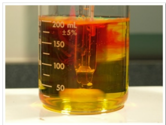
Set of Realistic Fluid-Particle Parameters

- Stokes law gives an estimate of the fluid-particle relaxation time scale
- This provides an additional time-step constraint





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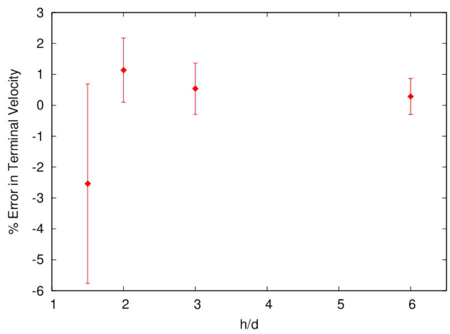
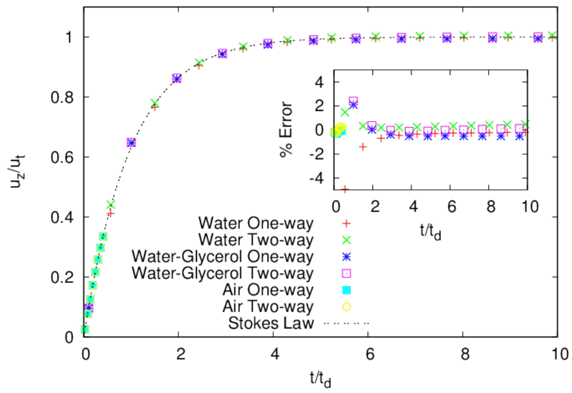
Re_p	0.65 – 3.19	0.15 – 0.85	0.002 – 0.011
Relaxation Time	7.5x10^{-2} s	1.56x10^{-3} s	1.56x10^{-4} s

Single Particle Sedimentation (SPS)

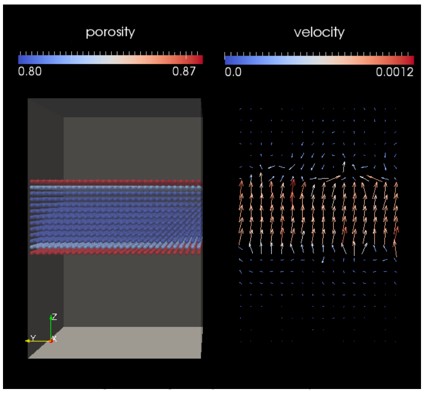
- Tested a range of particle Reynolds numbers ($0.01 \leq Re_p \leq 10$) and fluid resolution ($1 \leq h/d \leq 6$)
- SPH-DEM results consistently less than 2% error
- Provided the fluid resolution *h* is sufficiently coarse vs. particles ($h > 2d$)

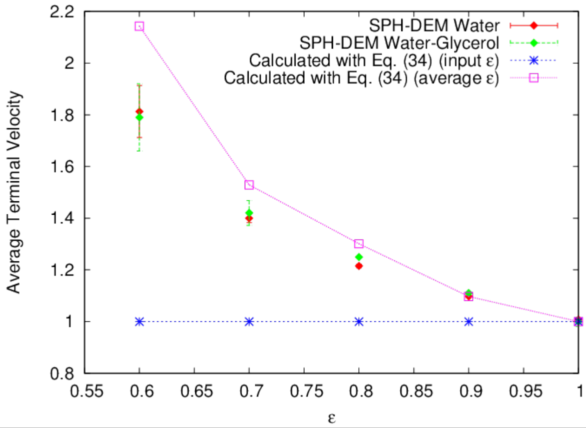


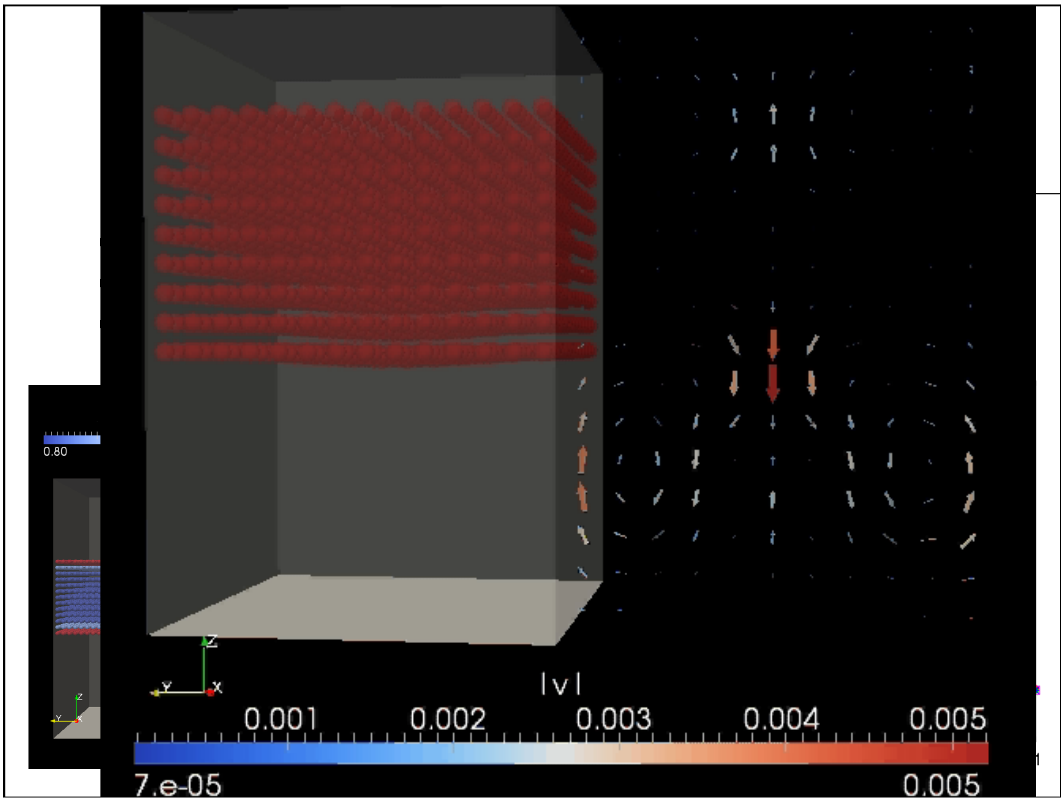


Sedimentation of a constant porosity block (CPB)

- Reproduces terminal velocity well over a range of porosity ($0.5 \le \varepsilon \le 1.0$)
- Fluid resolution reduces porosity at CPB edge
- Some errors at high porosity gradients - working with alternate drag calculation to minimize this.







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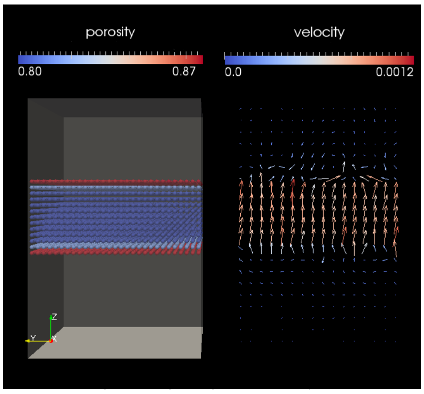
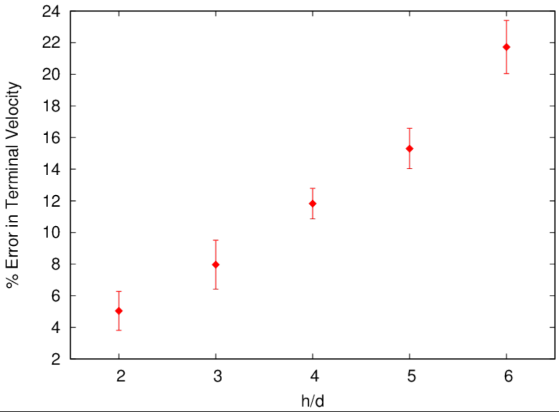
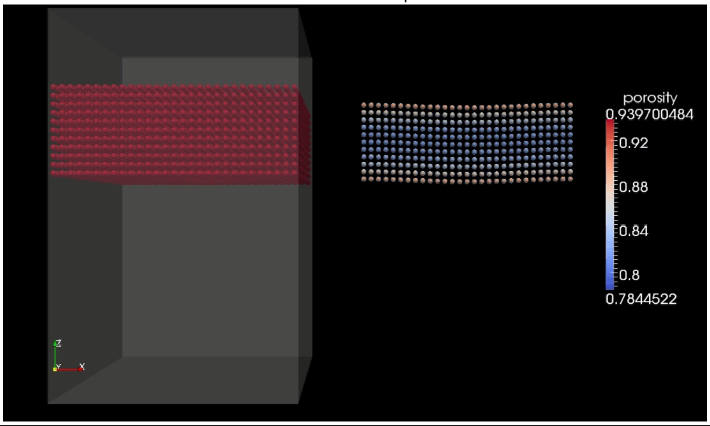
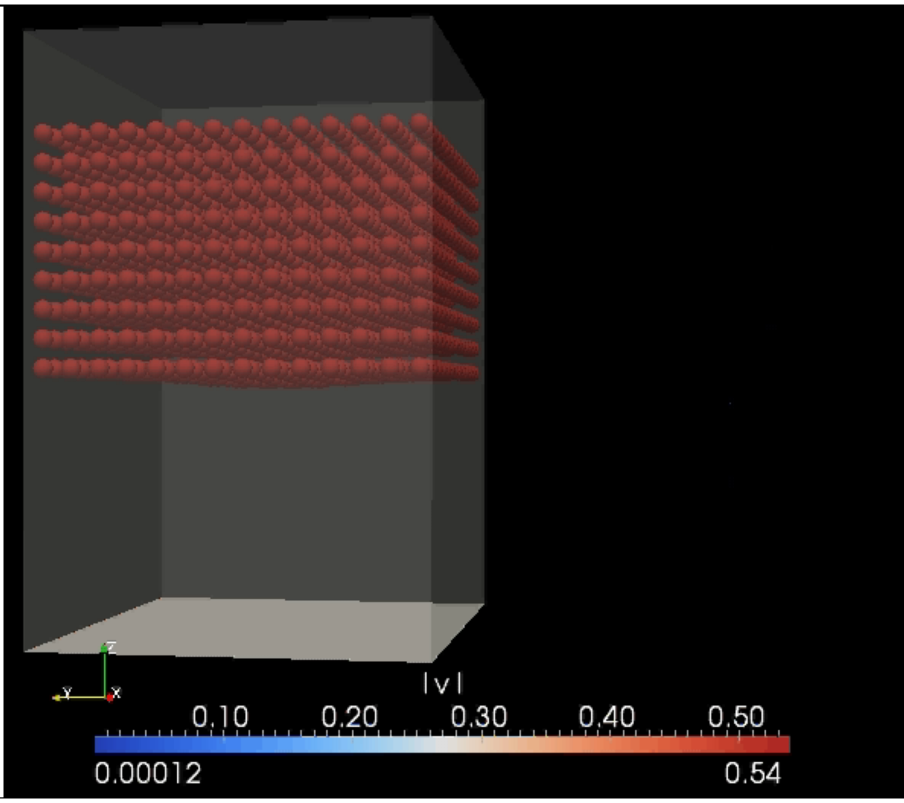




Rayleigh Taylor Instability (RTI)

- Compare the motion of lowest DEM particle to the predicted growth of the interface in a two-fluid model of RT (Chandrasekhar, 1961)
- Differences from fluid RT – sedimentation velocity & lower ε near boundary
- Good results seen for $Re_p \sim 0.01$ (glycerol-water). Difficulties at higher Re_p.





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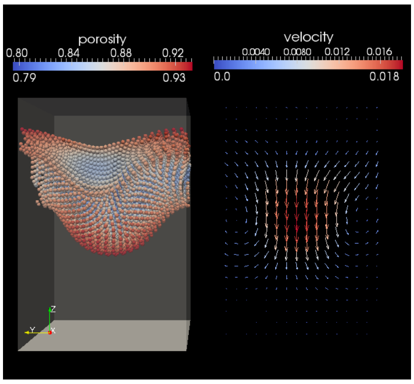
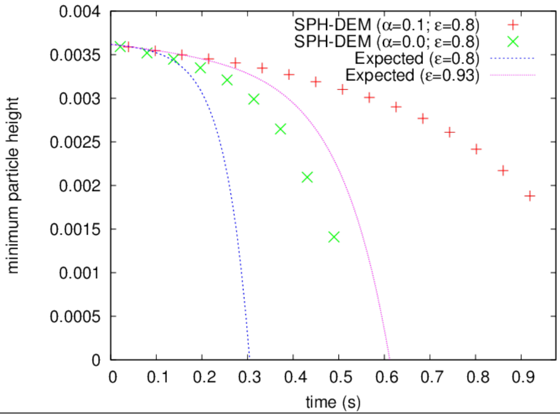
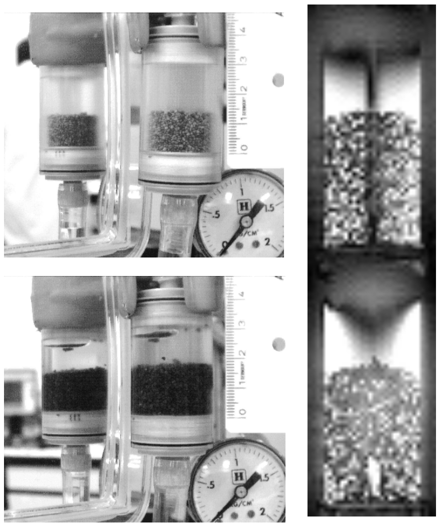




Realistic simulation of powder dispersion by a liquid jet

- The dispersion of grains by a liquid jet of interest to food processing industry (Nestlé)
- Experimental (camera, pressure sensor & MRI) data available* for poppy seeds and glass beads dispersed in water
- Goals:
  1. Validate SPH-DEM results to experiments
  2. Investigate dynamics of dispersed bed
  3. Map process parameters to different dynamical regimes



*Courtesy **Dr Marco Ramaioli** & **Chloe Trarieux**

Parameters

Name	Symbol	Value
Cell radius	R	11.5 mm
Cell height	H	31 mm
Inlet radius	R_i	0.5 mm
Inlet radius (simulated)	$\mathbf{R_{is}}$	**1.9 mm**
Inlet flow rate	F_i	100 ml/min
Inlet velocity	$\mathbf{V_i}$	$\mathbf{(1e^{-06} \ast F_i) / (60 \pi R_i^2) = 2.12}$ **m/s**
Fluid density	ρ	997 kg/m^3
Fluid viscosity	μ	8.93 Pa·s
Particle diameter	d	1.10 mm (change to 2.2mm)
Particle density	ρ_p	1160 kg/m^3

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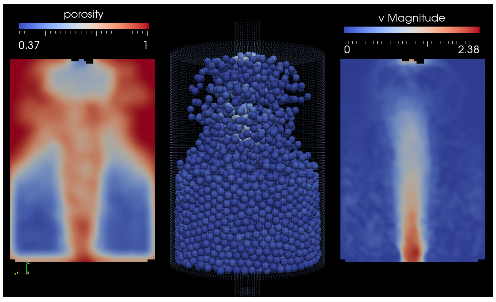
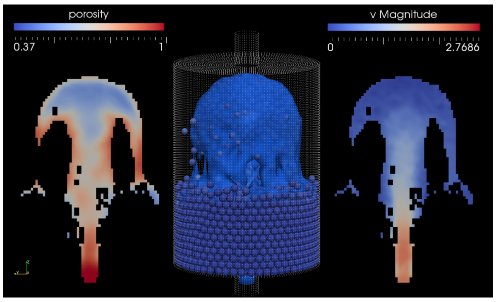
Two initial conditions…

- Wet
  1. Cylinder filled with water.
  2. Particles allowed to settle (mass*3)
  3. Particle mass reset to input parameter
  4. Jet starts



- Dry
  1. Particles allowed to settle
  2. Jet starts



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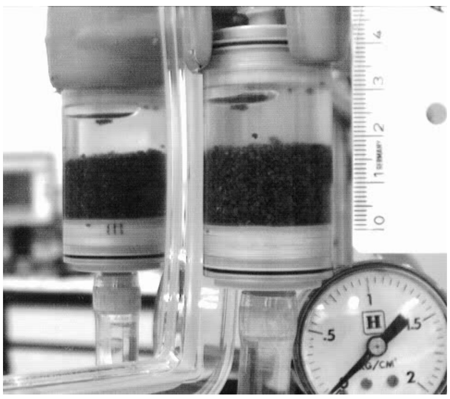
Wet Start



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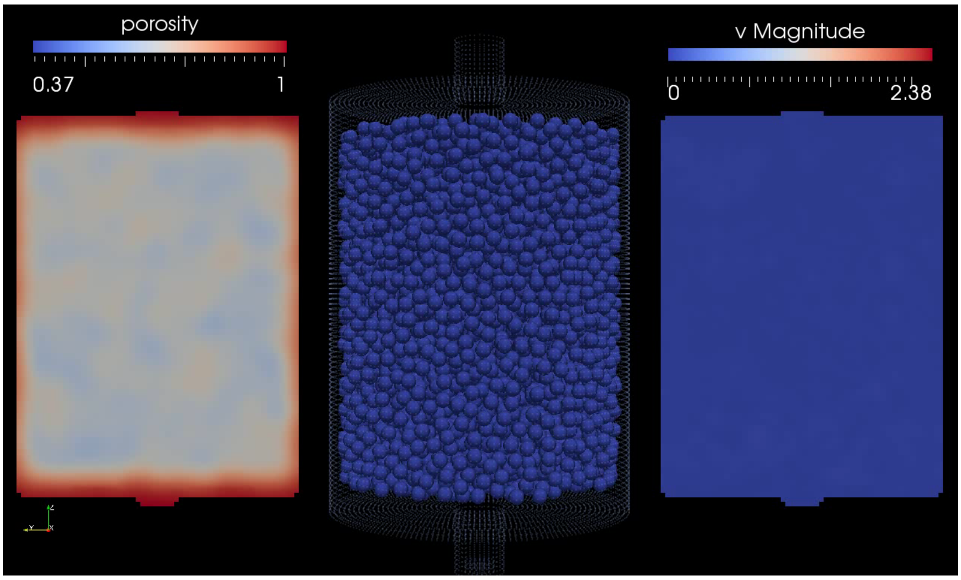
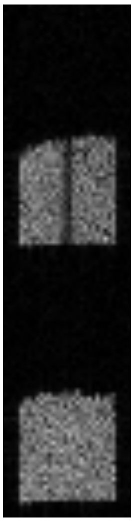
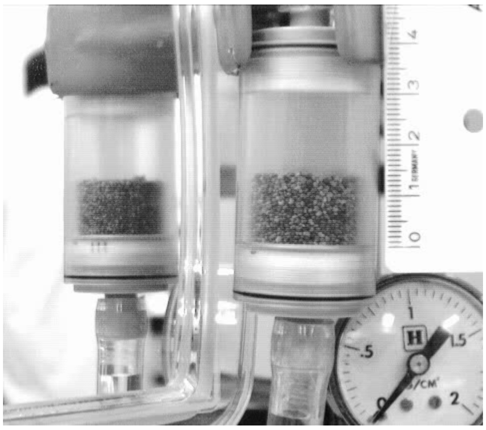
Wet Start



Dry Start





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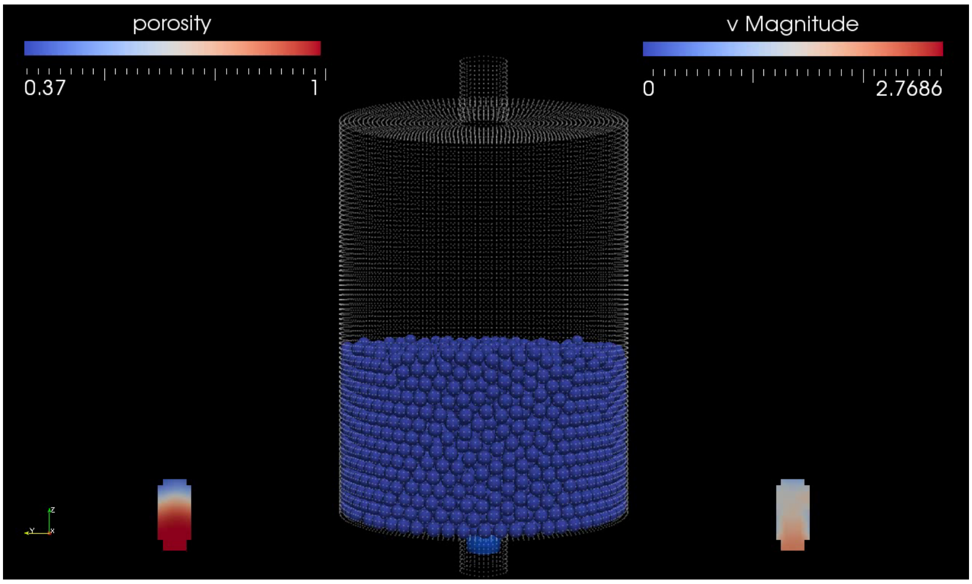
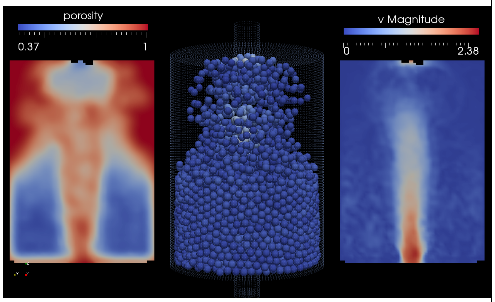
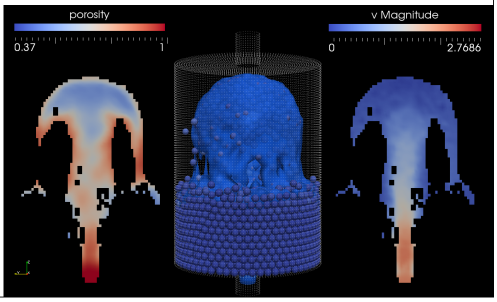
Dry Start



Realistic simulation of powder dispersion by a liquid jet

- Initial results:
  - **Wet** – Recovers some qualitative features from experiment. Correcting inlet/outlet condition and exploring parameter space
  - **Dry** – Fails to recover some major features (eg. Bed lift). Surface tension effects important but not modelled.





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Thank you for your attention.
Questions….?

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